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Review IJFMDS

Fracture Analysis of Functionally Graded Material (FGM) Plates Using Extended Finite Element Method: A Review

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Abstract

Functionally Graded Materials (FGMs), have drawn a lot of interest in various engineering applications due to their superior mechanical properties and ability to withstand extreme conditions. Fracture analysis in FGMs focuses on understanding how cracks initiate and propagate within these complex materials. The stress distribution becomes irregular due to spatial property variations which produces different crack paths than what occurs in homogeneous materials. The examination of FGM plates under fracture conditions requires immediate research because it applies crucially to biomedicine together with automotive and aerospace industries. The true engineering world contains multiple unknowns regarding material properties and load conditions together with dimensional specifications. The uncertainties are evaluated through probabilistic fracture analysis to make precise predictions on FGM plate failure behaviours. This review focuses on analysing how XFEM determines fracture behaviours and propagating cracks in FGM plates. This study details advancements in XFEM technology and explains its implementation challenges when studying FGMs. XFEM performance needs testing against other numerical approaches for determining its accuracy in predicting fracture properties. The writer highlights the need for enhanced computer models as the final point in describing the research paths ahead. In addition to its theoretical significance, fracture modelling of FGM plates has substantial practical implications in the design of high-performance structural components. FGMs are typically characterized by a continuous gradation of material properties, such as Young's modulus, thermal conductivity, and fracture toughness, often following exponential or power-law distributions. This gradation minimizes stress concentrations at interfaces and enhances resistance to thermal and mechanical loading. However, it also complicates crack-tip stress field evaluation, making conventional fracture mechanics approaches insufficient for accurate prediction.

Keywords: Functionally graded materials, XFEM, probabilistic analysis, fracture mechanics, monte carlo simulation

INTRODUCTION

Industrial engineers create Functionally Graded Materials that have their features dynamically changing through their entire volume between two limits. The property grading system of FGMs enhances operational durability under mechanical thermal and chemical conditions which makes them highly beneficial for aerospace applications along with mechanical and biomedical engineering applications [1, 2].

The fracture behavior of FGMs remains difficult to predict because their heterogeneous composition leads to unique material characteristics spread throughout the material structure. Extended Finite Element (XFEM) serves as an improved method

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over standard FEM which enables effective studying of crack propagation without requiring mesh adjustments [3, 4]. The evaluation of structural reliability needs probabilistic fracture analysis due to real-life variations found in material properties and loads [5].

In addition to their graded microstructure, FGMs are often manufactured using advanced processing techniques such as powder metallurgy, centrifugal casting, additive manufacturing, and thermal spraying. These fabrication methods allow controlled variation in volume fractions of constituent materials, thereby tailoring mechanical stiffness, thermal resistance, and fracture toughness according to specific design requirements. However, manufacturing imperfections, porosity, and residual stresses may introduce additional uncertainties that influence crack initiation and growth.

From a fracture mechanics perspective, the presence of continuous material gradients alters the stress intensity factors (SIFs) and energy release rates at the crack tip. Unlike homogeneous materials where crack paths are relatively predictable, FGMs exhibit crack deflection, branching, or curvilinear propagation due to spatial variations in elastic modulus and toughness. This behavior necessitates advanced numerical modeling approaches capable of accurately capturing discontinuities and material heterogeneity.

XFEM addresses these challenges by incorporating enrichment functions into the displacement approximation, allowing discontinuities such as cracks to be modeled independently of the mesh topology. This significantly reduces computational complexity associated with remeshing during crack growth simulations.

Furthermore, XFEM can be combined with cohesive zone models to simulate progressive damage and fracture process zones within FGM plates under mixed-mode loading conditions.

FUNCTIONALLY GRADED MATERIALS (FGMs)

FGMs are distinguished by a slow shift in composition that results in an ongoing fluctuation in characteristics such:

- Elastic modulus
- Thermal conductivity
- Poisson's ratio
- Fracture toughness

Typically, FGMs consist of a metal-ceramic combination where ceramic content enhances thermal resistance while metal improves ductility [6].

Orthotropic FGMs

The material properties of Orthotropic FGMs differ perpendicularly in three distinct directions. FGMs exhibit material behavior changes in space and direction which increases the design complexity but enables specifications for particular mechanical or thermal conditions [7].

Applications of FGMs

- *Aerospace:* Thermal barrier coatings for engines.
- *Biomechanics:* Implants and prosthetics simulating bone-like structures.
- *Mechanical Engineering:* Turbine blades and pressure vessels withstanding high thermal and mechanical stresses [8].

FRACTURE ANALYSIS IN FGMs

Fracture analysis focuses on crack initiation and propagation within FGMs. Due to varying material properties, stress fields near crack tips are non-uniform, altering crack growth paths compared to homogeneous materials [9].

Static Loading Conditions

- *Mode I:* Opening of Tensile crack
- *Mode II:* In-plane shear
- *Mixed-mode:* Combination of tensile and shear loading

Material gradation significantly influences the propagation path under these conditions [10].

EXTENDED FINITE ELEMENTS METHOD (XFEM)

XFEM makes the modelling possible by discontinuities (cracks) without requiring mesh conformations. It extends the FEM formulation using enrichment functions and is ideal for FGM fracture analysis [11].

Key Advantages

- **No remeshing required**
- **Captures curved crack paths**
- **Handles continuously varying material properties**

Intensity Factors of Stress (SIFs)

SIFs are crucial fracture parameter computed accurately by XFEM for FGMs, considering the gradient-induced stress field variations near crack tips [12].

PROBABILISTIC FRACTURE ANALYSIS

Material and load uncertainties necessitate probabilistic methods to achieve realistic predictions in fracture analysis [13].

Uncertainty Sources

- Material gradation inconsistencies
- Variations in initial crack size and geometry
- Loading fluctuations

Monte Carlo Simulation

Monte Carlo Simulation (MCS) handles variability by simulating multiple input scenarios using random sampling. It computes:

- Probability of crack initiation
- Crack growth path probabilities
- Failure probabilities under static loads [14]

XFEM-Based Simulation Procedure

1. *Material Property Definition:* Model Young's modulus, toughness, etc., across the thickness.
2. *Crack Modeling:* Initialize cracks and apply XFEM under static conditions.
3. *Probabilistic Inputs:* Introduce uncertainty through distributions (e.g., Gaussian).
4. *Simulation:* Perform multiple MCS runs to assess variability.
5. *Output Analysis:* Examine SIFs, crack paths, energy release rates, and failure probabilities [15].

Application Scenarios

- *Aerospace:* FGM coatings modeled under complex thermal and stress conditions.
- *Mechanical Engineering:* Turbine and pressure components assessed for structural integrity.
- *Biomechanics:* Simulate fatigue and fracture in implants under physiological conditions [16].

STOCHASTIC METHODS FOR FRACTURE ANALYSIS OF FGM PLATES

The analysis of fractures in FGMs becomes uncertain due to changes in material placement, crack geometry and load distribution. Through stochastic methods researchers can integrate these uncertain aspects while obtaining probabilistic information about crack behaviors. The evaluation techniques for

FGMs use Response Surface Methods (RSM), Polynomial Chaos Expansion (PCE), Perturbation Methods alongside Monte Carlo Simulation (MCS).

Monte Carlo Simulation

The implementation of MCS includes generating random test input samples for unknown parameters such as Young's modulus and crack size to run successive fracture simulations. Random variable definition follows steps that incorporate sampling methods (for instance LHS) with XFEM-based simulations and statistical computation.

- *Advantages:* High flexibility; handles complex nonlinear problems
- *Disadvantages:* Computationally intensive with slow convergence [17]

Perturbation Methods

Taylor series approximation calculates system responses when input parameters show small variations through these procedures. The system sensitivity measures are calculated using first- and second-order methods to determine mean and variance outputs.

- *Advantages:* Computationally efficient; suitable for small uncertainties
- *Disadvantages:* Not accurate for large variations or nonlinear systems [18]

Polynomial Chaos Expansion (PCE)

PCE expresses uncertain inputs and responses using orthogonal polynomials (e.g., Hermite, Legendre). It computes statistical measures efficiently for smooth systems.

- *Advantages:* High accuracy with fewer simulations
- *Disadvantages:* Complex implementation; less effective for highly nonlinear behavior [19]

Response Surface Methods (RSM)

RSM approximates fracture response using polynomial regression models built from limited simulations (e.g., DOE). Probabilistic analysis is then performed on the surrogate model.

- *Advantages:* Simple and computationally cheap
- *Disadvantages:* Accuracy depends on the surface fit; limited for nonlinear systems [20, 21]

XFEM Formulation for Fracture Analysis of Orthotropic or Laminated Composite Plate: ***Deterministic and Probabilistic Approaches***

The general formulation for fracture analysis of orthotropic material provided by Mohammadi [6] is applied in this study. Examine an orthotropic material that has been broken, where the local polar coordinate (r,θ) and global Cartesian coordinates (X,Y) correspond with the axes of elastic symmetry (x_1=x,y_1=y) on the crack tip, as shown in Figure 1. In the presence of generic displacement and traction boundary conditions, the body is exposed to arbitrary forces.

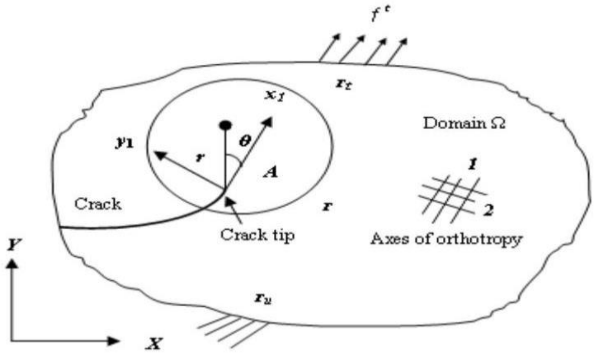

Figure 1. shows an arbitrary orthotropic body with a crack that is subject to traction f^t. Its global Cartesian coordinates are X, Y, and r, θ, while its internal area A has arbitrary boundary conditions and the fracture tip is encircled by contour r.

An orthotropic strain-stress relationship's general form is as follows:

$$\varepsilon_i = S_{ij}\sigma_j \ (i,j = 1,2,6) \tag{1}$$

where the pertinent coefficients of the orthotropic material's compliance in the x_1 and y_1 directions are S_ij (i,j=1,2,6). The definition of the stress-strain equations is

$$\sigma_{x_1} = S_{11}\frac{\partial u_{x_1}}{\partial x_1} + S_{12}\frac{\partial v_{y_1}}{\partial y_1} \tag{2}$$

$$\sigma_{y_1} = S_{12}\frac{\partial u_{x_1}}{\partial x_1} + S_{22}\frac{\partial v_{y_1}}{\partial y_1} \tag{3}$$

$$\tau_{x_{x_1}} = S_{66}\left(\frac{\partial u_{x_1}}{\partial x_1} + \frac{\partial v_{y_1}}{\partial y_1}\right) \tag{4}$$

The set of equations for an in-plane electrostatic issue may now be expressed as follows:

$$\frac{\partial^2 u}{\partial x_1^2} + \left(\frac{S_{66}}{S_{11}}\right)\frac{\partial^2 u}{\partial y_1^2} + 2\left(\frac{S_{12}+S_{66}}{2S_{11}}\right)\frac{\partial^2 v}{\partial x_1 \partial y_1} = 0 \tag{5}$$

$$\frac{\partial^2 v}{\partial x_1^2} + \left(\frac{S_{22}}{S_{66}}\right)\frac{\partial^2 v}{\partial y_1^2} + 2\left(\frac{S_{12}+S_{66}}{2S_{66}}\right)\frac{\partial^2 u}{\partial x_1 \partial y_1} = 0 \tag{6}$$

Using compatibility and equilibrium assumptions, the characteristic equation of the controlling fourth-order partial differential equation is as follows:

$$S_{11}\mu^4 - 2S_{16}\mu^3 + (2S_{12} + S_{66})\mu^2 - 2S_{26}\mu + S_{22} = 0 \tag{7}$$

In conjugate pairings, μ_1,¯(μ_1) and μ_2,¯(μ_2) occur, and the roots of Eq. (7) are always complex or entirely imaginary (μ_k=μ_(kx_1)+iμ_(ky_1) (k=1,2)). The two-dimensional displacement and stress fields near the crack tip were evaluated using complex variable methods and analytical functions, represented as $z_k = x_1 + \mu_k y_1$for $k = 1,2$.

For pure mode I, the stress components are described as follows:

$$\sigma_{11}^I = \frac{K_I}{\sqrt{2\pi r}}\mathrm{Re}\left[\frac{\mu_1\mu_2}{\mu_1-\mu_2}\left(\frac{\mu_2}{\sqrt{\cos\,\theta+\mu_2\sin\,\theta}} - \frac{\mu_1}{\sqrt{\cos\,\theta+\mu_1\sin\,\theta}}\right)\right] \tag{8}$$

$$\sigma_{22}^I = \frac{K_I}{\sqrt{2\pi r}}\mathrm{Re}\left[\frac{1}{\mu_1-\mu_2}\left(\frac{\mu_1}{\sqrt{\cos\,\theta+\mu_2\sin\,\theta}} - \frac{\mu_2}{\sqrt{\cos\,\theta+\mu_1\sin\,\theta}}\right)\right] \tag{9}$$

$$\tau_{12}^I = \frac{K_I}{\sqrt{2\pi r}}\mathrm{Re}\left[\frac{\mu_1\mu_2}{\mu_1-\mu_2}\left(\frac{1}{\sqrt{\cos\,\theta+\mu_1\sin\,\theta}} - \frac{1}{\sqrt{\cos\,\theta+\mu_2\sin\,\theta}}\right)\right] \tag{10}$$

and the displacements are

$$u^I = K_I\sqrt{\frac{2r}{\pi}}\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left[\mu_1 p_2\sqrt{\cos\,\theta + \mu_2\sin\,\theta} - \mu_2 p_1\sqrt{\cos\,\theta + \mu_1\sin\,\theta}\right]\right\} \tag{11}$$

$$v^I = K_I\sqrt{\frac{2r}{\pi}}\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left[\mu_1 q_2\sqrt{\cos\,\theta + \mu_2\sin\,\theta} - \mu_2 q_1\sqrt{\cos\,\theta + \mu_1\sin\,\theta}\right]\right\} \tag{12}$$

$$w^I = 0 \tag{13}$$

The same definition applies to the stress and displacement components for pure mode II, which are as follows.

The stresses that are

$$\sigma_{11}^{II} = \frac{K_{II}}{\sqrt{2\pi r}} \mathrm{Re}\left[\frac{1}{\mu_1-\mu_2}\left(\frac{\mu_2^2}{\sqrt{\cos\,\theta+\mu_2\sin\,\theta}} - \frac{\mu_1^2}{\sqrt{\cos\,\theta+\mu_1\sin\,\theta}}\right)\right] \tag{14}$$

$$\sigma_{22}^{II} = \frac{K_{II}}{\sqrt{2\pi r}} \mathrm{Re}\left[\frac{1}{\mu_1-\mu_2}\left(\frac{1}{\sqrt{\cos\,\theta+\mu_2\sin\,\theta}} - \frac{1}{\sqrt{\cos\,\theta+\mu_1\sin\,\theta}}\right)\right] \tag{15}$$

$$\tau_{12}^{II} = \frac{K_{II}}{\sqrt{2\pi r}} \mathrm{Re}\left[\frac{1}{\mu_1-\mu_2}\left(\frac{\mu_1}{\sqrt{\cos\,\theta+\mu_1\sin\,\theta}} - \frac{\mu_2}{\sqrt{\cos\,\theta+\mu_2\sin\,\theta}}\right)\right] \tag{16}$$

and displacement are

$$u^{II} = K_{II}\sqrt{\frac{2r}{\pi}}\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left[p_2\sqrt{\cos\,\theta+\mu_2\sin\,\theta} - p_1\sqrt{\cos\,\theta+\mu_1\sin\,\theta}\right]\right\} \tag{17}$$

$$v^{II} = K_{II}\sqrt{\frac{2r}{\pi}}\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left[q_2\sqrt{\cos\,\theta+\mu_2\sin\,\theta} - q_1\sqrt{\cos\,\theta+\mu_1\sin\,\theta}\right]\right\} \tag{18}$$

$$w^{II} = 0 \tag{19}$$

where p_k,q_k can be stated as follows: re indicates the real portion of the statement, and K_I and K_II are stress intensity factors for modes I and II, respectively.

$$p_k = S_{11}\mu_k^2 + S_{12} - S_{16}\mu_k \tag{20}$$

$$q_k = S_{12}\mu_k + \frac{S_{22}}{\mu_k} - S_{26} \tag{21}$$

CONCLUSION

This analysis concludes by emphasizing the value of applying the Extended Finite Element Method (XFEM) for predictable and probabilistic fracture analysis of Functionally Graded Material (FGM) plates under static loading conditions. FGMs, with their spatially varying material properties, pose unique challenges in crack modeling, particularly in orthotropic and laminated configurations. XFEM effectively simulates crack propagation without remeshing and accurately evaluates stress intensity factors (SIFs). To account for real-world uncertainties in material gradation, crack geometry, and loading, stochastic methods such as Monte Carlo Simulation (MCS), Perturbation Methods, Polynomial Chaos Expansion (PCE), and Response Surface Methods (RSM) are employed. These approaches enable a more reliable prediction of fracture behavior and support the robust design of FGM components in aerospace, mechanical, and biomedical applications.

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