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Research TEE

Higher Order Sliding Controller Parameter Determining Method: A Research

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Abstract

In this study, it is generally believed that sliding switching surfaces should reflect the design parameters of the system in variable structure control, and the control characteristics are the way to constrain the state on sliding switching surfaces. In the literature, classical sliding (primary sliding) is considered that sliding-switching surface is the same as the characteristic equation of the closed-loop system, and the design of the control system is reflected in accordance with the characteristic equation of the sliding-switching surface. The corresponding modes are referred to as SMs, and the technique is centered on high-frequency (or theoretically infinite frequency) control switching. SMs are often established in a finite amount of time, are accurate, and are insensitive to matching disturbances. The primary disadvantage of the technique is the potential for hazardous system vibrations caused by the control switching, sometimes known as the "chattering effect". Additionally, sliding variables of relative degree 1 and relay control form the foundation of standard SMs. But it is very difficult to set up to reflect the characteristics of the control system due to the differentiability because the switching surface is represented by discrete nonlinear (natural nonlinear) differential equations. Controller structures of high order sliding mode were introduced but the method of determining the controller parameters was not presented in the studies carried out in this field. Therefore, we propose the method of determining parameters in the use of high order sliding mode which is superior to classical sliding mode for free vibration and the shortest time problems with the help of simple mathematical knowledge. As a result, this study will concretize the design of controllers more exactly.

Keywords: Nonlinear control system, Sliding mode (SM), robustness, High-order sliding mode (HOSM), Single-input single-output (SISO), Multiple-input multiple-output (MIMO)

INTRODUCTION

One of the main focusses of contemporary control theory is control under highly uncertain circumstances [1].

The solution for this challenge, known as sliding mode control, relies on using high-frequency control switching to maintain an accurately selected constraint [2–4]. While the method is highly accurate and reliable, it has several limitations.

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Only when the relative degree of the constraint is 1, that is, when control must manifest itself directly in the first total time dependency of the constraint function, can the usual sliding mode be applied. Furthermore, the so-called chattering effect may result from high-frequency control flipping [5, 6]. To counteract the chattering effect that approximates the sign function in a boundary layer adjacent to the switching manifold, high-gain management with saturation is employed [7]. For controlling disturbed linear time-invariant systems, the sliding-sector method is appropriate [8].

This work considers the sliding-mode-order strategy, which preserves the sliding-mode properties and improves accuracy by eliminating stuttering and relative-degree constraints [9, 10].

Stated differently, sliding-mode (SM) control is a precise, disturbance-insensitive method of controlling systems in highly unpredictable circumstances [2].

The primary goal is to permanently eliminate uncertainty while maintaining a set of carefully selected constraint functions (sliding variables) at zero.

The controllers of HOSM are introduced as follows.

$$u = -\alpha \cdot sign\sigma \tag{1}$$

$$u = -\alpha \cdot sign\big(\dot{\sigma} + k_1 \cdot |\sigma|^{1/2} \cdot sign\sigma\big) \tag{2}$$

$$u = -\alpha \cdot sign\left(\ddot{\sigma} + k_2(|\dot{\sigma}|^3 + \sigma^2)^{1/6} sign\big(\dot{\sigma} + k_1|\sigma|^{2/3} \cdot sign\sigma\big)\right) \cdot \tag{3}$$

$$u = -\alpha \cdot sign(\dddot{\sigma} + k_3(\ddot{\sigma}^6 + \dot{\sigma}^4 + |\sigma|^3)^{1/12} sign(\ddot{\sigma} + k_2(\dot{\sigma}^4 + |\sigma|^3)^{1/6} \cdot sign\big(\dot{\sigma} + k_1|\sigma|^{3/4} \cdot sign\sigma\big))) \tag{4}$$

$$u = -\alpha \cdot sign(\sigma^{(4)} + k_4(\sigma^{12} + |\dot{\sigma}|^{15} + \ddot{\sigma}^{20} + \dddot{\sigma}^{30})^{\frac{1}{60}} \cdot sign(\dddot{\sigma} + k_3(\sigma^{12} + |\dot{\sigma}|^{15} + \ddot{\sigma}^{20})^{\frac{1}{30}} \cdot sign(\ddot{\sigma} + k_2(\sigma^{12} + |\dot{\sigma}|^{15})^{1/20} \cdot sign(\dot{\sigma} + k_1|\sigma|^{4/5} \cdot sign\sigma)))). \tag{5}$$

However, the method of determining the controller parameters was not proposed.

DETERMINATION OF THE HIGH ORDER SLIDING CONTROLLER PARAMETERS

The problem of selecting the parameters of the HOSM controller arises to provide the characteristics and design requirements of the plant and to satisfy the sliding condition.

According to the relative order of the control object [11–14], the HOSM controllers are as follows:

$$u = -\alpha \cdot sign\sigma \tag{6}$$

$$u = -\alpha \cdot sign\big(\dot{\sigma} + k_1 \cdot |\sigma|^{1/2} \cdot sign\sigma\big) \tag{7}$$

$$u = -\alpha \cdot sign\left(\ddot{\sigma} + k_2(|\dot{\sigma}|^3 + \sigma^2)^{1/6} sign\big(\dot{\sigma} + k_1|\sigma|^{2/3} \cdot sign\sigma\big)\right) \cdot \tag{8}$$

$$u = -\alpha \cdot sign(\dddot{\sigma} + k_3(\ddot{\sigma}^6 + \dot{\sigma}^4 + |\sigma|^3)^{1/12} sign(\ddot{\sigma} + k_2(\dot{\sigma}^4 + |\sigma|^3)^{1/6} \cdot sign\big(\dot{\sigma} + k_1|\sigma|^{3/4} \cdot sign\sigma\big))) \tag{9}$$

Sliding mode features the constraint of the characteristics of the control object to the sign function internal equation of the control signal. In HOSM control, it is important to know how to properly select the feedback gain factor selection. By solving the problem of selecting coefficient, the HOSM controller can be determined.

Therefore, in this study, the problem of selecting coefficient waiting for an important and robust solution for HOSM control is solved reasonably from the viewpoint of optimal control [15].

In the introduction section, the relationship between the shortest-time control and the HOSM controller is discussed in detail.

Based on this, the HOSM controller coefficients are also solved by graphically matching the optimal phase trajectory of the system and the higher order sliding mode switching line trajectory.

First, let us analyze the internal function characteristics of the sign function of the control law Eq. (2) corresponding to the 2-order system.

State equation of control object is as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -a_1 x_1 - a_2 x_2 + bu. \end{cases}$$

The equation of the switching line is given in the following form:

$$\Sigma_1 = \dot{\sigma} + k_1 \cdot |\sigma|^{1/2} \cdot sign\sigma = 0. \tag{10}$$

At that time, the form of control signal is as follows:

$$u = -\alpha_1 sign(\Sigma_1).$$

From the internal function characteristics of the sign function, we find the parameters of the switching line.

$$\sigma > 0, \dot{\sigma} + k_1 \cdot |\sigma|^{1/2} = 0 \rightarrow \dot{\sigma} = -k_1 |\sigma|^{1/2}. \tag{11}$$

$$\sigma < 0, \dot{\sigma} - k_1 \cdot |\sigma|^{1/2} = 0 \rightarrow \dot{\sigma} = k_1 |\sigma|^{1/2}. \tag{12}$$

The blue curve in Figure 1 is the optimal phase trajectory and the red curve is the higher order sliding mode switching line.

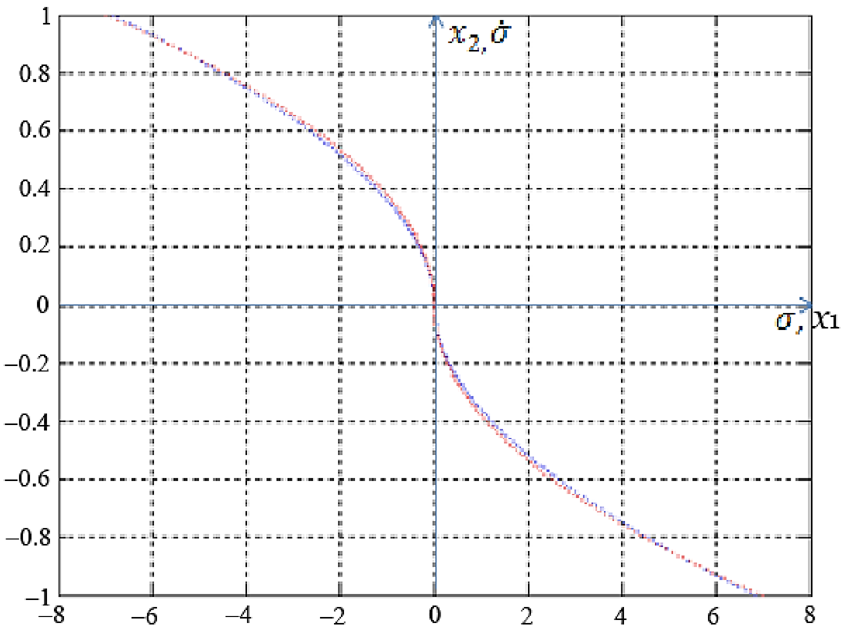

Figure 1. Optimal phase trajectory and switching line curve of $\dot{\sigma} + k_1 |\sigma|^{2/3} \cdot sign\sigma = 0$.

As you can see from the simulation curve, the coefficient k1 can be chosen in a geometric way, i.e. to match the optimal phase-trajectory and the switching line of the system [16–19].

Based on this, the transient time is determined.

$$\dot{\sigma} = -k_1|\sigma|^{1/2} \rightarrow |\sigma|^{-1/2}d\sigma = -k_1 dt$$
$$2\left|\sigma\left(t_f\right)\right|^{1/2} - 2|\sigma(0)|^{1/2} = -k_1 t_f.$$

For example, $\sigma(t) = 0$, initial value is 1^0 for roll deviation angle of vehicle, while required transient time is:

$$t_f = \frac{2}{k_1}\sigma(0)^{1/2}.$$

It is discovered that the controller parameter satisfies the sliding mode's existence criterion [20, 21]. Sliding Mode (SM) existence condition is $\dot{\Sigma}_1 sign(\Sigma_1) < 0$.

$$\dot{\Sigma}_1 = \ddot{\sigma} + \frac{1}{2}k_1|\sigma|^{-1/2}\dot{\sigma}.$$

Where $\sigma = r - y = r - x_1$, r: setting quantity, y: Output control volume.

$$\ddot{\sigma} = -\dot{x}_2 = a_1 x_1 + a_2 x_2 - bu.$$

Since the control signal u is discontinuous nonlinear, we consider the stability condition of Filippov.

Where the state quantity is assumed to be bounded (the physical quantity of the real system is bounded when the system is stable), i.e., $\|a_1 x_1 + a_2 x_2\| < \eta$ (Here η is random constant).

$$(a_1 x_1 + a_2 x_2) \in [-C_1 C_1], b \in [K_m K_M]$$

$$\ddot{\sigma} \in [-C_1 C_1] - [K_m K_M]u$$

From the equation of switching line when$\Sigma_1 = 0$, $\dot{\sigma} = -k_1|\sigma|^{1/2} sign\sigma$.

Thus,

$$\dot{\Sigma}_1 = [-C_1 C_1] - \alpha_1[K_m K_M]sign(\Sigma_1) - \frac{1}{2}k_1^2 sign\sigma.$$

The existence condition of sliding is considered.

$$\begin{cases} \Sigma_1 > 0, \alpha_1 K_m - C_1 > \frac{1}{2}k_1^2 \\ \Sigma_1 < 0, \alpha_1 K_m - C_1 > \frac{1}{2}k_1^2. \end{cases}$$

From this equation, the sliding controller parameter α_1 can be calculated as follows:

$$\alpha_1 > \left(\frac{1}{2}k_1^2 + C_1\right)/K_m.$$

This method is one of changing the sliding controller parameter k1 in a trial-and-error manner, given the assumption that the final value of the phase trajectory is 0 and the maximum initial deviation, which matches the shortest time trajectory and determines the transient time. To eliminate trial and error methods, we can solve this problem by setting the required transient time in addition to the above conditions.

However, since no more than three sliding controller structures are applicable, the internal function of the sign function can be solved by changing the relative order to 1-order. Next, consider the method of determining the parameters of the sliding controller for the 3-order system.

State equation of system is as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -a_1 x_1 - a_2 x_2 - a_3 x_3 + bu. \end{cases}$$

Sliding surface is given as follows:

$$\Sigma_2 = \ddot{\sigma} + k_2(|\dot{\sigma}|^3 + \sigma^2)^{1/6} sign\big(\dot{\sigma} + k_1|\sigma|^{2/3} \cdot sign\sigma\big) = 0 \tag{13}$$

Let the control signal be $u = -\alpha_2 sign(\Sigma_2)$.

First, consider the inner equation of the sign function:

$\sigma > 0, \dot{\sigma} + k_1 \cdot |\sigma|^{2/3} = 0 \rightarrow \dot{\sigma} = -k_1|\sigma|^{2/3}$

$\sigma < 0, \dot{\sigma} - k_1 \cdot |\sigma|^{2/3} = 0 \rightarrow \dot{\sigma} = k_1|\sigma|^{2/3}$

k_1 and the technique described in Figure 2 can be used to find transitory time.

The transient time is then calculated as $t_f = \frac{3}{k_1}\sigma(0)^{2/3}$.

Next, consider the method of determining k_2.

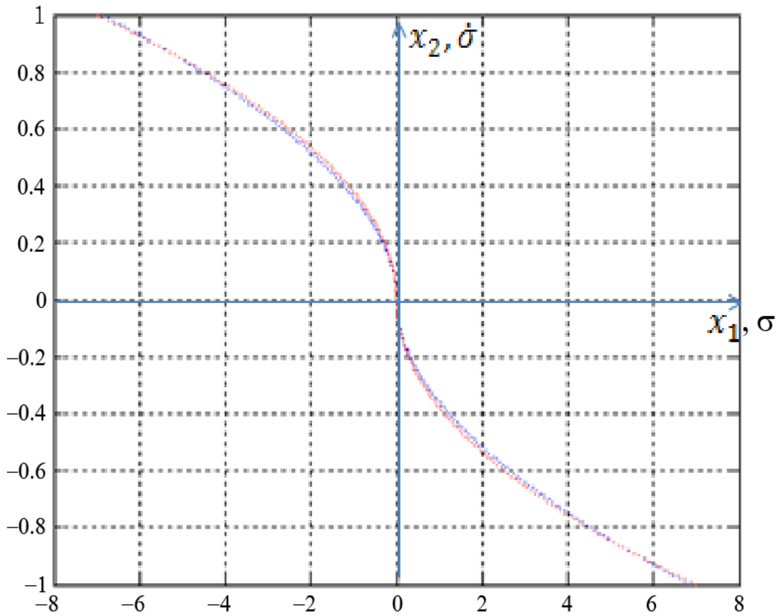

Figure 2. Optimal phase trajectory and curve of $\dot{\sigma} + k_1|\sigma|^{2/3} \cdot sign\sigma$ =0.

Assuming $\Sigma_1 = \dot{\sigma} + k_1|\sigma|^{2/3} \cdot sign\sigma$, Eq. (14) can be written as:

$$\begin{cases} \Sigma_1 > 0, \ddot{\sigma} + k_2(|\dot{\sigma}|^3 + \sigma^2)^{1/6} = 0 \\ \Sigma_1 < 0, \ddot{\sigma} - k_2(|\dot{\sigma}|^3 + \sigma^2)^{1/6} = 0. \end{cases} \tag{14}$$

Substituting $\sigma^2 = \frac{1}{{k_1}^3}\dot{\sigma}^3$ in $\Sigma_1 = 0$ into the first expression of Eq. (9) gives:

$$\ddot{\sigma} + k_2\left(|\dot{\sigma}|^3 + \frac{1}{{k_1}^3}|\dot{\sigma}|^3\right)^{1/6} = 0.$$
$$\ddot{\sigma} + k_2\left(1 + \frac{1}{{k_1}^3}\right)|\dot{\sigma}|^{1/2} = 0.$$

Let $\dot{\sigma} = x$, the above equation is as follows: (considering ${k_1}^3 >> 1$)

$$\dot{x} = -k_2 x^{1/2}$$

Hence, we can determine k_2 by geometrically matching the optimal phase trajectory and the trajectory of the switching line.

In order to find the transient time, we set $x(t_f) = 0$ from the idea of 2-order sliding to $\sigma, \dot{\sigma} \to 0$ in the integration interval $(t_f, 0)$ in the equation solution.

At that time,

$$\int_0^{t_f} x^{-\frac{1}{2}}\, dx = -\int_0^{t_f} k_2\, dt \to 2 \cdot x^{\frac{1}{2}}(0) = k_2 t_f.$$

Considering the maximum velocity when s=0 is reached as $x(0) = \dot{\sigma}(0)$, the maximum roll velocity of the present vehicle is considered.

$$t_f = \frac{2 \cdot \sqrt{x}}{k_2}$$

Let us additionally identify the controller quantity that meets the sliding mode's existence requirement.

SM existence condition is given as follows:

$$\dot{\Sigma}_2 sign(\Sigma_2) < 0$$

Since $\Sigma_1 = 0$, we have $\Sigma_2 = \ddot{\sigma} + k_2\left(1 + \frac{1}{k_1^3}\right)|\dot{\sigma}|^{1/2}$.

Thus,

$$\dot{\Sigma}_2 = \dddot{\sigma} + \frac{1}{2}k_2\left(1 + \frac{1}{k_1^3}\right)|\dot{\sigma}|^{-1/2}\ddot{\sigma}.$$

From:

$$\Sigma_1 = \Sigma_2 = 0, \ddot{\sigma} = -k_2\left(1 + \frac{1}{k_1^3}\right)|\dot{\sigma}|^{1/2}$$

$$\dddot{\sigma} = -\dot{x}_3 = a_1x_1 + a_2x_2 + a_3x_3 - bu$$

Since the control signal is discontinuous nonlinear, according to Filippov's stability theorem, the state quantity is limited.

$$\ddot{\sigma} \in [-C_2 C_2] - [K_m K_M]u$$

$$\dot{\Sigma}_2 = [-C_2 C_2] - \alpha_2 [K_m K_M] sign(\Sigma_2) - \frac{1}{2}\left[k_2\left(1+\frac{1}{k_1^3}\right)\right]^2 sign\sigma$$

$$\begin{cases} \Sigma_2 > 0, \dot{\Sigma}_2 = [-C_2 C_2] - \alpha_2 [K_m K_M] - \frac{1}{2}\left[k_2\left(1+\frac{1}{k_1^3}\right)\right]^2 sign\sigma < 0 \\ \Sigma_2 < 0, \dot{\Sigma}_2 = [-C_2 C_2] + \alpha_2 [K_m K_M] - \frac{1}{2}\left[k_2\left(1+\frac{1}{k_1^3}\right)\right]^2 sign\sigma > 0 \end{cases}$$

By considering the sliding controller parameter α_2, the following obtained:

$$\alpha_2 > \left\{\frac{1}{2}\left[k_2\left(1+\frac{1}{k_1^3}\right)\right]^2 + C_2\right\} / K_m.$$

Thus, the parameter k_1, k_2 of the 3-order sliding is set such that the characteristics of the object and the desired system design index are satisfied.

Next, let us choose the sliding controller parameters of the 4-order system in the same way. State equation of control object is as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -a_1 x_1 - a_2 x_2 - a_3 x_3 - a_4 x_4 + bu \end{cases}$$

Sliding surface is given as follows:

$$\Sigma_3 = \dddot{\sigma} + k_3(\ddot{\sigma}^6 + \dot{\sigma}^4 + |\sigma|^3)^{\frac{1}{12}} \cdot sign[\ddot{\sigma} + k_2 \cdot (\dot{\sigma}^4 + |\sigma|^3)^{\frac{1}{6}} \cdot sign\left(\dot{\sigma} + k_1 |\sigma|^{\frac{3}{4}} \cdot sign\sigma\right)] = 0. \quad (15)$$

Let the control signal be:

$$u = -\alpha_3 sign(\Sigma_3).$$

First, we find k_1 by solving the inner equation of the code function above.

We consider in:

$$\Sigma_1 = \dot{\sigma} + k_1 |\sigma|^{\frac{3}{4}} \cdot sign\sigma = 0.$$

$$\begin{cases} \sigma > 0, \dot{\sigma} = -k_1 \sigma^{\frac{3}{4}} \\ \sigma < 0, \dot{\sigma} = k_1 |\sigma|^{\frac{3}{4}}. \end{cases} \quad (16)$$

Next, let us determine k_2.

We consider in Figure 3:

$$\Sigma_2 = \ddot{\sigma} + k_2 \cdot (\dot{\sigma}^4 + |\sigma|^3)^{\frac{1}{6}} \cdot sign\left(\dot{\sigma} + k_1 |\sigma|^{\frac{3}{4}} \cdot sign\sigma\right) = 0. \quad (17)$$

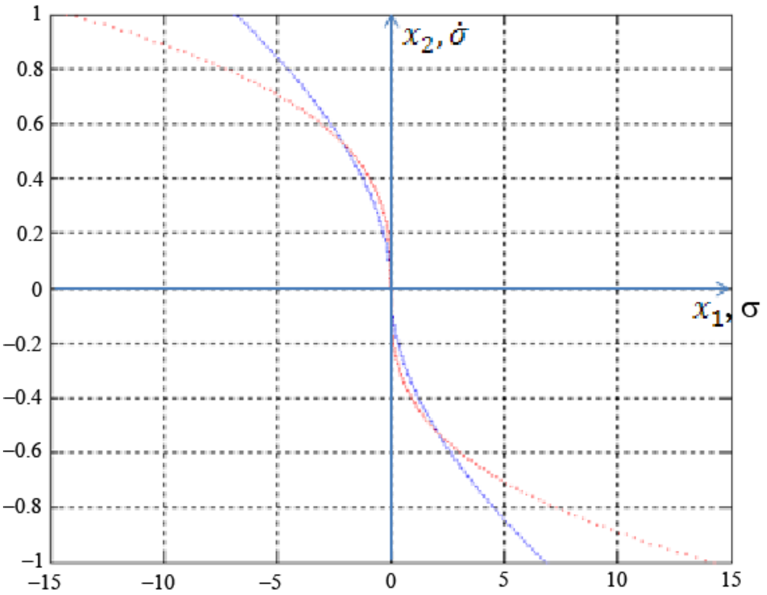

Figure 3. Optimal phase trajectory and curve of $\dot{\sigma} + k_1|\sigma|^{\frac{3}{4}} \cdot sign\sigma = 0$.

Assuming that:

$$\Sigma_1 = \dot{\sigma} + k_1|\sigma|^{\frac{3}{4}} \cdot sign\sigma,$$

When $\Sigma_1 = 0$, $|\sigma|^3 = -\frac{1}{k_1^4}\dot{\sigma}^4$ Eq. (12) is following that:

$\ddot{\sigma} + k_2\left(1 + \frac{1}{k_1^4}\right)\dot{\sigma}^{\frac{2}{3}} = 0$, When $k_1 > 1$, $k_1^4 >> 1$

Thus:

$$\ddot{\sigma} + k_2\dot{\sigma}^{\frac{2}{3}} = 0 \tag{18}$$

Determine k_2 in the same way as above, say $\dot{\sigma} = x$.

Determination of k_3 in Figure 4.

Considering $|\sigma|^3 = \frac{1}{k_1^4}\dot{\sigma}^4$ in Σ_3 and $\dot{\sigma}^4 = \frac{1}{k_2^6}\ddot{\sigma}^6$ in Eq. (13), we obtain:

$$\begin{cases} \Sigma_2 > 0, \dddot{\sigma} + k_3(\ddot{\sigma}^6 + \dot{\sigma}^4 + |\sigma|^3)^{\frac{1}{12}} = 0 \\ \Sigma_2 < 0, \dddot{\sigma} - k_3(\ddot{\sigma}^6 + \dot{\sigma}^4 + |\sigma|^3)^{\frac{1}{12}} = 0. \end{cases} \tag{19}$$

Considering the above equations in the first equation of Eq. (12), we have。

$$\dddot{\sigma} + k_3\left(\ddot{\sigma}^6 + \frac{1}{k_2^6}\ddot{\sigma}^6 + \frac{1}{k_1^4}\dot{\sigma}^4\right)^{\frac{1}{12}} = \dddot{\sigma} + k_3\left(1 + \frac{1}{k_2^6} + \frac{1}{k_1^4k_2^6}\right)|\ddot{\sigma}|^{1/2} = 0.$$

$k_1 > 1, k_2 > 1$ and $k_1^4k_2^6 >> 1, k_2^6 >> 1$.

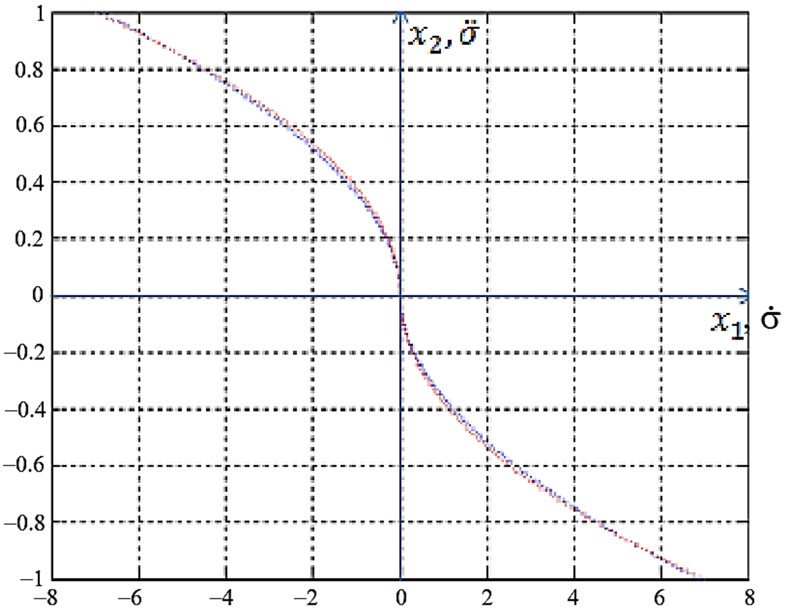

Figure 4. Optimal Phase Trajectory and Curve of $\ddot{\sigma} + k_2\dot{\sigma}^{\frac{2}{3}} = 0$.

Taking this into account, the above expression is:

$$\dddot{\sigma} + k_3|\ddot{\sigma}|^{1/2} \approx 0 \tag{20}$$

If $\ddot{\sigma} = x$, the higher case becomes $\dot{x} + k_3x^{1/2} = 0$.

Similarly, we may ascertain k_3.

Calculate the controller parameter α_3 from the following sliding existence condition:

$\dot{\Sigma}_3 sign(\Sigma_3) < 0$.

$\dot{\Sigma}_3 = \sigma^{(4)} + \frac{1}{2}k_3|\ddot{\sigma}|^{-\frac{1}{2}}\dddot{\sigma}$.

$\sigma^{(4)} = -\dot{x}_4 = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 - bu$.

Since the control signal u is represented by a discrete nonlinear function, the differential inclusion relation is expressed by the stability theory of Filippov as follows:

$\sigma^{(4)} \in [-C_3C_3] - [K_mK_M]u$

Here $C_3 = \|a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4\|, b \in [K_mK_M]$,

In $\Sigma_1 = \Sigma_2 = \Sigma_3 = 0$, $\dddot{\sigma} = -k_3\ddot{\sigma}^{1/2}$.

From this:

$\dot{\Sigma}_3 = [-C_3C_3] - \alpha_3[K_mK_M]sign(\Sigma_3) + \frac{1}{2}k_3^2 sign\sigma$.

$$\begin{cases} \Sigma_3 > 0, \dot{\Sigma}_3 = [-C_3 C_3] - \alpha_3 [K_m K_M] + \frac{1}{2} k_3^2 sign\sigma < 0 \\ \Sigma_3 < 0, \dot{\Sigma}_3 = [-C_3 C_3] + \alpha_3 [K_m K_M] + \frac{1}{2} k_3^2 sign\sigma > 0. \end{cases}$$

$$\alpha_3 K_m - C_3 > \frac{1}{2} k_3^2.$$

Thus:

$$\alpha_3 > \left(\frac{1}{2} k_3^2 + C_3\right) / K_m.$$

In this way, the parameters of the higher order sliding controller can be determined. This method perfectly reflects the optimal control characteristics of the plant as mentioned in the Figure 5 and is an innovative alternative to the parameter determination problem which has been a very robust problem in HOSM.

SIMULATION EXAMPLE

As a roll stabilization system design method, roll stabilization system is designed based on the air speed maximum point of vehicle which is the aerodynamic feature point, the source of altitude, the initial control stage (less than 1 Ma) and the turning point and control start point of control stage. First, we consider the change relationship between the transfer function and parameter of roll stabilization system. The roll motion transfer function of the aircraft is as follows:

$$W_\delta^\gamma(s) = \frac{K_{DX}}{s(T_{DX}s+1)}$$

Here:

$$K_{DX} = -\frac{C_{13}}{C_{11}} = -\frac{M_{x_1}^{\delta_y}}{M_{x_1}^{\omega_{x_1}}}:$$

Roll transfer coefficient of the aircraft

$$T_{DX} = \frac{1}{C_{11}} = -\frac{J_{x_1}}{M_{x_1}^{\omega_{x_1}}}:$$

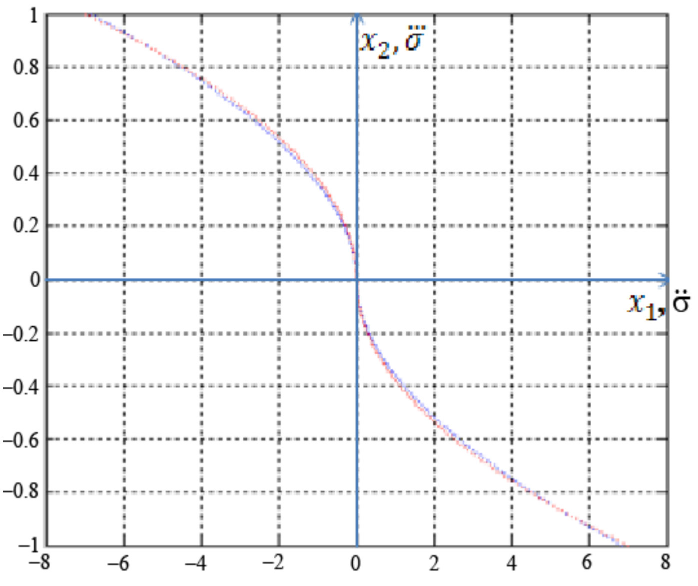

Figure 5. Optimal Phase Trajectory and Curve of $\ddot{\sigma} + k_3|\ddot{\sigma}|^{1/2} = 0$.

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Roll time constant of the aircraft

$$C_{11} = -\frac{M_{x_1}^{\omega x_1}}{J_{x_1}}, C_{13} = -\frac{M_{x_1}^{\delta \gamma}}{J_{x_1}}$$

The changing band of the high sawing drag coefficient is as follows:

$$C_{11} \in [0.03, 1.5], C_{13} \in [80, 8000]$$

Following this, the range of parameter change of the vehicle is:

$$K_{DX} \in [2000, 19333.3], T_{DX} \in [0.67, 33.4]$$

The model of transfer function of the rudder tracking system using electric rudder mechanism is as follows:

$$W_{Eҏ}(s) = \frac{K_r}{{T_r}^2 s^2 + 2\xi_r T_r s + 1}.$$

Analyzing the frequency characteristics of the open system as shown in Figure 6, it is unstable as follows:

$$W(s) = \frac{K_p K_r}{T_p T_r^2 s^4 + (2T_p \xi_r T_r + T_r^2)s^3 + (T_p + 2\xi_r T_r)s^2 + s}.$$

State space model of control object is formulated as follows:

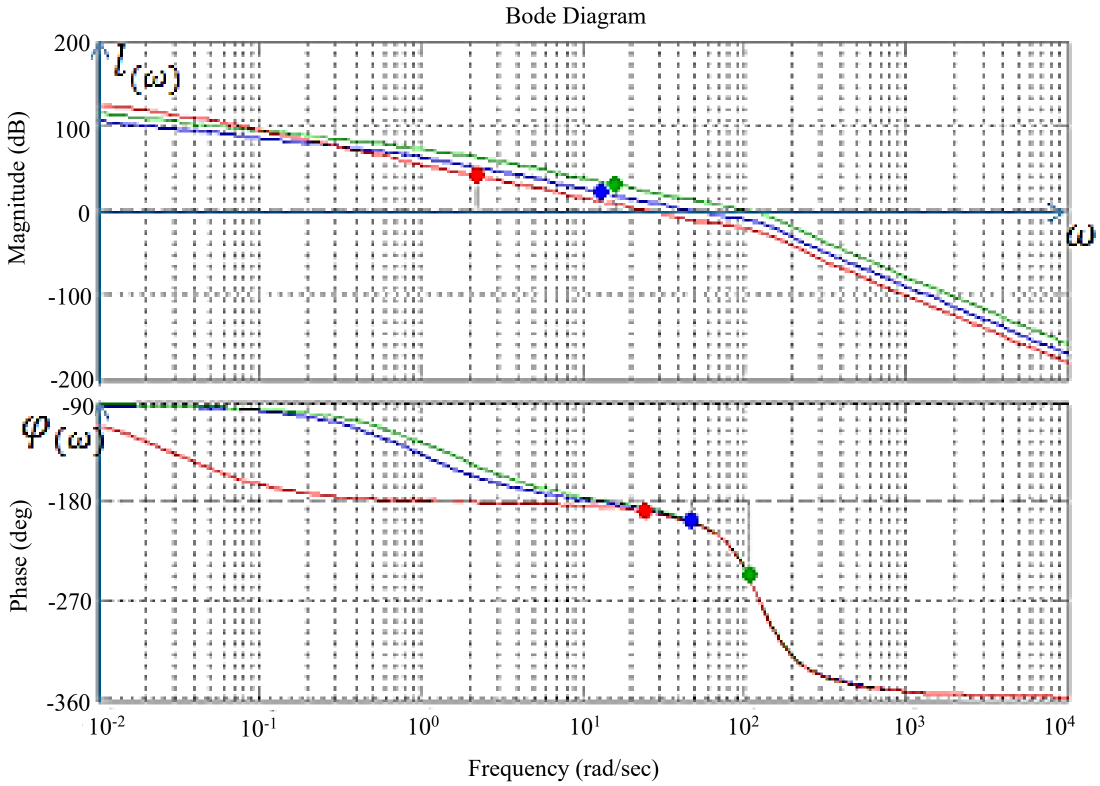

Figure 6. Frequency characteristics of roll control system.

$$\begin{cases} \dot{X} = AX + Bu \\ y = CX. \end{cases}$$

$$X = [x_1 x_2 x_3 x_4]^T, x_1 = y, x_2 = \dot{y}, x_3 = \ddot{y}, x_4 = \dddot{y}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -a_2 & -a_3 & -a_4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b \end{bmatrix}, C = [1 \quad 0 \quad 0 \quad 0]$$

$$a_2 = \frac{1}{T_p T_r^2}, a_3 = \frac{T_p + 2\xi_r T_r}{T_p T_r^2}, a_4 = \frac{2T_p \xi_r T_r + T_r^2}{T_p T_r^2}, b = \frac{K_r K_p}{T_p T_r^2}$$

$$T_r = 0.008, \xi_r = 0.37, K_p = 1$$

$$K_p \in [2000{,}19333.3], T_p \in [0.67{,}33.4]$$

$$a_2 = [467.8,\ \ 23320.9] a_3 = [15628,\ \ 15763], a_4 = [92.5,\ \ 94]$$
$$b = [9 \times 10^6,\ \ 1.24 \times 10^8]$$

This analysis shows in Figure 7 that the roll control system has a good but unstable character because the roll transfer coefficient of the aircraft is very big. Also, the variation of roll transfer coefficient and time constant is big, so robustness should be ensured.

Therefore, a sliding controller design is designed to ensure both robustness and adaptability simultaneously.

Since the relative order is 4, a 4-order sliding controller is designed. General form of 4-order sliding mode controller is set as follows:

$$u = -\alpha \cdot sign\{\dddot{\sigma} + k_3(\ddot{\sigma}^6 + \dot{\sigma}^4 + |\sigma|^3)^{\frac{1}{12}}$$
$$sign\left(\ddot{\sigma} + k_2(\dot{\sigma}^4 + |\sigma|^3)^{\frac{1}{6}} sign\left(\dot{\sigma} + k_1|\sigma|^{\frac{3}{4}} \cdot sign\sigma\right)\right)\}$$

$$u = -\alpha \cdot sign\Sigma$$

$$\Sigma = \dddot{\sigma} + k_3(\ddot{\sigma}^6 + \dot{\sigma}^4 + |\sigma|^3)^{\frac{1}{12}} \cdot sign\left[\ddot{\sigma} + k_2(\dot{\sigma}^4 + |\sigma|^3)^{\frac{1}{6}} sign\left(\dot{\sigma} + k_1|\sigma|^{\frac{3}{4}} sign\sigma\right)\right]$$

Where $\sigma = \gamma_c - \gamma$: roll error angle

The 2-order SM controller is also considered, but according to k_1, k_2, k_3, the rate of variation of the switching surface is changed and the dynamic process of the system along this switching surface is changed, so that the design requirements are reflected in parameter k_1, k_2, k_3 to ensure the desired transient response depending on the characteristics of the object, and α is set considering the actuator saturation and sliding conditions.

The sliding controller parameters are determined by the determining method considered above. The results obtained from the optimal phase trajectory and switching line curve of the object are as follows:

$k_1 = 27$
$k_2 = 300$
$k_3 = 15500$

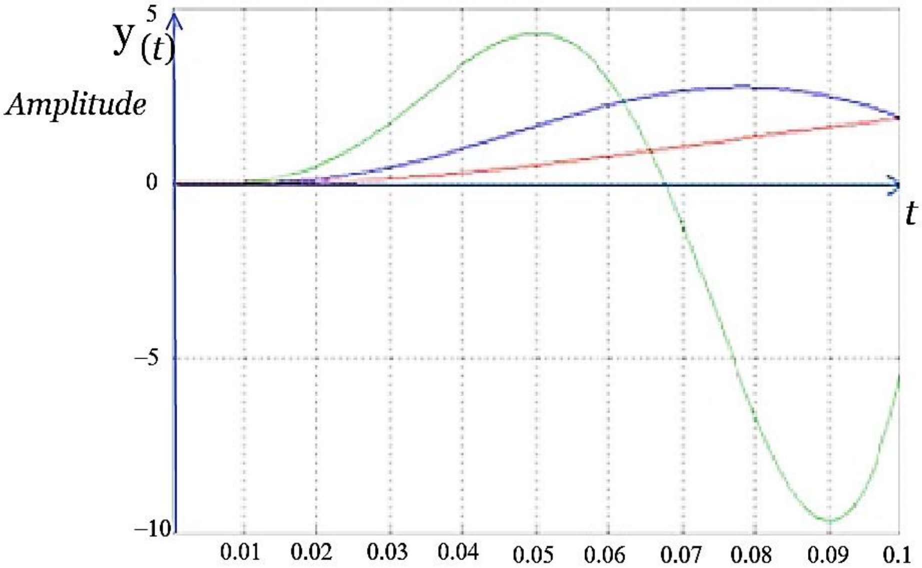

Figure 7. Transient response characteristics of roll control system.

Next, the convergent time t_{f_1} of σ, the convergence time t_{f_2} of $\dot{\sigma}$ and the convergence time t_{f_3} of $\ddot{\sigma}$ is calculated under the condition that the initial roll angle is 1^0, the maximum roll velocity is $100\ °/s$, the maximum roll acceleration is set to $60\ °/s^2$, and $\sigma, \dot{\sigma}, \ddot{\sigma}, \dddot{\sigma}$ converge to zero.

$$t_{f_1} = \frac{4}{k_1}|\sigma(0)|^{\frac{1}{4}} = \frac{4}{27}\sqrt[4]{1} = 0.15(s)$$

$$t_{f_2} = \frac{3}{k_2}|\dot{\sigma}(0)|^{\frac{1}{3}} = \frac{3}{300}\sqrt[3]{100} = 0.01(s)$$

$$t_{f_3} = \frac{2|\ddot{\sigma}|^{\frac{1}{2}}}{k_3} = \frac{2\sqrt{60}}{15500} = 0.001(s)$$

α shall be taken from rudder constraint characteristic of rudder mechanism, actuator $(-15^0 \sim 15^0)$ to $\alpha = 15$.

This value should also satisfy the sliding existence condition $\dot{\Sigma} sign \Sigma < 0$.

Considering the parameter varying range of the object, the SM existence condition is found to be satisfied enough.

The simulation scheme is as follows in Figure 8.

From the simulation results, the transient and frequency-pass characteristics are considered as follows, as shown in Figure 9.

From the simulation curve in Figure 10, the transient time is 0.06 s to meet the required quality index.

Next, consider the settling time of the roll velocity ω_x proposed in BTT vehicle control (Figure 11).

As shown in the characteristic curves, the pass characteristics of the roll control system are about 30~40 rad/s, which meets the design specifications and is completely separated from the passband of pitch and yaw, so that the effect of cross coupling is completely weakened.

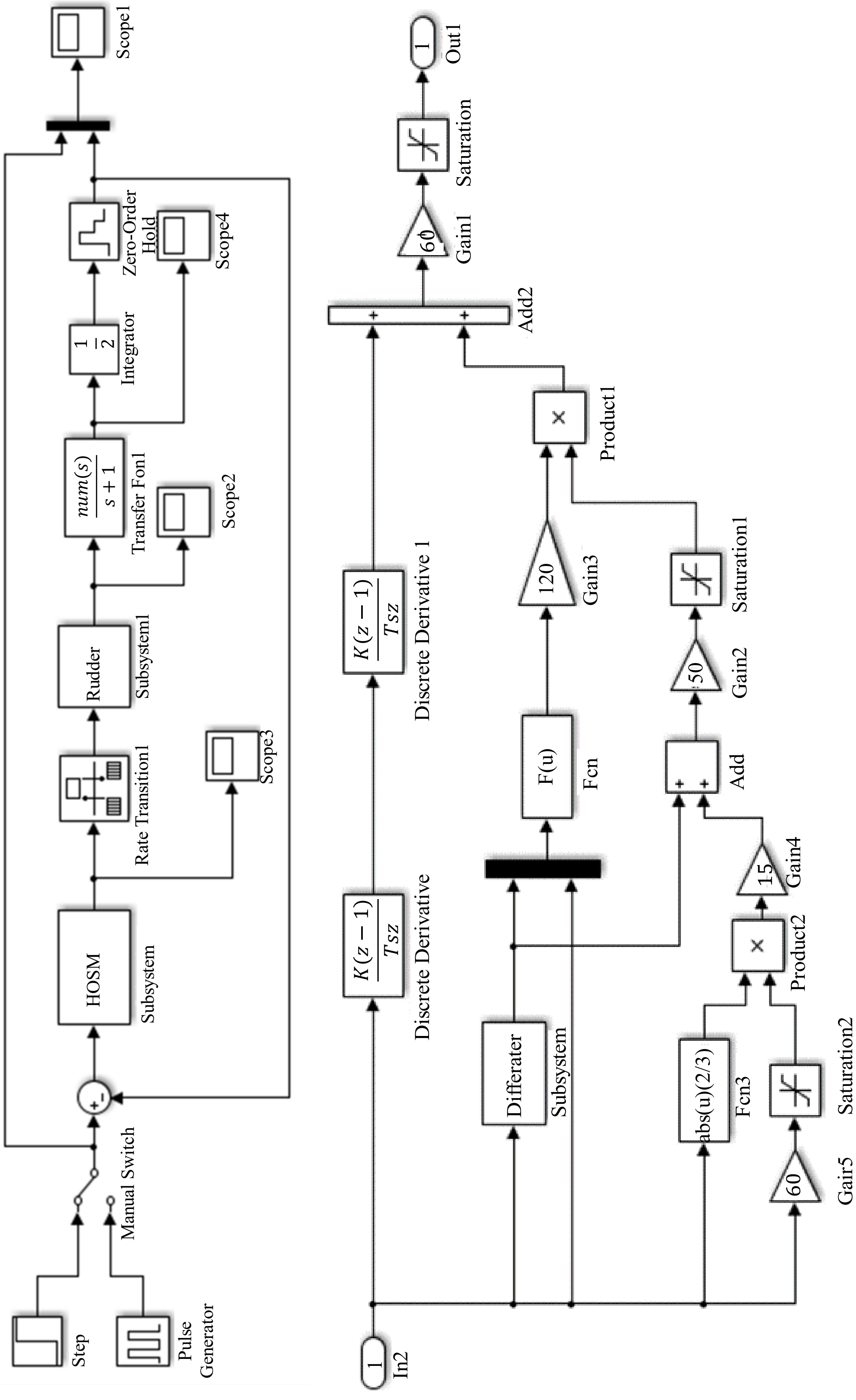

Figure 8. Simulation scheme of roll stabilization system.

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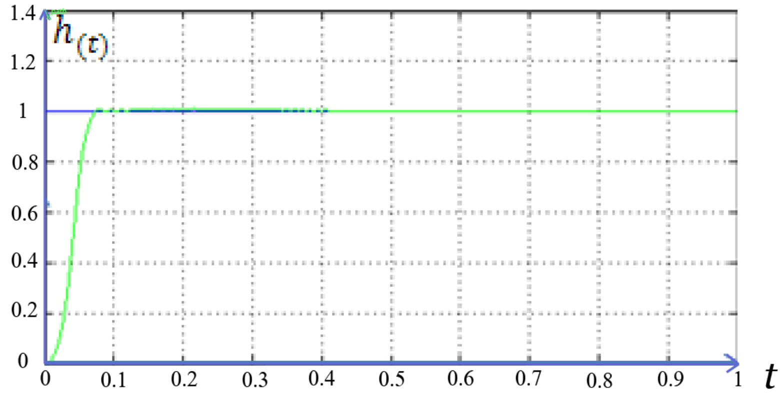

Figure 9. Transient characteristic curve.

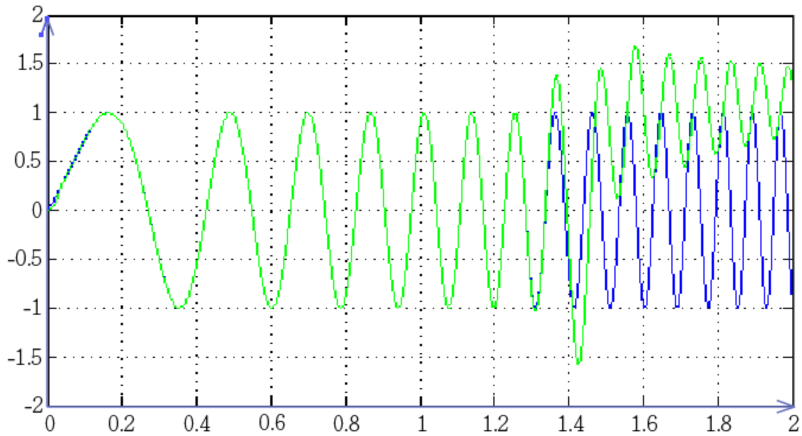
Figure 10. Frequency tracking characteristic curve.

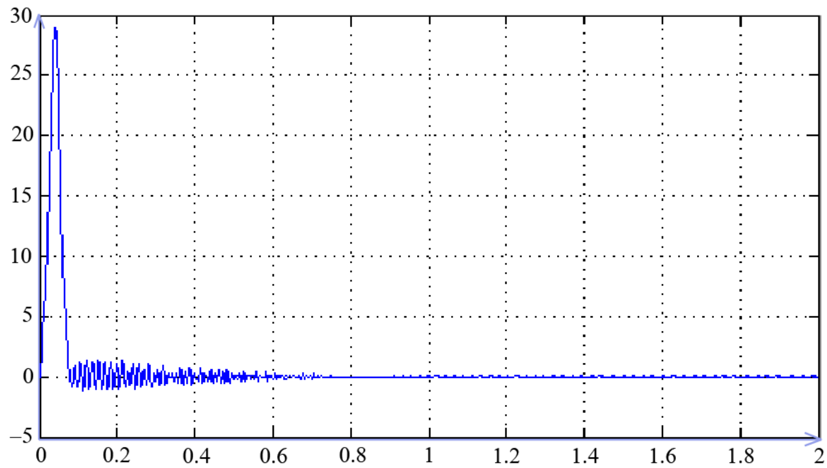
Figure 11. Roll velocity characteristic curve.

CONCLUSION

This study presents and validates a method for calculating the controller variables for a given HOSM controller topology in order comply with design for control systems standards. The method is used to the layout of the vehicle roll management system. The basic idea is that the internal characteristic equation of the higher order sliding-switching surface equation can be solved by transforming the relative 1-order linear differential equation solution with relative degree 1. The problem presented here is that the transient time related to the initial condition and inertial time constant should be reasonably set to suit the characteristics of the object.

REFERENCES

1. Levant A. Homogeneity approach to high-order sliding mode design. Automatica. 2005; 41(5): 823–830.
2. Utkin VI. Sliding Modes in Optimization and Control Problems. New York: Springer, 1992.
3. Zinober ASI. An introduction to sliding mode variable structure control. In: Zinober ASI, editor. Variable Structure and Lyapunov Control. Lecture Notes in Control and Information Sciences, vol 193. Berlin, Heidelberg: Springer; 1994. https://doi.org/10.1007/BFb0033676
4. Edwards C, Spurgeon SK. Sliding Mode Control: Theory and Applications. CRC Press; Florida, United States. 1998. https://doi.org/10.1201/9781498701822
5. Fridman E, Shaked U. An LMI approach to stability of discrete delay systems. *2003 European Control Conference (ECC)*, Cambridge, UK. 2003; 1477–1482. doi: 10.23919/ECC.2003.7085170.
6. Fridman E. A LMI approach to stability of descriptor systems with delay. *2001 European Control Conference (ECC)*, Porto, Portugal. 2001; 2871–2876. doi: 10.23919/ECC.2001.7076368.
7. Slotine JJ, Li W. Applied nonlinear control. Englewood Cliffs, NJ: Prentice Hall; 1991 Jan.
8. Pan Y, Furuta K, Suzuki S, Hatakeyama S. Design of variable structure controller-from sliding mode to sliding sector. In Proceedings of the 39th IEEE Conference on Decision and Control (Cat. No. 00CH37187). 2000 Dec 12; 2: 1685–1690.
9. Emel'yanov Stanislav V, Korovin Sergey K, Levantovskii LV. Higher-order sliding modes in binary control systems. Comput Math Model. 1996; 7(3): 294-318.
10. Levant A. Sliding order and sliding accuracy in sliding mode control. Int J Control. 1993; *58*(6): 1247–1263. https://doi.org/10.1080/00207179308923053
11. Levant A. Universal SISO sliding-mode controllers with finite-time convergence. IEEE Trans Autom Control. 2001 Sep; 46(9): 1447–1451.
12. Levant A. Principles of 2-sliding mode design. Automatica. 2007; 43(4): 576–586.
13. Levant A. Finite-time stability and high relative degrees in sliding-mode control. Lecture Notes Control Inform Sci. 2012; 412: 59–92.
14. Levant A. Construction principles of 2-sliding mode design. Automatica. 2007; 43(4): 576–586.
15. Utkin V, Lee H. Chattering analysis. In: Edwards C, Fossas Colet E, Fridman L, editors. Advances in Variable Structure and Sliding Mode Control, Lecture Notes in Control and Information Sciences. Berlin, Germany: Springer Verlag; 2006; 334: 107–123.
16. Levant A. Sliding order and sliding accuracy in sliding mode control. Int J Control. 1993; 58(6): 1247–1263.
17. Levant A. Finite-time stability and high relative degrees in sliding mode control. Lecture Notes Control Inform Sci. 2012; 412: 59–92.
18. Levant A. Quasi-continuous high-order sliding-mode controllers. IEEE Trans Autom Control. 2005 Nov; 50(11): 1812–1816.
19. Levant A. Higher-order sliding modes, differentiation and output feedback control. Int J Contr. 2003; 76(9/10): 924–941.
20. Shtessel YB, Shkolnikov IA. Aeronautical and space vehicle control in dynamic sliding manifolds. Int J Contr. 2003; 76(9/10): 1000–1017.
21. Shtessel YB, Shkolnikov IA, Brown MDJ. An asymptotic second-order smooth sliding mode control. Asian J Control. 2004; 5(4): 498–504.

