STM JOURNALS

International Journal of
Mechanical Dynamics and Systems Analysis

Volume 3, Issue 1, 2025
January–June
DOI (Journal): 10.37591/IJMDSA

https://journals.stmjournals.com/ijmdsa

Case Study IJMDSA

Parameter Identification of Vibrating MIMO Mass-Spring-Damper Mechanical Systems with Coulomb Friction

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Abstract

This study presents a time-domain method for identifying parameters in multi-degree-of-freedom (DOF) linear mass-spring-damper systems that include Coulomb friction, where the friction coefficient varies within a limited range, using measurements of position and control force. For mass identification in single-degree-of-freedom (DOF) vibrating mechanical systems with stiffness and no damping but Coulomb friction, the friction term is eliminated by first derivative processing. In addition, the identification of the coefficient of friction is not considered, and friction is treated as measurement noise by reference trajectory search control. A method of implementing online algebraic parameter identification by measuring position and control force in the time domain for MIMO (Multiple-Input, Multiple-Output) mass-spring-damper systems with limited Coulomb friction is proposed. The results of the mathematical model and study of the differential flatness of an n-DOF mass-spring-damper system with friction were obtained. An output feedback control scheme for trajectory tracking tasks considering Coulomb friction was obtained. The friction coefficient and parameters of mass, stiffness, and damping coefficient have been algebraically estimated from real-time position and input control force measurements to extend the application range of the online algebraic parameter identification method. By determining the velocity sign in two short time intervals when Coulomb friction exists and by processing the friction term in the manner of linearizing the sign function, in the case of an integral near a singular point, the reference trajectory tracking problem is realized. Based on this, a six-DOF mechanical system, for example, mass, stiffness, and damping factor without friction, was identified in real-time, and mass, stiffness, damping factor, and Coulomb friction coefficient in the presence of friction were identified. When friction exists and does not, parameter identification of a six-DOF vibrating mechanical system and reference trajectory tracking have been realized correctly. The method is of great help in realizing the static motion control of multi-DOF MIMO mechanical systems, including robot manipulators, medical devices, and precision tracking mechanisms. As a result, control costs are reduced.

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Received Date: March 26, 2025
Accepted Date: June 12, 2025
Published Date: June 23, 2025

Citation: KangSok Wi., KonKi Ri., KwangIl Ri., YongIl Ri. Parameter Identification of Vibrating MIMO Mass-Spring-Damper Mechanical Systems with Coulomb Friction. International Journal of Mechanical Dynamics and Systems Analysis. 2025; 3(1): 20–34p.

Keywords: Coulomb friction, vibrating mechanical systems, parameter identification, control force measurements, vibration control

INTRODUCTION

This method has been proposed to identify various methods, including modal analysis, experiments, dynamic system identification, time-frequency conversion, mathematical modeling and numerical simulation, identification by limited sensors, identification by band filter, identification by input-output characteristic analysis, identification of dynamic and linear systems,

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identification of systems, and signal processing [1–15]. In relation to algebraic parameter identification, model-free control, the GPI balancing control of an uncertain Jeffcott rotor model, nonlinear feedback control, operational calculus, adaptive-like vibration control in mechanical systems with unknown parameters and signals, active unbalance control of rotor systems using online algebraic identification methods, evaluation of online algebraic modal parameter identification methods, and dynamic vibration absorbers have been proposed [16–25].

In addition, semi-active control of forced oscillations in power transmission lines via optimum tunable vibration absorbers, vibration control of active structures, vibration control of active structures, machinery vibration and rotor dynamics, experimental and theoretical investigations on the dynamic properties of tuned particle dampers, and new parameter identification methods for calibrating the stiffness of AFM probes have been suggested [26–30]. In these studies, a new online algebraic identification method in the time domain for MIMO linear mass-spring-damper mechanical systems using position measurements was proposed.

In the theory and examples of nonlinear systems, vibration control optimization, generalized proportional integral output feedback controller for robust disturbance rejection in mechanical systems, and vibration control of mechanical systems have been discussed [31–34]. Beltrán-Carbajal and Silva-Navarro [35] identified a method that determines the mass, damping coefficient, and stiffness of n-DOF (n-degree-of-freedom) vibrating mechanical systems during the initial search tracking of the reference trajectory and realizes reference trajectory tracking. For the parameter identification of vibrating mechanical systems, proper mechanical parameters are given before tracking, and mechanical parameter identification is realized by measuring the input control force and the displacement value of the output n. Based on this, the reference trajectory tracking was completed. The accuracy of the parameter identification and reference trajectory tracking is verified in an example of a six-DOF vibrating mechanical system.

In addition, parameter identification and reference trajectory tracking are considered when white noise [36] is comprehensively considered in the identification of unknown parameters in linear, nonlinear, and discrete systems. For the parameter identification of a vibrating mechanical system with friction, the mass was identified by measuring the control force and displacement of a single-DOF vibrating system with no damping and stiffness. For mass identification of single-DOF vibrating mechanical systems with no damping and stiffness but Coulomb friction, the friction term is eliminated by first derivative processing. In addition, the identification of the coefficient of friction was not considered, and friction was treated as measurement noise by reference trajectory search control. This study describes the parameter identification of a multi-DOF mass-spring-damper system with Coulomb friction, which has a limited coefficient changing range, and the tracking of one or more reference trajectories that are of interest. Based on a previous study [35, 36], a method of implementing online algebraic parameter identification by measuring the position and control force in the time domain for MIMO mass-spring-damper systems with limited Coulomb friction was proposed.

Here, the Coulomb friction coefficient and parameters of mass, stiffness, and viscous damping coefficient are algebraically identified simultaneously through the behavior of the system in the time domain. The proposed identification method can be used in active control and search control tasks of MIMO mechanical systems. For the parameter identification of vibrating mechanical systems, proper mechanical parameters are given before tracking, and the mechanical parameters of mass, stiffness, viscous damping coefficient, and Coulomb friction coefficient are identified using measurements of one or more input control forces and n output displacement values. Based on this, the reference trajectory tracking was completed. The accuracy of parameter identification and reference trajectory tracking when Coulomb friction exists is verified as an example of a six-DOF vibrating mechanical system. Furthermore, it is suggested that the control input can be chosen so that one or more reference position controls of interest can be realized, and for only parameter identification, one reference trajectory search control input can be chosen.

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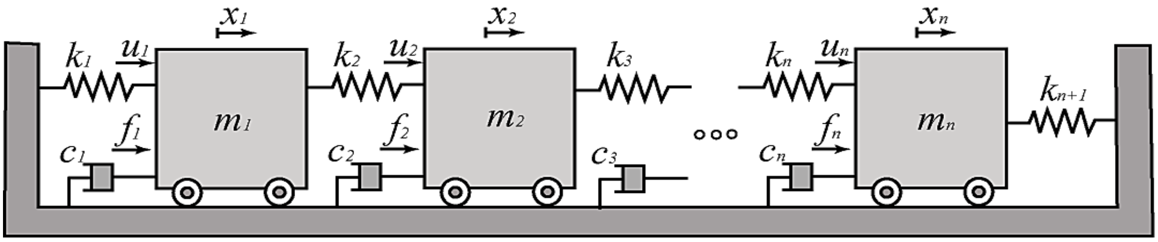

Figure 1. Structural scheme of an n-DOF vibrating mechanical system with friction.

The identification and control problem when noise exists in measurements and control was suggested [35, 36]; however, it is not considered in this study.

MATHEMATICAL MODEL AND STUDY OF THE DIFFERENTIAL FLATNESS OF N-DOF MASS-SPRING-DAMPER SYSTEM WITH FRICTION

Mathematical Model

Figure 1 shows an n-DOF vibrating mechanical system with friction.

The generalized coordinates are n positions of mass x_i, $i = 1,2, \dots n.$, and m_i, k_i, c_i denote the mass, stiffness, and viscous damping coefficient of the system, respectively, and u_i denotes the control force on mass. And f_i is the Coulomb frictional force, which is expressed as

$$f_i = -\mu_i m_i g \operatorname{sign} \dot{x}_i$$

Where, μ_i is the Coulomb frictional coefficient, and sign $\dot{x}_i$ is the sign function.

The mathematical model of this vibrating mechanical system can be described using the following coupled ordinary differential equations:

$$\begin{gathered}
m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = u_1 - \mu_1 m_1 g \operatorname{sign}\dot{x}_1 \\
m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2(x_2 - x_3) + k_3(x_2 - x_3) = u_2 - \mu_2 m_2 g \operatorname{sign}\dot{x}_2 \\
\vdots \\
m_{n-1}\ddot{x}_{n-1} + c_{n-1}\dot{x} + k_{n-1}(x_{n-1} - x_{n-2}) + k(x_{n-1} - x_n) = u_{n-1} - \mu_{n-1} m_{n-1} g \operatorname{sign}\dot{x}_{n-1} \\
m_n\dot{x}_n + c_n\dot{x} + k_n(x_n - x_{n-1}) + kx_n = u_n - \mu_n m_n g \operatorname{sign}\dot{x}_n
\end{gathered} \tag{1}$$

This study investigates not only the case in which all masses are acted upon by the control force, but also the case in which some masses among n masses are acted upon by the control force.

In the case where the mass m_i, is acted upon by the control force, Equation (1) is described in matrix form as follows:

$$M\ddot{X} + C\dot{X} + \mathrm{KX} = \mathrm{BU} + F; \quad X, U, F \in R \tag{2}$$

Where, M, C, K are the mass, damping, and stiffness matrices of the system, respectively, X, U, F are the displacement, control force, and frictional force vectors, respectively. B is $n \times n$ control force coefficient matrix. For example, if two masses of i = 3 and 5 are acted upon by the control force, $B_{33} = B_{55} = 1$ and the other elements $B_{\mathrm{ij}} = 0$.

The vibrating system (1) can be completely controlled, observed, and stable if K is defined as positive definite and C=0, U=0. If C is positive, it is asymptotically stable.

Study of Differential Flatness

The MIMO vibrating system exhibited differential flatness properties. The flat output is given by the position of mass objects $y_i = x_i$, $i = 1,2, \dots, n$, and all state variables $x_{i,}$ \{$\dot{x}_i$ and the control input $u_i$can be parameterized into a flat output y_i and a finite number of derivatives:

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$$y_i = x_i$$
$$\dot{y}_i = \dot{x}_i$$
$$u_i = m_i\ddot{y} + c_i\dot{y}_i + k_i(y_i - y_{i-1}) + k_{i+1}(y_i - y_{i+1}) + \mu_i m_i g \text{sign}\dot{x}_i,\ i = 1,2,\dots,n \tag{3}$$
$$y_0 = y_{n+1} = 0$$

Where, y_i describes the position output signal of the vibrating system used for the parameter identification and control.

Now, express the differential flatness controller for searching the reference position trajectory y_i as

$$u_i = m_i v_i + c_i\dot{y} + k_i(y_i - y_{i-1}) + k_{i+1}(y_i - y_{i+1}) + \mu_i m_i g \text{sign}\dot{x}_i,\ i = 1,2,\dots,n \tag{4}$$

where v_i is an auxiliary control input combined to get the required asymptotic output by searching reference position y_i:

$$v_i = \ddot{y}_i - \beta_{1i}(\dot{y}_i - \dot{y}_i) - \beta_{0i}(y_i - y^i),\ i = 1,2\dots,n \tag{5}$$

Where, β_{1i}, β_{0i} are the parameters chosen to ensure asymptotic output searching with closed loop dynamic properties. Therefore, the closed loop search dynamic error is:

$$e_i = y_i - y_i(t) \tag{6}$$

and satisfies the following equation.

$$\ddot{e}_i + \beta\dot{e}_i + \beta_{0i}e = 0$$

The differential flatness control in Equation (4) is used for the search control solution of the closed-loop reference position trajectory for an n-DOF MIMO vibrating mechanical system. However, this control scheme increases the completion cost and attenuates self-ability owing to the differential in the position signal relative to time, and requires velocity and position measurements.

In Equation (4), a discrete problem is suggested in the neighborhood of the last term $\dot{y} = 0$.

AN OUTPUT FEEDBACK CONTROL SCHEME FOR TRAJECTORY TRACKING TASKS IN CONSIDERATION OF COULOMB FRICTION

From Equation (1), velocity signals in a vibrating system can be obtained as

$$\dot{y}_i = \frac{-c_i}{m_i}y_i - \frac{k_i}{m_i}\int_0^t (y_i| \quad |-y_{i-1})\text{dt} - \frac{k_{i+1}}{m_i}$$
$$\int_0^t (y_i| \quad |-y_{i+1})\text{dt} + \frac{1}{m_i}\int_0^t u_i\,\text{dt} + \mu_i g\int_0^t \quad \text{sign}\dot{y}_i\text{dt} + p_i,\ i = 1,2,\dots,n \tag{7}$$

Not considering a constant depending on initial conditions, Equation (7) can be expressed as

$$\hat{\dot{y}}_i = \frac{-c_i}{m_i}y_i - \frac{k_i}{m_i}\int_0^t (y_i| \quad |-y_{i-1})\text{dt} - \frac{k_{i+1}}{m_i}$$
$$\int_0^t (y_i| \quad |-y_{i+1})\text{dt} + \frac{1}{m_i}\int_0^t u_i\,\text{dt} + \mu_i g\int_0^t \quad \text{sign}\dot{y}_i\text{dt},\ i = 1,2,\dots,n \tag{8}$$

In the case of friction at a singular point ($\dot{y}_i = 0$), the processing problem of $\mu_i g\int_0^t \quad \text{sign}\dot{y}_i\text{dt}$ is suggested.

By measuring the values of $y_{i,\tau}, y_{i,t-\tau}, \dot{y}_i \approx \frac{y_{i,\tau}\text{-}\,y_{i,t-\tau}}{\tau}$ is calculated for sampling time τ and when $-\varepsilon < \dot{y}_i < \varepsilon$ change of sign $\dot{y}_i$ is linearly approximated, that is, approximated as $\text{sign}\dot{y}_i = \frac{1}{\varepsilon}\dot{y}_i$ $(|\dot{y}_i| < \varepsilon)$

Now supposing

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$$\int_0^t \mathrm{sign}\dot{y}_i \mathrm{dt} = \int_0^{t_{1i}} \mathrm{sign}\dot{y}_i \mathrm{dt} + \int_{t_{1i}}^{t_{2i}} \mathrm{sign}\dot{y}_i \mathrm{dt} + \int_{t_{2i}}^{t_{3i}} \mathrm{sign}\dot{y}_i \mathrm{dt} + \int_{t_{3i}}^{t} \mathrm{sign}\dot{y}_i \mathrm{dt}$$

when $\varepsilon \to 0$ is given by $\Delta t_{1i} \to 0, \Delta t_{2i} \to 0$. $\Delta t_{1i} = t_{2i} - t_{1i}, \Delta t_{2i} = t_{3i} - t_{2i}$ are time intervals corresponding to domainε, where t_{1i} is time for $\dot{y}_i = \varepsilon$, t_{2i} is time for $\dot{y}_i = 0$, and t_{3i} is time for $\dot{y}_i = -\varepsilon$.

In this time domain ε,$\int_{t_{1i}}^{t_{2i}} \mathrm{sign}\dot{y}_i \mathrm{dt} \approx \int_{t_{2i}}^{t_{3i}} \mathrm{sign}\dot{y}_i \mathrm{dt}\, 0$. Taking the time interval $0 - t$satisfying $\dot{y}_i > 0$ in $0 - t_{1i}$ and $\dot{y}_i < 0$ in $t_{3i} - t$ and considering $\int_0^{t_{1i}} \mathrm{sign}\dot{y}_i \mathrm{dt} = t_{1i,}$ $\int_0^{t_{3i}} \mathrm{sign}\dot{y}_i \mathrm{dt} = t_{3i}$, it can be written as $\int_0^t \mathrm{sign}\dot{y}_i \mathrm{dt} = t_{1i} - (t - t_{3i})$.

Therefore, in the control process, the time interval of $|\dot{y}_i| < \varepsilon$ can be determined, and Equation (8) can then be integrated. Here, according to the choice of ε control error and tracking error, it is determined to be near $y_i = 0$, and if $\varepsilon \to 0$, the error also converges to zero. This assumes that y_i is bounded and continuous.

The expressions of the output feedback control scheme and auxiliary control input for tracking the reference position trajectory y_i are as follows:

$$u_i = m_i v_i + c_i \hat{\dot{y}}_i + k_i(y_i - y_{i-1}) + k_{i+1}(y_i - y_{i+1}) + \mu_i m_i g \mathrm{sign}\hat{\dot{x}}_i, \; i = 1,2, \dots, n \tag{9}$$

$$v_i = \ddot{y}_i - \beta_{2i}(\hat{\dot{y}}_i - \dot{y}_i) - \beta_{1i}(y_i - y_i) - \beta_{0i} \int_0^t (y_i| \quad | - y_i) \mathrm{dt}, \quad i = 1,2 \dots, n \tag{10}$$

In Equation (10), the integral term is used to compensate for the tracking error of the velocity signal, and the integral-type differential equation of error tracking can be obtained from Equation (9) of the control as follows:

$$\ddot{e}_i + \beta_{2i}\dot{e}_i + \beta_{1i}e_i + \beta_{0i} \int_0^t e_i \,\mathrm{dt} = a_i, i = 1,2, \dots .n \tag{11}$$

where a_i is an unknown constant for error tracking. Differentiating Equation (11)

$$e_i^{(3)} + \beta_{2,i}\ddot{e}_i + \beta_{1,i}\dot{e}_i + \beta_{0,i}e_i = 0, i = 1,2,\dots,n \tag{12}$$

Design parameter $\beta_{\mathrm{ji}}(j = 0,1,2)$ can be chosen to make the error zero〔35〕. In addition, parameters can be chosen to improve the quality of control [37].

ONLINE ALGEBRAIC PARAMETER IDENTIFICATION OF A VIBRATING MECHANICAL SYSTEM WITH COULOMB FRICTION

Fundamental Relational Expression

Equation (1) is described in the notation of Mikusiński operational calculus by

$$m_i s^2 Y_i(s) + c_i s \mathrm{Y}_i(s) + k_i[Y_i(s) - Y_{i-1}(s)] + k_{i+1}[Y_i(s) - Y_{i+1}(s)] = \grave{U}_i(s) \\ + R_i(s), i = 1,2, \cdots, n \tag{13}$$

where $Y_0(s) = Y_{n+1}(s) \equiv 0, R_i(s) = m_i y_{0,i} s + m_i \dot{y}_{0,i} + c_i y_{0,i}$, $y_{0,i} = y_i(t_0)$ and $\dot{y}_{0,i} = \dot{y}_i(t_0)$ are unknown constants denoting the system's initial conditions at time $t_0 \geq 0$.

On the other hand, in the intervals $\dot{y}_i > 0$ and $\dot{y}_i < 0$, $\bar{U}_i(s)$ is as follows:

$\bar{U}_i(s) = U_i(s)$ -$\mu_i m_i g$, \{$\dot{y}_i > 0$

$\bar{U}_i(s) = U_i(s) + \mu_i m_i g$, \{$\dot{y}_i < 0$

The control force u in the domain $\dot{y}_i \neq 0$ can be regarded as $\bar{U}_i(s)$, and the transformed control force $\grave{U}_i$ always acts on the frictional force.

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To eliminate the influence of the initial conditions, Equation (13) is differentiated twice with respect to s and is multiplied by s^2. Then, transforming it to the time domain, the following expression can be obtained.

$$a_{11,i}(t)m_i + a_{12,i}(t)c_i + a_{13,i}(t)k_i + a_{14,i}(t)k_{i+1} = b_i(t),\ i = 1,2,\dots,n\dots \quad (14)$$

Coefficients of Equation (14) are given by

$$a_{11,i}(t) = 2\int_{t_0}^{t}\int_{t_0}^{\tau_2} y_i\,(\tau)d\tau_1 d\tau_2 - 4\int_{t_0}^{t}(\tau_1| \quad |-t_0)y_i(\tau_1)d\tau_1 +$$

$$a_{12,i}(t) = -2\int_{t_0}^{t}\int_{t_0}^{\tau_2}(\tau_1 - t_0)y_i\,(\tau)d\tau_1 d\tau_2 - \int_{t_0}^{t}$$

$$a_{13,i}(t) = \int_{t_0}^{t}\int_{t_0}^{\tau_2}$$

$$a_{14,i}(t) = \int_{t_0}^{t}\int_{t_0}^{\tau_2}$$

$$b_{1,i}(t) = \int_{t_0}^{t}\int_{t_0}^{\tau_2}$$

where

$$\bar{u}_i = u_i - \mu_i m_i g, \quad \backslash\{\dot{y}_i > 0$$

$$\bar{u}_i = u_i + \mu_i m_i g, \quad \backslash\{\dot{y}_i < 0.$$

Expressing Equation (14) in matrix form

$$A_i\theta_i = B_i,\ \mathrm{i} = 1.2.\dots,n \quad (15)$$

where $\theta = [m_i, c_i, k_i, k_{i+1}]^T$.

$$A_i = \left[a_{11,i}a_{12,i}a_{13,i}a_{14,i}\right]\left[a_{21,i}a_{22,i}a_{23,i}a_{24,i}\right]\left[a_{31,i}a_{32,i}a_{33,i}a_{34,i}\right]$$

Components of A_i, B are specified as 〔35〕.

$$a_{\mathrm{kh},i} = \int_{t_0}^{t} a_{k-1h,i}\,(\tau_1)d\tau_1,\ b_{k,i} = \int_{t_0}^{t} b_{k-1,i}\,(\tau_1)d\tau_1 \quad (16)$$

$$i = 1,2,\dots,n,\ \mathrm{k} = 2,3,4,\ \mathrm{h} = 1,2,3,4$$

Solving Equation (16), the parameter vector can be expressed by

$$\theta_i = A_i^{-1}B_i = \frac{1}{\Delta_i}\ 〔\Delta_{1,i}\Delta_{2,i}\Delta_{3,i}\Delta_{4,i}〕^T,\ i = 1,2,\dots,n \quad (17)$$

The solution of Equation (17) can be determined as follows.

To avoid singularities when $\Delta_i \equiv 0$, considering that m_i, c_i, k_i and k_{i+1} have to be positive constants, the following expressions are given by

$$/\Delta_i| \quad \backslash\{\hat{m}_i = /\Delta_{1,i}/,\ /\Delta_i/\hat{c}_i = /\Delta_{2,i}/,\ /\Delta_i/\hat{k}_i = /\Delta_{3,i}/,\ /\Delta_i/\hat{k}_{i+1} = /\Delta_{4,i}/,\ i = 1,2,\dots,n \quad (18)$$

Where, notation (^) is used to denote estimated parameters.

Integrating Equation (18) twice with respect to time, the following expression can be obtained:

$$\hat{m}_i = \frac{\int_{t_0}^{(2)} /\Delta_{1,i}/}{\int_{t_0}^{(2)} /\Delta_i/},\quad \hat{c}_i = \frac{\int_{t_0}^{(2)} /\Delta_{2,i}/}{\int_{t_0}^{(2)} /\Delta_i/},\quad \hat{k}_i = \frac{\int_{t_0}^{(2)} /\Delta_{3,i}/}{\int_{t_0}^{(2)} /\Delta_i/},\quad \hat{k}_{i+1} = \frac{\int_{t_0}^{(2)} /\Delta_{4,i}/}{\int_{t_0}^{(2)} /\Delta_i/},\ i = 1,2,\dots,n \quad (19)$$

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where
$\int_{t_0}^{(2)} \phi(t)$=$\int_{t_0}^{t} \int_{t_0}^{t_2} \phi(\tau_1| \quad | \quad) d\tau_1 d\tau_2$.

An Identification Method for the Friction Coefficient

In this study, an identification method for the friction coefficient in two cases was developed considering the sign change of $\dot{y}_i$ in two adjacent time intervals.

First, in the case of a sign change of $\dot{y}_i$ in two time intervals, the identification method of the friction coefficient is as follows: Matrices A_i, B_i are expressed as A_{1i}, A_{2i}, B_{1i}, B_{2i} corresponding to the time interval $[t_0, t_1]$ of $\dot{y}_i > 0$ and time interval $[t_2, t_3]$ of $\dot{y}_i < 0$. In addition, the components of the new column vectors D_{1i}, D_{2i} are calculated by the coefficients.

$D_{1i,1} = \frac{1}{12}$ and $D_{2i,j} = \int_{t_0}^{t} D_{2i,j-1} (\tau_1) d\tau_1, j = 2,3,4.$

Then, in the domains of $\dot{y}_i > 0$ and $\dot{y}_i < 0$. Equation (15) can be expressed as follows:

$$A_{1i}\theta_i = B_{1i} - \mu_i m_i g D_{2i}$$
$$A_{2i}\theta_i = B_{2i} + \mu_i m_i g D_{2i}. \tag{20}$$

Eliminating term $\mu_i m_i g$ from the equations 19 and 20 can be given by

$$\bar{A}_i\theta_i = \bar{B}_i \tag{21}$$

where

$$\bar{A}_i = \bar{D}_{2i}A_{1i} + \bar{D}_{1i}A_{2i},\ \bar{B}_i = \bar{D}_{2i}B_{1i} + \bar{D}_{1i}B_{2i} \tag{22}$$

In Equation (22) $\bar{D}_{1i}, \bar{D}_{2i}$ are 4×4 matrices whose diagonal elements are ones of D_{1i}, D_{2i} and the others are zero.

By solving Equation (21) with the method suggested [35], θ_i is found, and by using Equation (20), μ_i was calculated.

Next, consider the identification method of the friction coefficient when the sign of $\dot{y}_i$ does not change in adjacent time intervals.

As in the first case, in two time intervals $[t_0, t_1]$, $[t_2, t_3]$ $(t_0 < t_1 < t_2 < t_3)$, where the sign of $\dot{y}_i$ does not change, matrices A_{1i}, A_{2i}, B_{1i}, B_{2i}, D_{1i}, D_{2i} are calculated by

$$A_{1i}\theta_i = B_{1i} \pm \mu_i m_i g D_{1i}$$
$$A_{2i}\theta_i = B_{2i} \pm \mu_i m_i g D_{2i} \tag{24}$$

Where
+:$\dot{y}_i < 0$, - :$\dot{y}_i > 0$.

Considering this, Equation (22) can be expressed as

$$\bar{A}_i = \bar{D}_{2i}A_{1i} - \bar{D}_{1i}A_{2i}$$
$$\bar{B}_i = \bar{D}_{2i}B_{1i} - \bar{D}_{1i}B_{2i} \tag{25}$$

The succeeding identification process is the same as in the first case.

Among these two methods, the second is more convenient for realizing quick identification and ensuring the accuracy of the reference trajectory tracking after parameter identification.

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Identification Methods for Some Reference Trajectory Tracking

Next, algebraic parameter identification is realized when only the i[th] mass, not all n masses, is controlled (Figure 2). In mass m_{i-1}, because $u_{i-1} = 0$ and k_i is a known quantity; the parameter we have to identify is $\theta_{i-1} =$. Therefore, in $\bar{A}_{i-1}$, $\bar{B}_{i-1}$ transfers a known quantity k_i to the right side and eliminates the 4[th] row and column, and a new matrix is $\bar{A}'_{i\text{-}1}(3 \times 3)$ and vector $\bar{B}'_{i\text{-}1}(3 \times 1)$ are composed. Then from the resulting equation $\bar{A}'_{i\text{-}1}\bar{\theta}_{i\text{-}1} = \bar{B}'_{i\text{-}1}$, $\bar{\theta}_{i\text{-}1}$ can be found.

In a similar method, it is possible to realize algebraic parameter identification when only some masses are controlled among n masses.

A CASE STUDY: SIMULATION RESULTS

Here, an identification example is given in the case of the reference trajectory tracking of m_2 in a six-DOF vibrating mechanical system with Coulomb friction.

The mechanical parameters of the six-DOF vibrating mechanical system are listed in Table 1. For comparison with the case of no friction, the values are given as described by Beltrán-Carbajal and Silva-Navarro [35]. Measurement values are low pass filtered to use in data processing.

Here, a reference trajectory of m_2 is given as follows:

$x_2\ (t) = \{0.05\sin\frac{\pi}{2t_{02}}t$ for $0 \leqq t \leqq t_{02}$

$\{0.05$ for $t_{02} \leqq t \leqq t_{12}\{0.05\text{-}0.05\psi_2(t, t_{12}, t_{22})$ for $t_{12} \leqq t \leqq t_{22}$

$\psi_2(t, t_{12}, t_{22})$ is a Bezier polynomial, with $\psi_2(t_{12}, t_{12}, t_{22})$=0, $\psi_2(t_{22}, t_{12}, t_{22})$=1, and expressed as

$\psi_2(t, t_{12}, t_{22}) =$

Where,

r_1=252, r_2=1050, r_3=1800, r_4=1575, r_5=700, r_6=126.

For parameter identification, it is integrated in the corresponding two time intervals chosen, considering the natural frequency of each mass, and the initial values of the parameters are given as $m_i = 2.5$ kg, $c_i = 5.0$ Ns$/m$, $\mu_i = 0.20$,$i = 1,2, \cdots,6$;$k_i = 700\,N/m$, $i = 1,2,\cdots,7$; $t_{02} = 0.1s$, $t_{12} = 5s$, $t_{22} = 10s$.

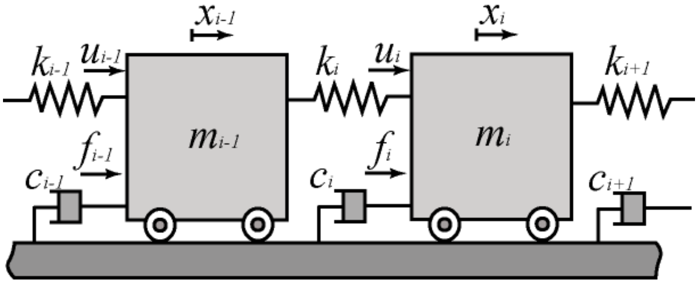

Figure 2. Model of a vibrating system controlling only the i[th] mass.

Table 1. Mechanical parameters of a six-DOF vibrating mechanical system.

$m_1 = 2.0$ kg	$c_1 = 5.5$ Ns$/m$	$\mu_1 = 0.20$	$k_1 = 1000\,N/m$
$m_2 = 2.5$ kg	$c_2 = 5.0$ Ns$/m$	$\mu_2 = 0.18$	$k_2 = 900\,N/m$
$m_3 = 3.0$ kg	$c_3 = 4.5$ Ns$/m$	$\mu_3 = 0.16$	$k_3 = 800\,N/m$
$m_4 = 3.5$ kg	$c_4 = 4.0$ Ns$/m$	$\mu_4 = 0.14$	$k_4 = 700\,N/m$
$m_5 = 4.0kg$	$c_5 = 3.5Ns/m$	$\mu_5 = 0.12$	$k_5 = 600N/m$
$m_6 = 4.5kg$	$c_6 = 3.0Ns/m$	$\mu_6 = 0.10$	$k_6 = 500N/m$
			$k_7 = 400N/m$

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To compare the results of the example from Beltrán-Carbajal and Silva-Navarro (2015) [35], the results of the parameter identification in the case of no friction are depicted in Figure 3(a), (b), and (c). Figure 4 presents six displacement measurements in the case of no friction, and Figure 5 presents the control force.

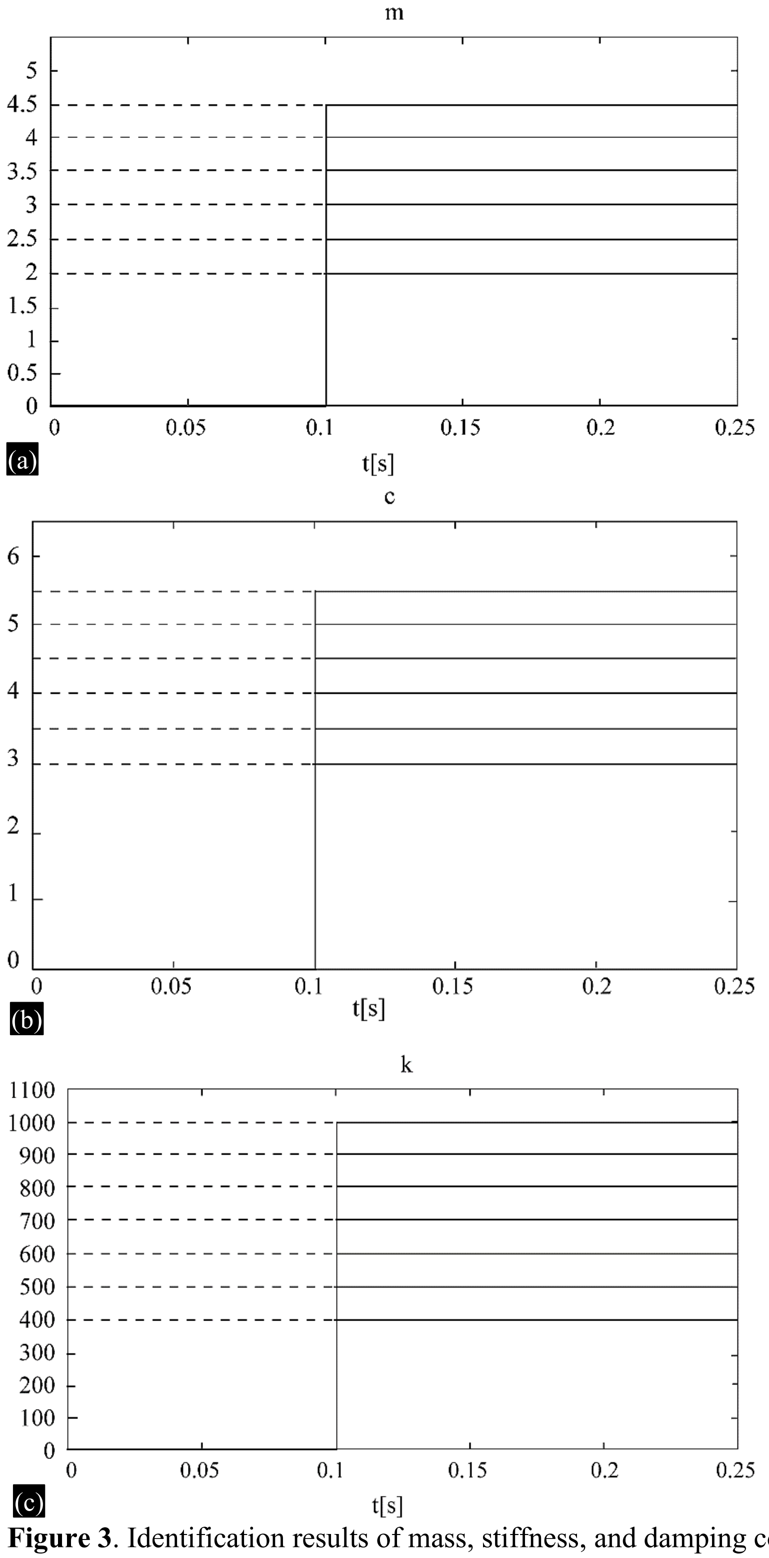

Figure 3. Identification results of mass, stiffness, and damping coefficient when no friction.
(a) Identification results of mass when no friction; (b) Identification results of stiffness coefficient when no friction; and (c) Identification results of damping coefficient when no friction.

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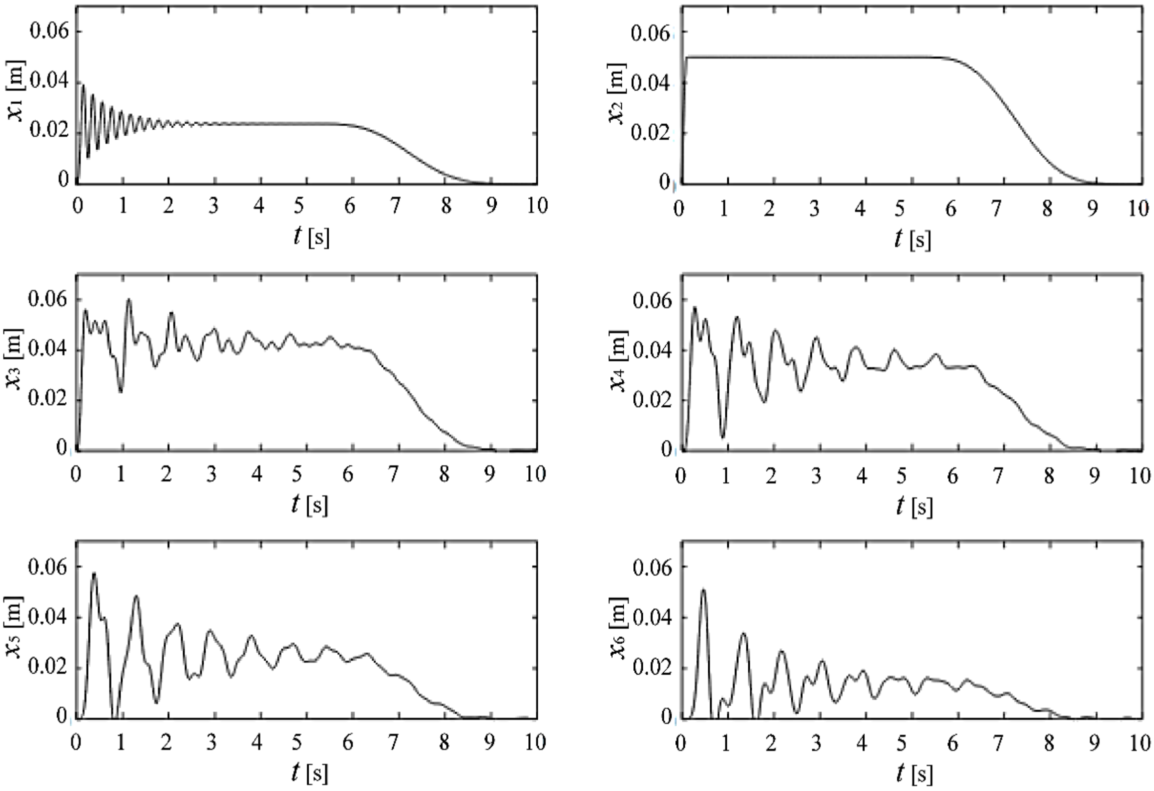

Figure 4. Result of six displacement measurements with no friction.

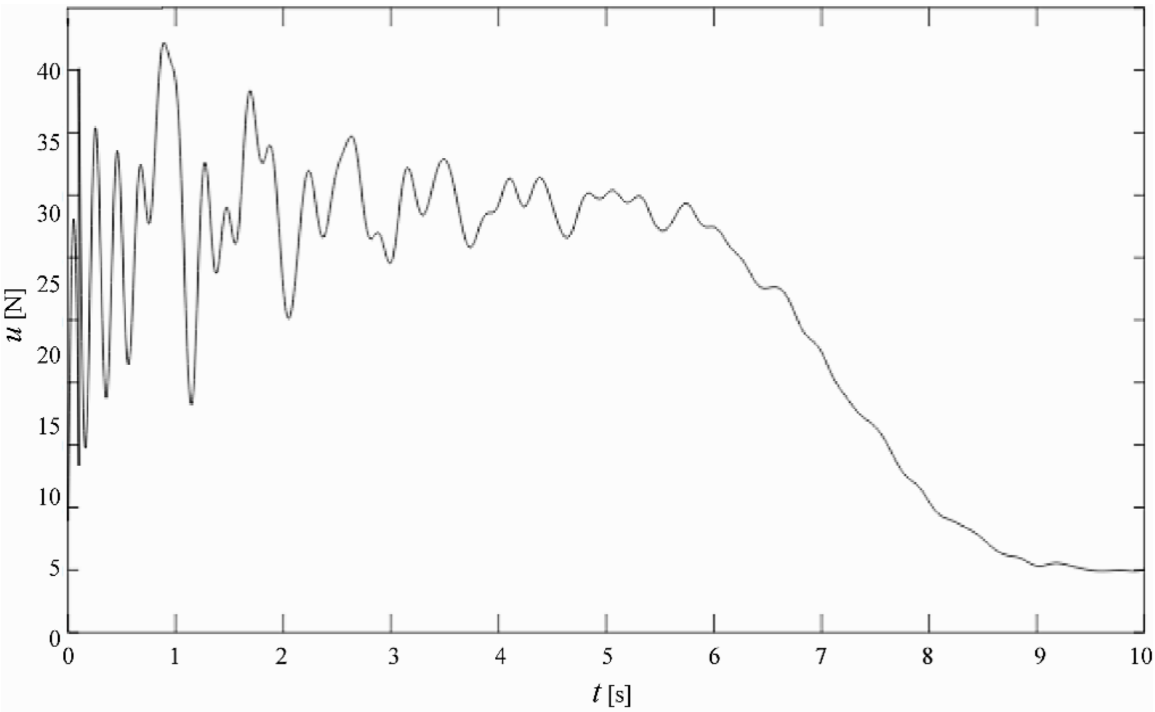

Figure 5. Result of control force measurement with no friction.

Figure 6(a) to (d) shows the individual results of mass, damping coefficient, stiffness, and ratio of friction identification. Figure 7 shows the result of six displacement measurements, and Figure 8 shows the result of control force measurement when friction exists. Here, two adjacent time intervals smaller than 0.05 s from the motion beginning moment are selected.

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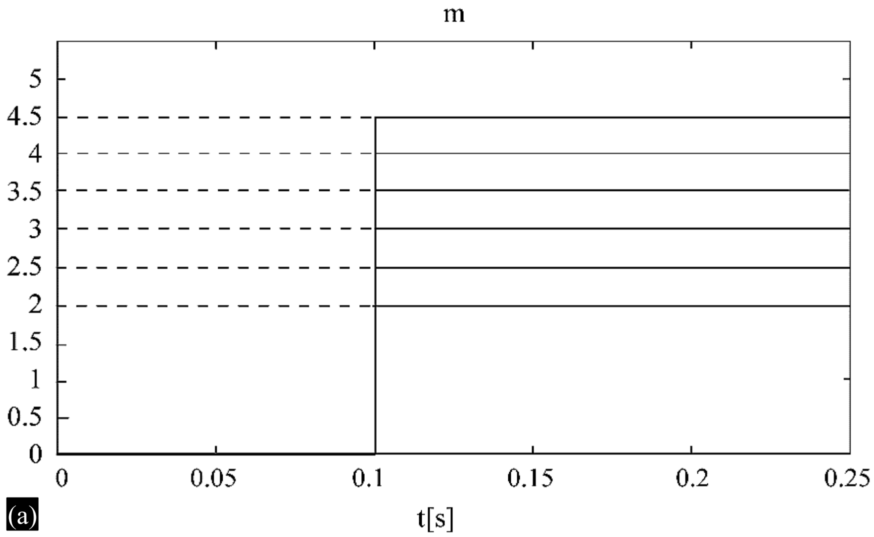
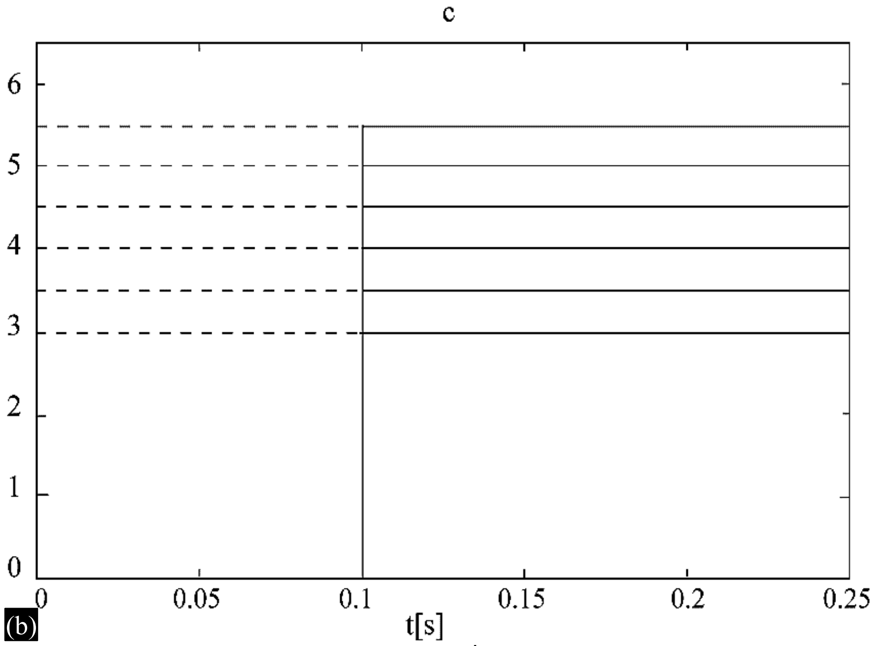
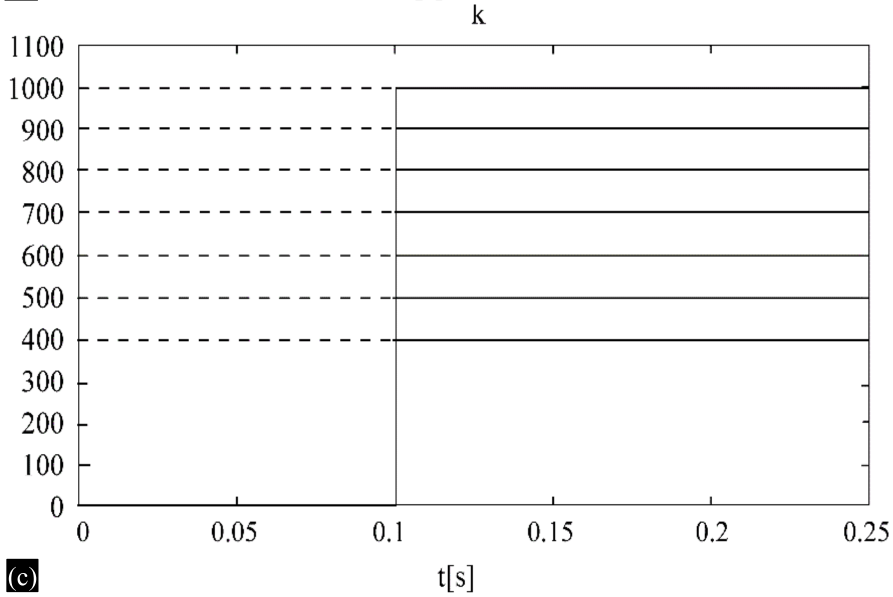


(a)



(b)



(c)

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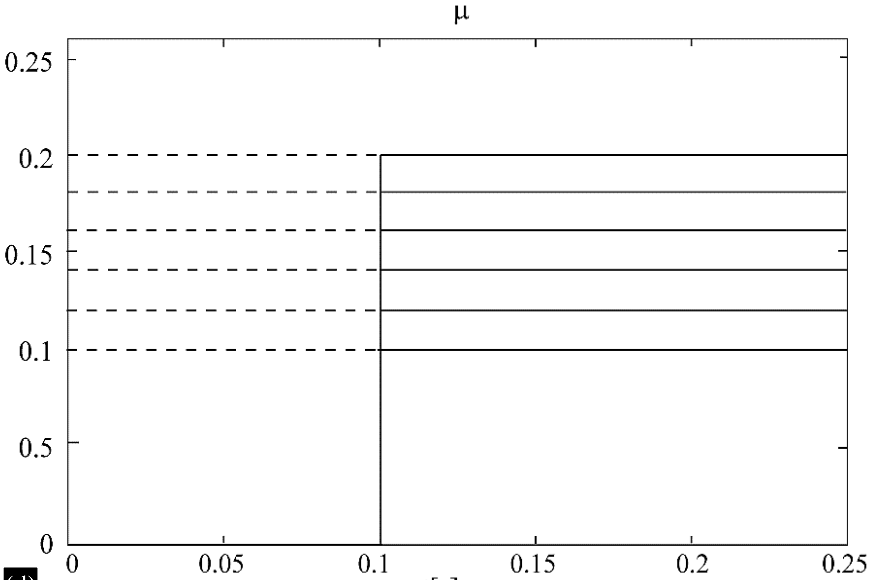

(d)

Figure 6. Identification result of mass, damping coefficient, stiffness and ratio of friction when friction exists; (a) Identification result of mass when friction exist, (b) Identification result of damping coefficient when friction exist, (c) Identification result of stiffness when friction exist, and (d) Identification result of ratio of friction when friction exist.

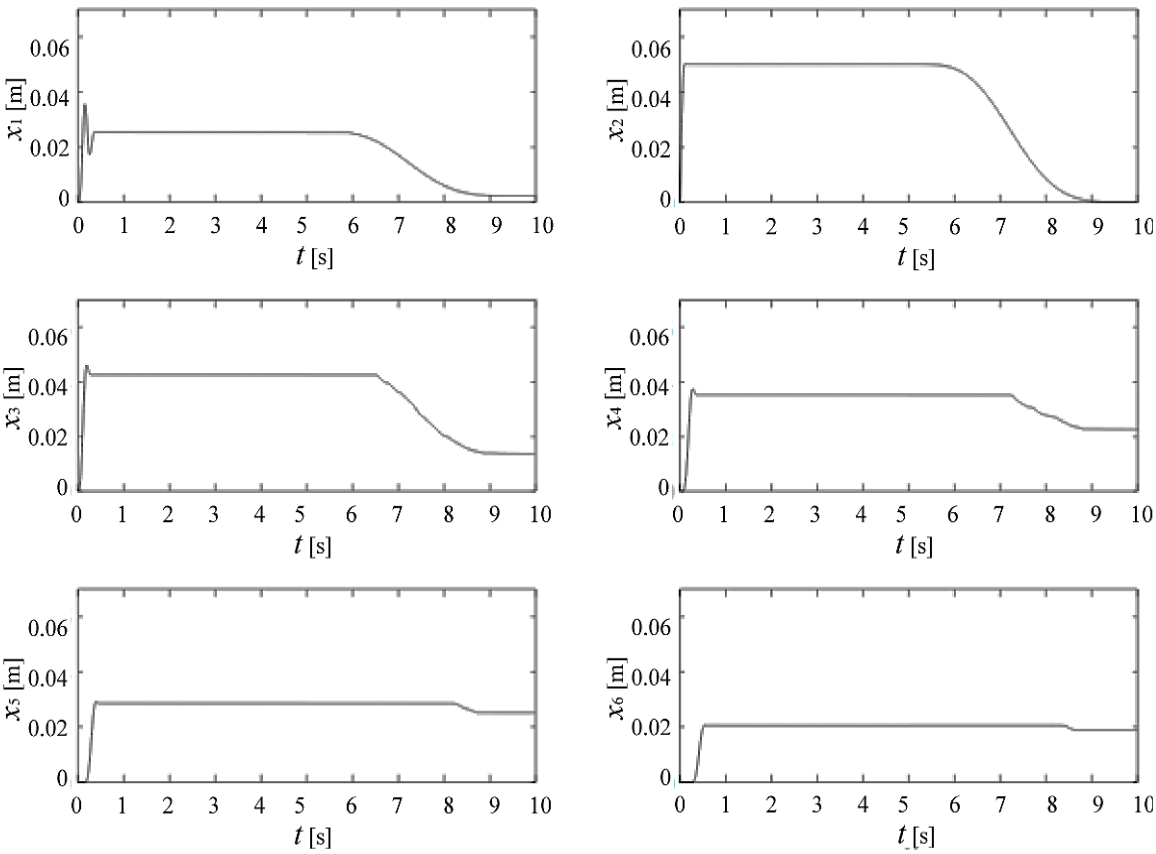

Figure 7. Result of six displacement measurements when friction exists.

When friction exists and does not, the parameter identification of the six-DOF vibrating mechanical system and reference trajectory tracking have been realized correctly.

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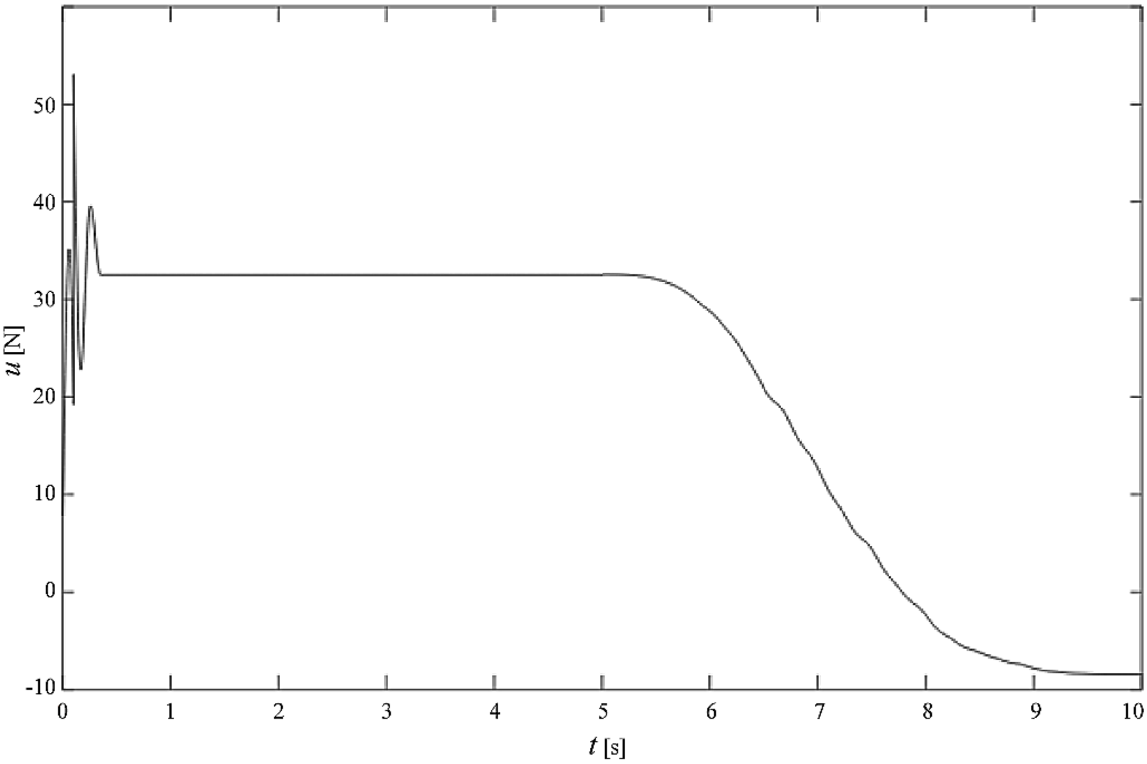

Figure 8. Result of control force measurement when friction exists.

CONCLUSION

Based on the results of a study by Beltrán-Carbajal and Silva-Navarro (2015) [35] for online algebraic parameter identification, an algebraic parameter identification method in the time domain for a multi-DOF vibrating mechanical system with friction, which has a limited coefficient changing range, is suggested.

For parameter identification of the vibrating mechanical system, proper mechanical parameter values are given before tracking, and the Coulomb friction coefficient and mechanical parameters of mass, stiffness, and damping coefficient are realized shortly before reference trajectory tracking by measuring one or more input control forces of interest and n output displacement values.

By determining the velocity sign in two short time intervals when Coulomb friction exists, and by processing the friction term in the manner of linearizing the sign function, in the case of an integral near a singular point, the reference trajectory tracking problem is realized.

The proposed method enables easy identification and reduces cost by realizing reference trajectory tracking and identification of one or more DOF in an n-DOF system. This method can be effectively applied to cases with and without friction; therefore, the efficiency of the online algebraic identification method is increased. The efficiency of algebraic parameter identification is verified as an example of identification and tracking control task for a six-DOF MIMO mass-spring-damper system with friction.

REFERENCES

1. Heylen W, Lammens S, Sas P. Modal Analysis, Theory and Testing. Leuven (BE): Katholieke Universiteit Leuven; 2003.
2. Le TP, Paultre P. Modal identification based on the time–frequency domain decomposition of unknown-input dynamic tests. Int J Mech Sci. 2013;71:41–50. doi:10.1016/j.ijmecsci.2013.03.005.
3. Lardies J, Gouttebroze S. Identification of modal parameters using the wavelet transform. Int J Mech Sci. 2002;44:2263–83. doi:10.1016/S0020-7403(02)00175-3.

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4. Lin RM. Development of a new and effective modal identification method – Mathematical formulations and numerical simulations. J Vib Control. 2011;17:741–58. doi:10.1177/107754631 0376426.
5. Vu VH, Thomas M, Laheurte F, Marcouiller L. Towards an automatic spectral and modal identification from operational modal analysis. J Sound Vib. 2013;332:213–27. doi:10.1016/j.jsv.2 012.08.019.
6. Yang Y, Nagarajaiah S. Output-only modal identification with limited sensors using sparse component analysis. J Sound Vib. 2013;332:4741–65. doi:10.1016/j.jsv.2013.04.004.
7. Liao Y, Wells V. Modal parameter identification using the log decrement method and band-pass filters. J Sound Vib. 2011;330:5014–23. doi:10.1016/j.jsv.2011.05.017.
8. Chakraborty A, Basu B, Mitra M. Identification of modal parameters of a MDOF system by modified L–P wavelet packets. J Sound Vib. 2006;295:827–37. doi:10.1016/j.jsv.2006.01.037.
9. Hung CF, Ko WJ, Peng YT. Identification of modal parameters from measured input and output data using a vector backward auto-regressive with exogeneous model. J Sound Vib. 2004;276:1043–63. doi:10.1016/j.jsv.2003.08.020.
10. Isermann R, Münchhof M. Identification of Dynamic Systems. Berlin (DE): Springer-Verlag; 2011. doi:10.1007/978-3-540-78879-9.
11. Ljung L. Systems Identification: Theory for the User. Upper Saddle River (NJ): Prentice Hall; 1987.
12. Söderström T, Stoica P. System identification. New York (NY): Prentice Hall; 1989.
13. Fliess M, Sira-Ramírez H. An algebraic framework for linear identification. ESAIM Control Optim Calc Var. 2003;9:151–68. doi:10.1051/cocv:2003008.
14. Fliess M, Mboup M, Mounier H, Sira-Ramirez H. Questioning some paradigms of signal processing via concrete examples. In: Sira-Ramirez H, Silva-Navarro G, editors. Algebraic Methods in Flatness, Signal Processing and State Estimation. Mexico: Lagares; 2003. p. 1–21.
15. Fliess M. Analyse non standard du bruit. Acad Sci Paris Ser I. 2006;342:797–802.
16. Fliess M, Join C. Model-free control. Int J Control. 2013;86:2228–52. doi:10.1080/00207179.2013. 810345.
17. Sira-Ramírez H, Silva-Navarro G, Beltran-Carbajal F. On the GPI balancing control of an uncertain Jeffcott rotor model. Proc 4th Int Conf Electr Electron Eng. ICEEE; 2007. p. 306–9.
18. Mboup M, Join C, Fliess M. A revised look at numerical differentiation with an application to nonlinear feedback control. 2007 Mediterranean Conference on Control & Automation, Athens, Greece. 2007. p. 1–6. doi:10.1109/MED.2007.4433728.
19. Mboup M, Join C, Fliess M. Numerical differentiation with annihilators in noisy environment. Numer Algorithms. 2009;50:439–67. doi:10.1007/s11075-008-9236-1.
20. Mikusiński J. Operational Calculus. 2nd ed. Oxford: PWN & Pergamon; 1983. Vol. 1.
21. Glaetske HJ, Prudnikov AP, Skrnik KA. Operational Calculus and Related Topics. London (GB): Chapman & Hall; 2006.
22. Beltrán-Carbajal F, Silva-Navarro G. Adaptive-like vibration control in mechanical systems with unknown parameters and signals. Asian J Control. 2013;15:1526–1613.
23. Beltrán-Carbajal F, Silva-Navarro G, Arias-Montiel M. Active unbalance control of rotor systems using on-line algebraic identification methods. Asian J Control. 2013;15:1627–37. doi:10.1002/asjc. 744.
24. Beltrán-Carbajal F, Silva-Navarro G, Trujillo-Franco LG. Evaluation of on-line algebraic modal parameter identification methods. In: Allemang R, editor. Topics in Modal Analysis II, Volume 8. Conference Proceedings of the Society for Experimental Mechanics Series. Cham: Springer; 2014. p. 145–52. doi:10.1007/978-3-319-04774-4_14.
25. Korenev BG, Reznikov LM. Dynamic Vibration Absorbers: Theory and Technical Applications. Hoboken (NJ): John Wiley & Sons; 1993.
26. Asmari Saadabad NA, Moradi H, Vossoughi G. Semi-active control of forced oscillations in power transmission lines via optimum tuneable vibration absorbers: With review on linear dynamic aspects. Int J Mech Sci. 2014;87:163–78. doi:10.1016/j.ijmecsci.2014.06.006.
27. Preumont A. Vibration Control of Active Structures: An Introduction. Berlin (DE): Springer-Verlag; 2011.

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28. Vance J, Zeidan F, Murphy B. Machinery vibration and rotordynamics. Hoboken (NJ): John Wiley & Sons; 2010. doi:10.1002/9780470903704.
29. Yao B, Chen Q, Xiang HY, Gao X. Experimental and theoretical investigation on dynamic properties of tuned particle damper. Int J Mech Sci. 2014;80:122–30. doi:10.1016/j.ijmecsci.2014.01.009.
30. Sheng CY, Lan YM, Li W, Su CC. New parameter identification method for calibrating stiffness of AFM probes. Int J Mech Sci. 2014;79:25–30. doi:10.1016/j.ijmecsci.2013.11.020.
31. Fliess M, Lévine J, Martin P, Rouchon P. Flatness and defect of non-linear systems: Introductory theory and examples. Int J Control. 1995;61:1327–61. doi:10.1080/00207179508921959.
32. Fliess M, Marquez R, Delaleau E, Sira-Ramirez H. Correcteurs proportionnels-intégraux généralisés. ESAIM Control Optim Calc Var. 2002;7:23–41.
33. Sira-Ramírez H, Beltran-Carbajal F, Blanco-Ortega A. A generalized proportional integral output feedback controller for the robust perturbation rejection in a mechanical system. SciTechnol Autom. 2008;5:24–32.
34. Inman DJ. Vibration with Control. Hoboken (NJ): John Wiley & Sons; 2006. doi:10.1002/0470010533.
35. Beltrán-Carbajal F, Silva-Navarro G. On the algebraic parameter identification of vibrating mechanical systems. Int J Mech Sci. 2015;92:178–86. doi:10.1016/j.ijmecsci.2014.12.006.
36. Sira-Ramírez H, García-Rodríguez C, Cortés-Romero J, Luviano-Juárez A. Algebraic identification and estimation methods in feedback control systems. Hoboken (NJ): Wiley and Sons Ltd; 2014. doi:10.1002/9781118730591.
37. Gerry JP, Shinskey FG. (2004 May). PID controller specification. [Online]. White paper. Available from: http://www.expertune.com/PIDspec.htm.

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