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Harmonically Varying Moving Load on time Dependent Uniform Beam Restng on Pasternak Foundation

Jimoh A.[1], A. Joge E.O.[2,*], Ajiola D.I.[3], Oni D.I.[4], Abiola Kolawole.[5], Adesanmi O.A.[6]

Abstract

In this paper, we investigated elastic beam whose properties does not varies with spatial coordinate but constant along the span L of the beam. The moving load considered in this work is harmonically varying moving load with non-classical boundary conditions, time dependent boundary conditions in particular. Also, considered in this work is two parameters foundation which are Winkler and Pasternak foundations. Closed form solutions in plotted form are obtained using Mindlin Goodman method, Fourier series transformation, integral transformation and theorem of convolution. Mindlin Goodman method is used to transformed the non-homogeneous boundary conditions to homogeneous boundary conditions, the Fourier series transformation is used to reduce the fourth order non-homogeneous partial differential equation with singular coefficient describing the dynamical system to second order ordinary differential equation. The resulting equation is solved using integral transformation alongside with the theorem of convolution. The results are presented graphically and it was revealed that, as the structural parameters such as Winkler foundation parameter (K), Pasternak foundation parameter (G), damping coefficient (E), axial force (N), and circular frequency (w), increases, the response amplitude of the time dependent uniform elastic beam reduces. Amongst all the structural parameters, shear modulus (G) and axial force (N) gives more noticeable effects compares to other structural parameters. Higher values of G and N are required in order to reduce the resonance effect in the dynamical system and this could guarantee the safety of lives.

***Author for Correspondence**
A. Joge E.O.
E-mail: eajoge@oauife.edu.ng

[1]Research Professor, Department of Mathematics and Statistics, Conference University of Science and Technology, Osara, Kogi State, Nigeria
[2]Research Scholar, Centre for Energy Research and Development, Obafemi Awolowo University, Ile-Ife, Osun State, Nigeria.
[3]Research Scholar, Department of Biochemistry, Chemistry, and Physics. College of Science and Mathematics, Georgia Southern University, 1332 Southern Drive Statesboro, GA 30458. USA.
[4]Research Scholar, Department of Informatics and Computer Engineering, Vietnam National University Internation School, Vietnam.
[5]Research Scholar, Department of Chemical Engineering, University of Hull, United Kingdom.
[6]Research Scholar, Department of Mathematics, Obafemi Awolowo University, Ile-Ife, Nigeria.

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INTRODUCTION

The dynamic behaviour of elastic structures such as beams, plates, and shells on elastic foundation subjected to moving load such as railroad rails to moving trains, response of bridges and elevated roadways to moving vehicles, rudimentary surfaces for aircraft and guided missiles etc has been investigated by many researchers in the field of mathematical physics, physics and applied mathematics. As a result of modern trends toward higher speed in the field of railway engineering in particular so as to accurately predicted the behaviour of the railway track has further intensify the researchers in these areas of research. In most of the studies, only one parameter foundation model [1–11] was considered. The one parameter foundation consists of closely-spaced linear strings subjected to a moving load and it has a short coning due to the independent of its vertical springs which allows unloaded parts in the springs. It does not take into account the bending deformation of the foundation itself.

To overcome the short coming of the one parameter foundation model is by introducing some kind of interaction between the independent springs with constant parameter which characterizes the interaction between the springs. This is called two parameter foundations (Pasternak foundation). The two parameters foundation model was proposed by Pasternak [12]. In view of this, Baran [13] used transfer-matrix method to obtained solution to dynamic response of simply supported Timoshenko beam resting on two parameter foundations subjected to moving load. He concluded from his findings that, the response amplitude of the one parameter foundation is slightly higher than the two parameter foundations. His finding also revealed that, the one parameters foundation model has lower natural frequency when compared with two parameter foundation model. Free vibration analysis of clamped-clamped and simply supported beams resting on two parameter foundations with axial materials graduation was investigated by Saurabh [14]. The governing equation describing the dynamical system was derived using Timoshenko beam theory and energy method alongside with Hamilton's principle. The effects of materials model, foundation parameter and boundary conditions on the dynamic behaviour of the beam was considered in his work. It was revealed from his findings that, the effects of other parameters significantly reduce as the values of Winkler foundation parameter increases. The dynamic response of thick beam resting on bi-parametric foundation subjected to moving load was carried out by Jimoh [15]. Modal asymptotic analysis and Strubbles asymptotic technique was used to obtain a closed form solution to the dynamical problem. The effects of prestress, foundation parameters and moving velocity on the dynamic response of the beam was considered. His results revealed that, the foundation stiffness and the excitation frequency affected the critical speed of the dynamical system and that the deformation shape of the beam strongly depend on the speed of the moving load.

In a very recent development, Giogio *et al* [16] used finite element modelling of a beam resting on an elastic foundation. They based their method on variable reduction approach. They considered Winkler foundation model, two parameter foundation model and Hetenyi foundation model. Jimoh *et al* [17] investigated the effect of two parameters foundation on elastic beam subjected to concentrated load with clamped-clamped ends conditions. They employed generalize integral transform approach alongside with a series representation of Dirac-Delta function, asymptotic method, integral transformation and convolution integral to obtain results in plotted graphs. They concluded from their findings that, the effect of shear modulus on the response amplitude of the beam is more noticeable compared to foundation stiffness and also resonance effect reached earlier in the moving mass problem than the moving force problem. Some of the other researchers that considered two parameters foundation in their works are: Lucas *et al* [18], Jimoh and Ajoge [19], Ajijola [20], Hammed et al [21], Awodola et al [22] to mention but a few.

It is interesting at this juncture to note that, all the aforementioned researchers considered only the classical boundary conditions such as simply supported, clamped-clamped, and clamped-free boundary conditions which are homogeneous in nature where as in real life situation, the boundary conditions are not homogeneous. It happened frequently that, problem that has to do with vibration deal with continuous system whose one or more boundaries are not stationeries but undergoes sort of displacement which varies with time. Such boundaries conditions are called non-classical boundary conditions which are time dependent boundary conditions and elastically supported boundary conditions. Research work involving non-classical boundary conditions, time dependent in particular where moving load is considered to be harmonically remained outstanding in the literature. This is because of the difficulties involved in finding solution to the governing equations describing such dynamical system.

In the present analysis, harmonically varying moving load on uniform elastic beam resting on two parameter foundations with time dependent boundary conditions has been investigated.

GOVERNING EQUATION

The dynamic response $V(x,t)$ of uniform elastic beam resting on two parameters foundation subjected to harmonically varying moving load with time dependent boundary conditions is governed by Frybal [23].

$$EI\frac{\partial^4 V(x,t)}{\partial x^4} + \mu\frac{\partial^2 V(x,t)}{\partial t^2} - N\frac{\partial^2 V(x,t)}{\partial x^2} + \alpha\frac{\partial V(x,t)}{\partial t} + KV(x,t) - G\frac{\partial^2 V(x,t)}{\partial x^2} = Pcos\gamma t\,\delta(x-ct) \quad (1)$$

Where

EI= flexural rigidity of the structures, μ = mass per unit length of the beam

K = foundation modulus, G = Shear modulus, N= axial force,

$Alpha$ (α) = damping coefficient, E= young's modulus, I = moment of inertia,

$V(x,t)$= transverse displacement, x = spatial coordinate and t = time coordinate

$Omega$ (γ) = circular frequency of the moving load. c = velocity of the moving load.

The boundary conditions corresponding to equation (1) are taken to be time dependent.

Thus, at each of the boundary points, there are two boundary conditions given as

$$D_i[V(0,t)] = f_i(t), i = 1,2 \quad (2)$$

$$D_i[V(L,t)] = f_i(t), i = 1,2 \quad (3)$$

Where D_i are differential operators of the order less than or equal to three.

For example, if the beam in question has simple support at both ends, then

$$D_1 = 1, \quad D_2 = \frac{\partial^2}{\partial x^2}, \quad D_3 = 1 \qquad D_4 = \frac{\partial^2}{\partial x^2} \quad (4)$$

The initial conditions of the motion are specified by the functions

$$V(x,0) = 0 = \frac{\partial V(x,0)}{\partial t} \quad (5)$$

METHODS OF SOLUTION

In order to solve the above initial boundary valued problem, one introduces the auxiliary variable $U(x,0)$ in the form

$$V(x,0) = \ U(x,0) + \sum_{i=0}^{4} f_i(t)\, g_i(x) \quad (6)$$

The substitution of (6) into the boundary valued problem (1), (2) and (3) transform the latter to a boundary valued problem interms of $U(x,0)$. The functions $g_i(x)$ are to be chosen so as to render the boundary conditions for the boundary valued problem in $U(x,0)$ homogeneous [24].

Substituting (6) into (1) after simplification yields

$$EI\frac{\partial^4 V(x,t)}{\partial x^4} + \mu\frac{\partial^2 V(x,t)}{\partial t^2} - N\frac{\partial^2 V(x,t)}{\partial x^2} + \alpha\frac{\partial V(x,t)}{\partial t} + KV(x,t) - G\frac{\partial^2 V(x,t)}{\partial x^2} = \ Mgcos\gamma t\,\delta(x-ct) \quad - \sum_{i=1}^{4}\left\{\begin{matrix} f_i(t)EI\frac{\partial^4 g_i(x)}{\partial x^4} + \mu g_i(x)\ddot{f}_i(\mathrm{t}) + \sum g_i(x)\dot{f}_i(t) \\ f_i(t)N\frac{\partial^2 g_i(x)}{\partial x^2} \ + f_i(t)Kg_i(x) - \ f_i(t)g\frac{\partial^2 g_i(x)}{\partial x^2} \end{matrix}\right\} \quad (7)$$

Where

$$\mathrm{P} = \mathrm{Mg} \quad (8)$$

Now, the assumed expression (6) must satisfy the boundary conditions (2) and (3).

Thus,

$$D_i[U(0,t)] + \sum_{j=0}^{4} f_i(t)\, D_i[g_i(0)] = f_i(t),\ i = 1,2 \tag{9}$$

$$D_i[U(L,t)] + \sum_{j=0}^{4} f_i(t)\, D_i[g_i(L)] = f_i(t),\ i = 3,4 \tag{10}$$

Substituting (6) into the initial conditions (5) to obtains

$$U(x,0) = -\sum_{i=0}^{4} f_i(0)\, g_i(x) \tag{11}$$

$$\frac{\partial U(x,0)}{\partial t} = -\sum_{i=0}^{4} f_i(0)\, g_i(x) \tag{12}$$

Using the Mindlin-Goodman method, the boundary conditions (9) and (10) in terms of $U(x,t)$ can be made homogeneous if the function $g_i(x)$ are chosen so that the conditions given by

$$D_i\left[g_j(0)\right] = \delta_{ij}(i = 1,2 \qquad j = 1,2,3,4) \tag{13}$$

$$D_i\left[g_j(L)\right] = \delta_{ij}(i = 1,2 \qquad j = 1,2,3,4) \tag{14}$$

are satisfied.

Where

$$\delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \tag{15}$$

Using equations (13) and (14) in the nonhomogeneous boundary conditions (9) and (10) to obtains the homogeneous boundary conditions as

$$D_i[U(0,t)] = 0,\ i = 1,2 \tag{16}$$

$$D_i[U(L,t)] = 0,\ i = 1,2 \tag{17}$$

The original problem now reduces to that of solving the nonhomogeneous partial differential equation (7) subject the homogeneous boundary conditions (16) and (17) with initial conditions (11) and (12).

To solve the initial boundary valued problem (7), (16) and (17) above, one employed the method of generalized finite integral transform defined by

$$U(m,t) = \int_0^L U(x,t) \sin\frac{m\pi x}{L}\, dx, m = 1,2,3 \tag{18a}$$

With the inverse

$$U(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} U(m,t) \sin\frac{m\pi x}{L} \tag{18b}$$

Now, employing (18a) into (7) after some simplification and rearrangement to obtains

$$\mu U_{tt}(m,t) + \alpha U_t(m,t) + \left(\alpha_a^2 + \alpha_b^2 + \alpha_c^2 + K\right) U(x,t) = Mg cos\gamma t\, \delta(x - ct)$$
$$\left[Q_a(t) + Q_b(t) + Q_c(t) + Q_d(t) + Q_e(t) + Q_f(t)\right] \tag{19}$$

Where

$$\alpha_a^2 = EI\frac{m^4\pi^4}{L^2}, \qquad \alpha_b^2 = N\frac{m^2\pi^2}{L^2}, \qquad \alpha_c^2 = G\frac{m^2\pi^2}{L^2} \tag{20}$$

$$Q_a(t) = \sum_{i=1}^{4} EI f_i(t) \int_0^L \frac{\partial^4 g_i(x)}{\partial x^4} Sin\frac{m\pi x}{L}\, dx \tag{21a}$$

$$Q_b(t) = \sum_{i=0}^{4} \mu \ddot{f}_i(\mathrm{t}) \int_0^L g_j(x) Sin\frac{m\pi x}{L}\, dx \tag{21b}$$

$$Q_c(t) = \sum_{i=1}^{4} E\, \dot{f}_i(t) \int_0^L g_j(x) Sin\frac{m\pi x}{L}\, dx \tag{21c}$$

$$Q_d(t) = \sum_{i=1}^{4} N f_i(t) \int_0^L \frac{\partial^2 g_i(x)}{\partial 2} Sin\frac{m\pi x}{L}\, dx \tag{21d}$$

$$Q_e(t) = \sum_{i=1}^{4} K\, f_i(t) \int_0^L g_j(x) Sin \frac{m\pi x}{L}\, dx \tag{21e}$$

$$Q_f(t) = \sum_{i=1}^{4} G f_i(t) \int_0^L \frac{\partial^2 g_i(x)}{\partial 2} Sin \frac{m\pi x}{L}\, dx \tag{21f}$$

Evaluating (21a-21f) and substituting the results into (19), after some simplification and rearrangement to obtains

$$U_{tt}(m,t) + A_1 U_t(m,t) + A_2 U(m,t) = \frac{Mg}{\mu} cos\gamma t\ Sin \frac{m\pi ct}{L} - (N_1 + N_2 + N_3) \tag{22}$$

Where

$$A_1 = \frac{E}{\mu}, \qquad A_2 = \frac{\alpha_a^2 + \alpha_b^2 + \alpha_c^2 + K}{\mu}, \qquad A_3 = \frac{K}{\mu} \tag{23a}$$

$$N_1 = \ddot{f}_i(\mathrm{t})\ (I_1 - \frac{I_2}{L}) + \dot{f}_3(t) \frac{I_2}{L} \tag{23b}$$

$$N_2 = A_1 \left(\dot{f}_1(t)(I_1 - \frac{I_2}{L}) + \dot{f}_3 \frac{I_2}{L} \right) \tag{23c}$$

$$N_3 = A_3 \left(f_i(t)(I_1 - \frac{I_2}{L}) + f_3(t) \frac{I_2}{L} \right) \tag{23d}$$

$$I_1 = \frac{L}{m\pi}(1 - cosm\pi) \tag{23e}$$

$$I_2 = \frac{L^2}{m\pi}((-1)^{m+1}) \tag{23f}$$

Equation (22) is the transformed equation of time dependent uniform elastic beam on two parameters Pasternak foundation subjected to harmonically varying moving load.

Analysis of the Transformed Equation

Recall that we can write

$$f_1(t) = Sin\Omega \mathrm{t} \ \text{ and } \ f_3(t) = e^{-\beta t} Sin\Omega \mathrm{t} \tag{24}$$

Where Ω and β are frequency and parameter respectively.

Differentiating (24) with respect to t to obtain

$$\left.\begin{array}{c} \dot{f}_1(t) = \Omega Cos\Omega \mathrm{t}, \qquad \ddot{f}_1(\mathrm{t}) = -\Omega^2 Sin\Omega \mathrm{t} \\ \dot{f}_3(t) = -\beta e^{-\beta t} Sin\Omega \mathrm{t} + \Omega e^{-\beta t} Cos\Omega \mathrm{t} \\ \ddot{f}_3(\mathrm{t}) = \beta^2 e^{-\beta t} Sin\Omega \mathrm{t} - 2\beta\Omega e^{-\beta t} Cos\Omega \mathrm{t} - \Omega^2 e^{-\beta t} Sin\Omega \mathrm{t} \end{array}\right\} \tag{25}$$

Using equation (25) in equation (22) yields

$$U_{tt}(m,t) + A_1 U_t(m,t) + A_2 U(m,t) = \frac{Mg}{\mu} cos\gamma t\ Sin \frac{m\pi ct}{L}$$

$$-\left\{ \begin{array}{c} -\mathrm{C_{f1}} \Omega^2 Sin\Omega \mathrm{t} + \mathrm{C_{f2}} \left(\beta^2 e^{-\beta t} Sin\Omega \mathrm{t} - 2\beta\Omega e^{-\beta t} Cos\Omega \mathrm{t} - \Omega^2 e^{-\beta t} Sin\Omega \mathrm{t} \right) \\ + A_1 \left[\mathrm{C_{f1}} \Omega Cos\Omega \mathrm{t} + \mathrm{C_{f2}} \left(\Omega e^{-\beta t} Cos\Omega \mathrm{t} - \beta e^{-\beta t} Sin\Omega \mathrm{t} \right) \right] \\ A_3 (\mathrm{C_{f1}} Sin\Omega \mathrm{t} + \mathrm{C_{f2}} e^{-\beta t} Sin\Omega \mathrm{t}) \end{array} \right\} \tag{26}$$

Taking the Laplace transform of (26) with initial conditions (5), after some simplification and rearrangement to obtains

$$\bar{U}(m,s) = \frac{1}{r_1 - r_2} \left[A_4 \left(\frac{\gamma_1}{S^2 + \gamma_1^2} - \frac{\gamma_2}{S^2 + \gamma_2^2} \right) + A_5 \frac{\Omega}{S^2 + \Omega^2} + A_6 \frac{\Omega}{(s+\Omega)^2 + \Omega^2} + A_7 \frac{\mathrm{s} + \beta}{(s+\beta)^2 + \Omega^2} + A_8 \frac{\mathrm{s}}{S^2 + \gamma^2} \right] \mathrm{x} \left[\frac{1}{s + r_2} - \frac{1}{s + r_1} \right] \tag{27}$$

Where

$$\left.\begin{array}{c} A_4 = \frac{\mathrm{Mg}}{2\mu}, A_5 = \mathrm{C}_{\mathrm{f1}}(\Omega^2 - A_3), A_6 = -\mathrm{C}_{\mathrm{f2}}(\Omega^2\beta^2 - A_3) + A_1\beta, \\ A_7 = \mathrm{C}_{\mathrm{f2}}(2\beta\Omega - A_1), A_8 = -A_1 Cf_1\Omega \\ r_1 = -\frac{1}{2}(A_1 + \sqrt{A_1^2 - 4A_1}) \quad r_2 = -\frac{1}{2}(A_1 - \sqrt{A_1^2 - 4A_1}) \end{array}\right\} \tag{28}$$

By adopting the following representation

$$F(S) = \frac{A_4\gamma_1}{S^2+\gamma_1^2} - \frac{A_4\gamma_2}{S^2+\gamma_2^2} + \frac{A_5\Omega}{S^2+\Omega^2} + \frac{A_6\Omega}{(S+\Omega)^2+\Omega^2} + \frac{A_7(S+\beta)}{(S+\beta)^2+\Omega^2} + \frac{A_8 S}{S^2+\Omega^2} \tag{29}$$

$$G_1(S) = \frac{1}{s+r_2}, \quad , \quad G_2(S) = \frac{1}{s+r_1} \tag{30}$$

The Laplace inversion of (27) can be obtained as

$$U(m,t) = \emptyset\begin{bmatrix} A_4(J_1 - J_2) + A_5J_3 + A_6J_4 + A_7J_5 + A_6J_4 + A_7J_5 \\ +A_8J_6 - A_4(J_7 - J_8) + A_5J_9 + A_6J_{10} + A_7J_{11} + A_8J_{12} \end{bmatrix} \tag{31}$$

Where after evaluation and simplification, the integrals $(J_1 - J_{12})$ becomes

$$J_1 = \frac{\gamma_1}{\gamma_1^2+r_2^2}(e^{-r_2 t} - Cos\gamma_1 t + \frac{r_2}{\gamma_1}Sin\gamma_1 t) \tag{32a}$$

$$J_2 = \frac{\gamma_2}{\gamma_2^2+r_2^2}(e^{-r_2 t} - Cos\gamma_2 t + \frac{r_2}{\gamma_2}Sin\gamma_2 t) \tag{32b}$$

$$J_3 = \frac{\gamma_2}{\Omega^2+r_2^2}(e^{-r_2 t} - Cos\Omega t + \frac{r_2}{\Omega}Sin\Omega t) \tag{32c}$$

$$J_4 = \frac{\Omega}{\Omega^2+r_2^2(1-\Omega)^2}(e^{-r_2\Omega t}\, Cos\Omega t - e^{-r_2 t} + e^{r_2\Omega t}\,\frac{r_2(1-\Omega)}{\Omega}Sin\Omega t) \tag{32d}$$

$$J_5 = \frac{\Omega}{\Omega^2+r_2^2(1-\beta)^2}(e^{r_2\beta t} + \frac{r_2(1-\beta)}{\Omega}(e^{r_2\beta t}Cos\Omega t - e^{-r_2 t})) \tag{32e}$$

$$J_6 = \frac{\Omega}{\Omega^2+r_2^2}(Sin\Omega t + \frac{r_2(1-\beta)}{\Omega}(e^{r_2\beta t}Cos\Omega t - e^{-r_2 t})) \tag{32f}$$

$$J_7 = \frac{\gamma_1}{\gamma_1^2+r_1^2}(e^{-r_1 t} - Cos\gamma_1 t + \frac{r_1}{\gamma_1}Sin\gamma_1 t) \tag{32g}$$

$$J_8 = \frac{\gamma_2}{\gamma_2^2+r_1^2}(e^{-r_1 t} - Cos\gamma_2 t + \frac{r_1}{\gamma_2}Sin\gamma_2 t) \tag{32h}$$

$$J_9 = \frac{\Omega}{\Omega^2+r_1^2}(e^{-r_1 t} - Cos\Omega t + \frac{r_1}{\Omega}Sin\Omega t) \tag{32i}$$

$$J_{10} = \frac{\Omega}{\Omega^2+r_1^2(1-\Omega)^2}(e^{r_1\Omega t}Cos\Omega t - e^{r_1 t} + \frac{r_1(1-\Omega)}{\Omega})e^{r_1\Omega t}Sin\Omega t \tag{32j}$$

$$J_{11} = \frac{\Omega}{\Omega^2+r_1^2(1-\beta)^2}(e^{r_1\beta t} + \frac{r_1(1-\beta)}{\Omega}(e^{r_1\beta t}Cos\Omega t - e^{r_1 t})) \tag{32k}$$

$$J_{12} = \frac{\Omega}{\Omega^2+r_1^2}(Sin\Omega t + \frac{r_1}{\Omega}(Cos\Omega t - e^{r_1 t})) \tag{32L}$$

Putting (31) into (18b) to obtains

$$U(x,t) = \frac{2}{L}\sum\sum_{m=1}^{\infty}\emptyset\begin{bmatrix} A_4(J_1 - J_2) + A_5J_3 + A_6J_4 + A_7J_5 + A_6J_4 + A_7J_5 \\ +A_8J_6 - A_4(J_7 - J_8) + A_5J_9 + A_6J_{10} + A_7J_{11} + A_8J_{12} \end{bmatrix}\sin\frac{m\pi x}{L} \tag{33}$$

Equation (33) is the response amplitude of time dependent uniform elastic beam resting on pasternak foundation subjected to harmonically varying moving load.

DISCUSSION OF RESULTS

The responses of the uniform time dependent elastic beam resting on two parameters foundation and subjected to harmonically varying moving load for various values of Winkler foundation parameter (K), Pasternak foundation parameter (G), axial force (N), damping coefficient (alpha), and circular frequency (omega) of the moving load are shown in the Figures (1-5).

Figures 1 depicts decreases in the response amplitude of the beam subjected to harmonic moving load as the values of Pasternak foundation parameter increases. The deflection profile of the beam subjected to harmonic moving load is shown in Figure 2 and it decreases as the values of the Winkler foundation parameter increases.

Figures 3, 4 and 5 shows the response amplitudes of the beam subjected to harmonic moving load for various values of the damping coefficient, axial force, and circular frequency respectively. From the figures, it is observed that increasing the values of each of the parameters lead to decreases in the response amplitudes of the beam.

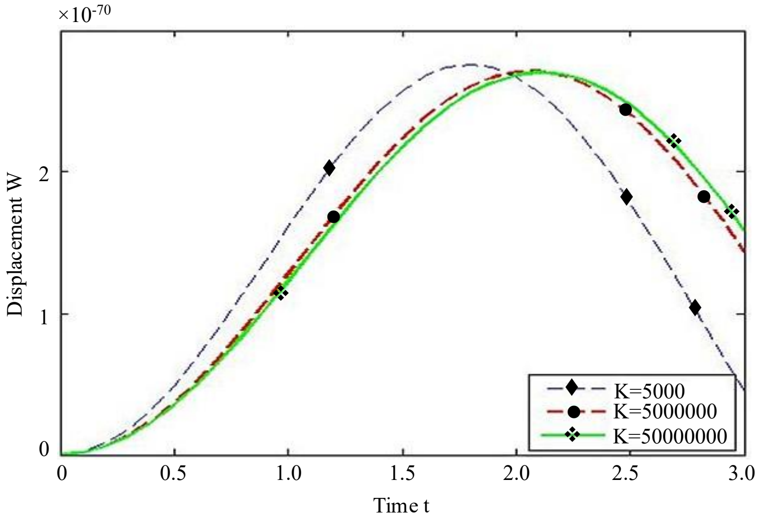

Figure 1. Response amplitude of time dependent uniform elastic beam resting on Pasternak foundation subjected to harmonically varying moving load with varying foundation modulous (K).

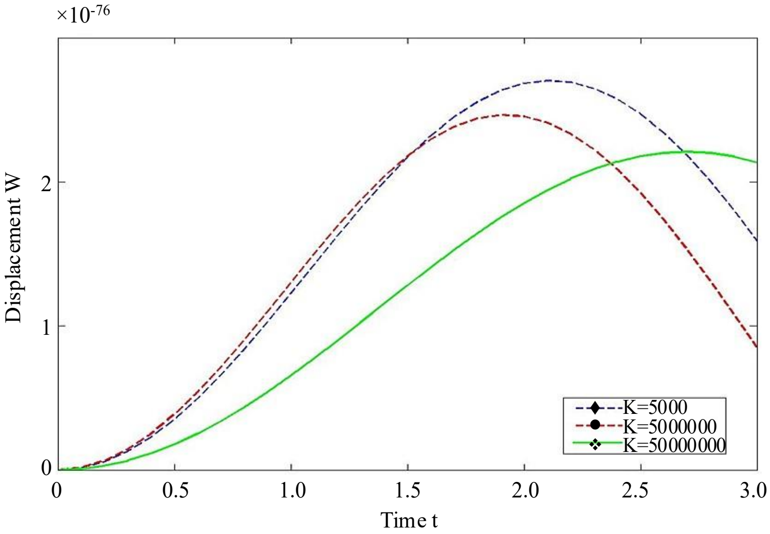

Figure 2. Response amplitude of time dependent uniform elastic beam resting on Pasternak foundation subjected to harmonically varying moving load with varying shear modulous (G).

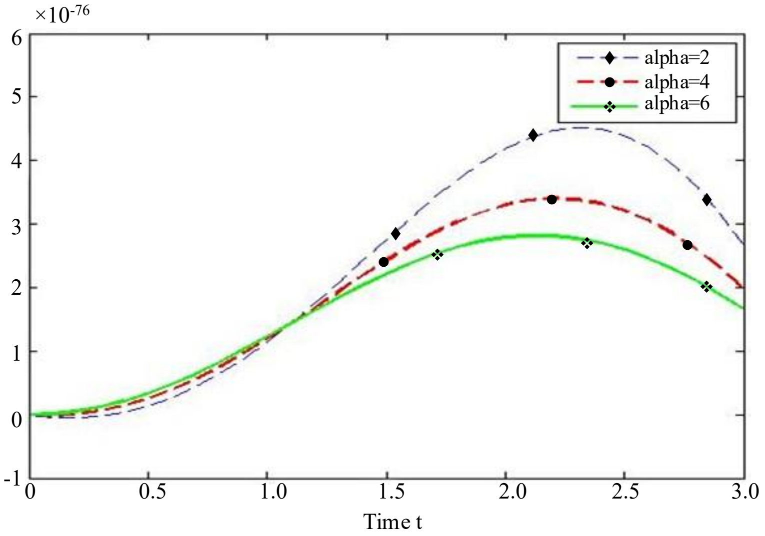

Figure 3. Response amplitude of time dependent uniform elastic beam resting on Pasternak foundation subjected to harmonically varying moving load with varying damping coefficient (alpha)

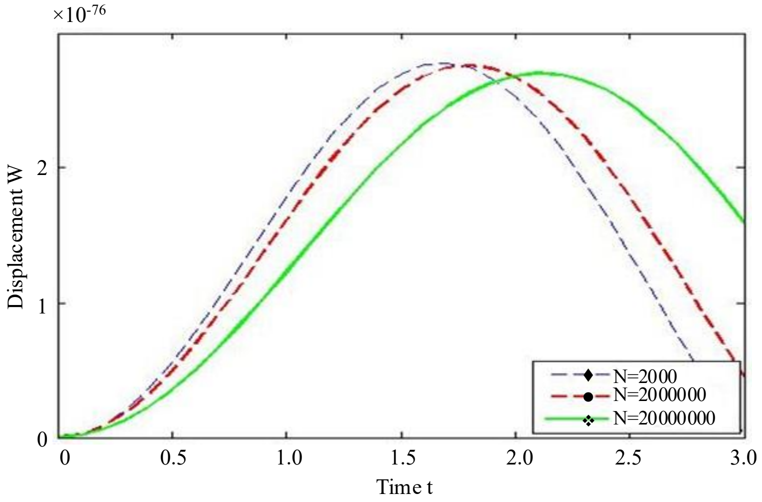

Figure 4. Response amplitude of time dependent uniform elastic beam resting on Pasternak foundation subjected to harmonically varying moving load with varying axial force (N),

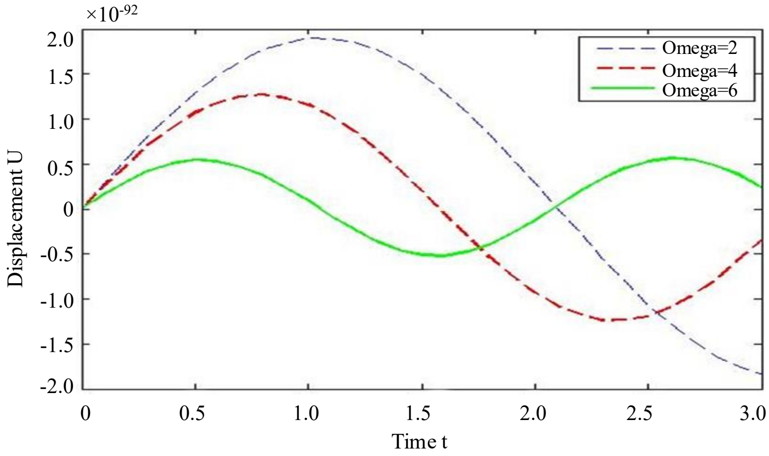

Figure 5. Response amplitude of time dependent uniform elastic beam resting on Pasternak foundation subjected to harmonically varying moving load with varying circular frequency (omega)

CONCLUSION

The dynamic analysis of uniform time dependent elastic beam resting on two parameters foundation subjected to harmonically varying moving load using Mindlin Goodman method, Fourier series transformation, Laplace transformation and theory of convolution has been investigated. Also, taking into consideration in the governing equation describing the dynamical system is the effect of damping.

The effects of the structural parameters on the beam has also been evaluated as shown in the graphs and it was observed that, for more noticeable effect, higher values of the Pasternak foundation parameter and Winkler foundation parameter are required. It was seen from the results that, damping coefficient and circular frequency of the moving load has more significant effects on the beam subjected to harmonically varying moving load compared to other structural parameters and these are shown in Figures 3 and 5 respectively.

Finally, higher values of the structural parameters especially damping coefficient and circular frequency of the moving load are required so that the resonance effects on the dynamical system could be reduced drastically and this could in effect guarantee the safety of life and properties.

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