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Intimate Mappings in Interval-Valued Fuzzy Metric Space: A Few Fixed-Point Findings

Naval Singh[1], Umashankar Singh[2], Ruchi Singh[3]*

Abstract

The purpose of this study is to extend some previously established fixed point results for interval valued fuzzy metric space. For this objective, various contractive criteria with respect to an intimate mapping are applied. We employ intimate mapping in interval valued fuzzy metric space (IVFMS) to validate some well-known fixed point results. Our findings complement and generalize the recent findings of the common fixed point theorem for intimate mapping. The primary aim of this study is to build upon and extend the existing fixed point results within the realm of interval-valued fuzzy metric spaces (IVFMS). Fixed point theorems are crucial in various mathematical and applied fields, providing solutions to equations where a function maps a point to itself. In this research, we focus on applying diverse contractive conditions specifically in the context of intimate mappings to achieve our objectives. Intimate mapping, a relatively novel concept in IVFMS, plays a key role in our analysis. By employing this type of mapping, we are able to verify and validate several well-established fixed point results within IVFMS. This approach not only supports the known results but also broadens their applicability, thereby enhancing the theoretical foundation of fixed point theorems in fuzzy metric spaces. Our findings make significant contributions by complementing and generalizing recent discoveries related to the common fixed point theorem for intimate mappings. In essence, this study provides a deeper and more comprehensive understanding of fixed point theorems, showcasing the versatility and robustness of intimate mappings in interval-valued fuzzy metric spaces. This advancement opens new avenues for future research and potential applications in various scientific and engineering disciplines where fuzzy metric spaces are utilized.

Keywords: Interval-valued fuzzy metric, fixed point, common fixed point, intimate mappings. Contractive condition

***Author for Correspondence**
Ruchi Singh
E-mail: ruchisingh0107@gmail.com

[1]Professor, Govt. Dr. SPM Science & Commerce College, Bhopal, Madhya, Pradesh, India
[2]Associate Professor, Sagar Institute of Research & Technology Excellence, Bhopal, Madhya, Pradesh India
[3]Professor Department of Mathematics, Pandit S.N. Shukla University, Shahdol, Madhya, Pradesh India

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INTRODUCTION

By 1975, Kromosil and Michalek [1]. proposed the idea of fuzzy metric space. Abu Osman [2]. employed fuzzy metric space membership functions in 1983 to determine the fuzzy metric between two fuzzy sets; this approach differed from that suggested by [3]. Fuzzy metric space was reformulated in 1994 when Veeramani and George [4] presented the revised idea of Continuous t-norm. Letter on, numerous authors and researchers have thoroughly and extensively examined a variety of issues pertaining to this space from a variety of perspectives, including compatible mapping, R-weakly computing mapping, Weak compatible mapping, CLR-Property, E.A. property, etc. and have produced new findings on fuzzy metric space. Sushil Sharma [5] first presented the idea of fuzzy 2-metric space in 2002 and proved a few fixed point theorems. Based on an interval valued membership function, Zadeh [6]. first the idea of an interval-valued fuzzy set in 1975. By inspiring the concept of compatible maps, Jungck, Murthi, and Cho presented the idea of compatible maps of type (A) in metric space and Banach space in 1993 [7]. In 2001, Sahu et al [8]. developed certain fixed point results and generalized the idea of intimate mapping in metric space. Three different types of distances between two interval-valued fuzzy sets on real line R were employed by C. Li [9]. in 2009. Using the concept of continuous interval-valued t-norm, it is possible to define interval-valued fuzzy metric space and describe the uncertainty of the distance between two points in a fuzzy metric space. V. Deshmukh et al. [10] recently established some common fixed point solutions for mapping on interval-valued fuzzy metric space that is occasionally weakly compatible. This was demonstrated in 2023 [11].

In this study, we validate certain well-known fixed point results using intimate mapping in interval valued fuzzy metric space (IVFMS). Our results generalize and extend the results of the common fixed point theorem for intimate mapping that were published recently.

Preliminaries

*Definition 2.1[6] I*n a non empty set Ʊ, a mapping $R_m\colon Ʊ \rightarrow [I]$ is called an interval- valued fuzzy set on Ʊ. Collection of all interval-valued fuzzy sets on Ʊ is denoted by $IVF(Ʊ)$. if $R_m \in IVF(Ʊ)$. , let $R_m(x) =$ for all $x \in Ʊ$, then the set ${R_m}^{-:Ʊ\rightarrow[I]\wedge {R_m}^{+:Ʊ\rightarrow[I]}}$ are called lower fuzzy set and upper fuzzy set of R_m and if $R^{-(x)={R_m}^{+(x)}}$ then is called degenerate fuzzy set for all $x \in Ʊ$.[12]

Definition 2.2[6] A binary operation of the form is an interval valued t_{norm} is $*_I\colon [I] \times [I] \rightarrow [I]$ on $[I]$ such that for all $\overline{u}, \overline{v}, \overline{w} \in [I]$ if satisfying following four conditions:

1. *Commutativity:* $\overline{u}_{ivf} *_I \overline{v}_{ivf} = \overline{v}_{ivf} *_I \overline{u}_{ivf},$
2. *Associativity:* $\left[\overline{u}_{ivf} *_I \overline{v}_{ivf}\right] *_I \overline{w}_{ivf} = \overline{u}_{ivf} *_I \left[\overline{v}_{ivf} *_I \overline{w}_{ivf}\right],$
3. *Monotonicity*: $\overline{u}_{ivf} *_I \overline{v}_{ivf} \leq \overline{u}_{ivf} *_I \overline{w}_{ivf} whenever \overline{v}_{ivf} *_I \overline{w}_{ivf},$
4. *Boundary condition*: $\overline{u}_{ivf} *_I \overline{1} = \overline{u}_{ivf}, \overline{u}_{ivf} *_I \overline{0} =$

Note: Each interval valued t_{norm}satisfies some additional boundary conditions for all $\overline{u} \in [I]$:
$\overline{u} *_I \overline{0} = \overline{0} *_I \overline{u} = \overline{0},$
$\overline{I} *_I \overline{u} = [0,1] *_I$
$\overline{1} *_I \overline{u} = \overline{1}.$

Example 2.3: (a) $\overline{u} *_I \overline{v} =$ (b)$\overline{u} *_I \overline{v} =$;

Definition 2.4[11]: Let $\{\overline{u}_i\} =$ be a sequence of interval numbers in *[I]*, $\overline{u}$ =if $lim_{i\rightarrow\infty} u_i^{-=u^-}$and $lim_{i\rightarrow\infty} u_i^{+=u^+}$ then the sequence $\{\overline{u}_i\}$ is convergent to$\overline{u}$ and denoted by$lim_{i\rightarrow\infty} \overline{u}_i = \overline{u}$.

Definition 2.5[11]: An interval valued $t_{norm} *_I$ is continuous if it is continuous in its first component, i.e for each$\overline{v} \in [I]$, if $lim_{i\rightarrow\infty} \overline{u}_i = \overline{u}$, then
$lim_{i\rightarrow\infty}(\overline{u}_i *_I \overline{v}) =$ Where $\{\overline{u}_i\} \subseteq [I], \overline{u} \in [I]$.

Definition 2.6[6]:A triplet $(X, M_{IVFMS}, *_I)$ is called interval valued fuzzy metric space *(IVFMS)* if X is an arbitrary set, $*_I$ is a continuous interval valued t_{norm} on *[I]* and $M_{IVFMS}$is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:

1. (1)$M_{IVFMS}(x, y, t_{norm}) > \overline{0}$;

2. $M_{IVFMS}(x, y, t_{norm}) = \overline{1} forallt > 0 iff x = y$;
3. $M_{IVFMS}(x, y, t_{norm}) = M(y, x, t_{norm})$;
4. $M_{IVFMS}(x, y, t_1) \quad {}_I M_{IVFMS}(y, z, t_2) \leq M_{IVFMS}(x, z, t_1 + t_2)$;
5. $\forall x, y, z \in \Xi \wedge t_1, t_2, > 0$
6. $M_{IVFMS}(x, y, *_I)$: $[0, \infty] \rightarrow$ is continuous;
7. $lim_{t\rightarrow\infty} M_{IVFMS}(x, y, t_{norm}) = \overline{1}$; $\forall x, y, z \in X, t_{norm} > 0$,

Note: Every metric can induce an interval valued fuzzy metric space.

Example: Let (X, d) be any general metric space. Denote interval valued $t - norm\ \overline{u} *_I \overline{v}$ =and let M_{IVFMS} be an interval value fuzzy metric define as follows:

$$M_{IVFMS}(x, y, t_{norm}) = \left[\frac{at^n}{at^n + hd(x, y)}, \frac{at^n}{at^n + kd(x, y)}\right]$$

For all $a, n, h \wedge k \in R^{+\wedge h \geq k}$ then $(X, M_{IVFMS}, *_I)$ is called interval value fuzzy metric space. It is also hold if interval valued $t - norm\ \overline{u} *_I \overline{v} =$ and $a = n = 1 \wedge h \geq k$ then [13].

$$M_{IVFMS}(x, y, t_{norm}) = \left[\frac{t}{t + hd(x, y)}, \frac{t}{t + kd(x, y)}\right]$$

Moreover, if $h = k = 1$ then it will me reduce in standard fuzzy metric space.

Definition 2.7[11]: Let $(X, M_{IVFMS}, *_I)$ is an *IVFMS*,

a. If $\beta t_{norm} > 0$ then $M_{IVFMS}(x, y, t_{norm}) \leq M_{IVFMS}(x, y, \beta)$for $x, y \in X$.

b. A sequence $\{x_n\}$ in X is referred to as a Cauchy sequence if for all $\overline{\epsilon} > \overline{0}$ and $t_{norm} > 0$ then there exists a $n_0 \in N \overline{\epsilon} M_{IVFMS}(x, y, t_{norm}) > 1 - \overline{\epsilon}$, for all $x, y \geq n_0$.

c. If every Cauchy sequence is convergent in $(X, M_{IVFMS}, *_I)$ then it is complete *IVFMS.*

Definition 2.8[11]: let two self mapping $\mathfrak{K}$ and $\mathfrak{L}$ on *IVFMS* $(X., M_{IVFMS}, *_I)$then

i. compatible if $lim_{n\rightarrow\infty} M_{IVFMS}(KLx_n, LKx_n| *_I |, t_{norm}) = \overline{1} forallt_{norm} > 0$ whenever a sequence $\{x_n\}$in X provided $lim_{n\rightarrow\infty} Kx_n = lim_{n\rightarrow\infty} Lx_n = u, forallu \in X$.

ii. Compatible of type (A) if $lim_{n\rightarrow\infty} M_{IVFMS}(KLx_n, \ll x_n| *_I |, t_{norm}) = \overline{1}, lim_{n\rightarrow\infty} M_{IVFMS}(LKx_n, KKx_n| *_I |, t_{norm}) = \overline{1}$

iii. for all$t_{norm} > 0$, whenever a sequence $\{x_n\}$in X provided$lim_{n\rightarrow\infty} Kx_n = lim_{n\rightarrow\infty} Lx_n = u, forallu \in X$.

Definition 2.9[5]: let two self mapping $\mathfrak{K}$ and $\mathfrak{L}$ on *IVFMS* $(X., M_{IVFMS}, *_I)$then $\mathfrak{K}$ and $\mathfrak{L}$ are called:

i. A −Intimate mappings: if$\alpha M_{IVFMS}(KLx_n, Kx_n, t_{norm}) \geq \alpha M_{IVFMS}(x_n, Lx_n, t_{norm})$ where$\alpha = lim_{n\rightarrow\infty} \vee lim_{n\rightarrow\infty} Inf$, for all$t_{norm} > 0$, whenever a sequence $\{x_n\}$in X provided$lim_{n\rightarrow\infty} Kx_n = lim_{n\rightarrow\infty} Lx_n = u, forallu \in X$.[14]

ii. S −Intimate mappings: if$\alpha M_{IVFMS}(LKx_n, Lx_n, t_{norm}) \geq \alpha M_{IVFMS}(KKx_n, Kx_n, t_{norm})$ where$\alpha = lim_{n\rightarrow\infty} \vee lim_{n\rightarrow\infty} Inf$, for all$t_{norm} > 0$, whenever a sequence $\{x_n\}$in X provided$lim_{n\rightarrow\infty} Kx_n = lim_{n\rightarrow\infty} Lx_n = u, forallu \in X$ [15].

Proposition 2.10[5]: let two self mapping $\mathfrak{K}$ and $\mathfrak{L}$ on *IVFMS* $(X.,M_{IVFMS},*_I)$and $\mathfrak{K}$ and $\mathfrak{L}$ are Compatible mapping of type (A) then the pair $\mathfrak{K}$ and $\mathfrak{L}$ are A −Intimate mappings and S −Intimate mappings.[16].

Proposition 2.11[5]: let two self mapping $\mathfrak{K}$ and $\mathfrak{L}$ on *IVFMS* $(X.,M_{IVFMS},*_I)$and $\mathfrak{K}$ and $\mathfrak{L}$ are A −Intimate mappings and $Ks = Ls = p, p \in X$ then

$$M_{IVFMS}(Kp, p, t_{norm}) \leq M_{IVFMS}(Lp, p, t_{norm})$$.[17]

Remark: Pair $\mathfrak{K}$ and $\mathfrak{L}$ are A −Intimate mappings or S −Intimate mappings but not Compatible mapping of type (A).

Lemma 2.12[Singh and Meade 1977: $\mathbb{y}(p) < p$ for every $p > 0$ if and only if $lim_{n\to\infty} \mathbb{y}^n(p) = 0$, where $\mathbb{y}^n$denote the n times composition of $\mathbb{y}$.[18]

Main Result

Theorem 3.1: Let $\mathfrak{K}$, $\mathfrak{L}$, $\mathfrak{D}$, and $\mathfrak{F}$ be a self mapping of a complete *IVFMS* $(X, M_{IVFMS},*_I)$ satisfying the following conditions:

i. $K(X) \subseteq F(X) \wedge L(X) \subseteq D(X)$.
ii. Pair $\{K, D\} isD - intimate \wedge \{L, F\} isF - intimate.$
iii. $(X$ is complete.
iv. For $\forall x, y \in X \ \ t_{norm} > 0, 0 < \xi < 1$

$$M_{IVFMS}(Kx, Ly, t_{norm}) \leq \xi \left[Max\begin{Bmatrix} M_{IVFMS}(Dx, Fy, t_{norm}), M_{IVFMS}(Kx, Dx, t_{norm}), \\ M_{IVFMS}(Ly, Fy, t_{norm}), M_{IVFMS}(Kx, Fy, t_{norm}) \end{Bmatrix}\right]$$

$thenK, L, D, \wedge\ FhaveauniqueCommonPoint.$

Proof: Let $x_0 \in X$ then there exists $x_{1,}x_2 \in X$such that$Kx_0 = Fx_1 = y_0 \wedge Lx_1 = Dx_2 = y_{1.}$. Inductively we establish two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $Kx_{2n} = Fx_{2n+1} = y_{2n} \wedge Lx_{2n+1} = Dx_{2n+2} = y_{2n+1.}$ for$n \geq 0$. Now we can prove $\{y_n\}$ is a Cauchy's sequence in X. Now by putting $x = x_{2n}, y = x_{2n+1}$ in inequality (iv)then

$$M_{IVFMS}(Kx_{2n}, Lx_{2n+1}, t_{norm}) \leq \xi \left[Max\begin{Bmatrix} M_{IVFMS}(Dx_{2n}, Fx_{2n+1}, t_{norm}), M_{IVFMS}(Kx_{2n}, Dx_{2n}, t_{norm}), \\ M_{IVFMS}(Lx_{2n+1}, Fx_{2n+1}, t_{norm}), M_{IVFMS}(Kx_{2n}, Fx_{2n+1}, t_{norm}) \end{Bmatrix}\right]$$

$$M_{IVFMS}(y_{2n}, y_{2n+1.}, t_{norm}) \leq \xi \left[Max\begin{Bmatrix} M_{IVFMS}(y_{2n-1.}, y_{2n}, t_{norm}), M_{IVFMS}(y_{2n}, y_{2n-1.}, t_{norm}), \\ M_{IVFMS}(y_{2n+1.}, y_{2n}, t_{norm}), M_{IVFMS}(y_{2n}, y_{2n}, t_{norm}) \end{Bmatrix}\right]$$

$$M_{IVFMS}(y_{2n}, y_{2n+1.}, t_{norm}) \leq \xi \left[Max\begin{Bmatrix} M_{IVFMS}(y_{2n-1.}, y_{2n}, t_{norm}), M_{IVFMS}(y_{2n}, y_{2n-1.}, t_{norm}), \\ M_{IVFMS}(y_{2n+1.}, y_{2n}, t_{norm}), \overline{1} \end{Bmatrix}\right]$$

If$M_{IVFMS}(y_{2n+1.}, y_{2n}, t_{norm}) > M_{IVFMS}(y_{2n}, y_{2n-1.}, t_{norm})$ then

$M_{IVFMS}(y_{2n}, y_{2n+1.}, t_{norm}) \leq M_{IVFMS}(y_{2n+1.}, y_{2n}, t_{norm})$, Which is contradiction.

Thus, $M_{IVFMS}(y_{2n}, y_{2n+1.}, t_{norm}) \leq \xi M_{IVFMS}(y_{2n-1}, y_{2n.}, t_{norm})$
In general

$$\begin{aligned} M_{IVFMS}(y_n, y_{n+1.}, t_{norm}) &\leq \xi M_{IVFMS}(y_{n-1}, y_{n.}, t_{norm}) \\ &\leq \xi^2 M_{IVFMS}(y_{n-2}, y_{n-1.}, t_{norm}) \end{aligned}$$

$$\leq \xi^{n} M_{IVFMS}(y_0, y_{1.}, t_{norm})$$

For every integer $m > 0$, we get

$$\begin{aligned} M_{IVFMS}(y_n, y_{n+m.}, t_{norm}) &\leq M_{IVFMS}(y_n, y_{n+1.}, t_{norm}) + M_{IVFMS}(y_{n+1}, y_{n+2.}, t_{norm}) \\ &\quad + M_{IVFMS}(y_{n+m-1}, y_{n+m.}, t_{norm}) \end{aligned}$$

$$\leq (1 + \xi + \xi^2 + \cdots \dots \dots . + \xi^{m-1}) M_{IVFMS}(y_n, y_{n+1.}, t_{norm})$$
$$\leq \left(\frac{\xi^m}{1-\xi}\right) M_{IVFMS}(y_n, y_{n+1.}, t_{norm}) \rightarrow 0 asn \rightarrow \infty.$$

Thus $M_{IVFMS}(y_n, y_{n+m.}, t_{norm}) \rightarrow 0$. therefore $\{y_n\}$ is a Cauchy's sequence in X and converges to$\sigma \in X$. Further all subsequence of $\{y_n\}$ is also converges to σ.

Form this argument,
$y_{2n} = Kx_{2n} = Fx_{2n+1} \rightarrow \sigma \wedge y_{2n+1} = Lx_{2n+1} = Dx_{2n+2} \rightarrow \sigma$ as $n \rightarrow \infty$.

Since (X is complete then ∃ a point $j \in X such that\ \mathfrak{D}j = \sigma$.

Now to claim $Kj = \sigma$, for this we put $x = j \wedge y = x_{2n+1}$ in inequality (iv)

$$M_{IVFMS}(Kj, Lx_{2n+1}, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(Dj, Fx_{2n+1}, t_{norm}), M_{IVFMS}(Kj, Dj, t_{norm}), \\ M_{IVFMS}(Lx_{2n+1}, Fx_{2n+1}, t_{norm}), M_{IVFMS}(Kj, Fx_{2n+1}, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(Kj, \sigma, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(\sigma, \sigma, t_{norm}), M_{IVFMS}(Kj, \sigma, t_{norm}), \\ M_{IVFMS}(\sigma, \sigma, t_{norm}), M_{IVFMS}(Kj, \sigma, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(Kj, \sigma, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} \overline{1}, M_{IVFMS}(Kj, \sigma, t_{norm}), \\ \overline{1}, M_{IVFMS}(Kj, \sigma, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(Kj, \sigma, t_{norm}) \leq \xi M_{IVFMS}(Kj, \sigma, t_{norm})$$

This is contradiction. So that $Kj = \sigma \Rightarrow Kj = \sigma = Dj$.

Since $K(X) \subseteq F(X)$ then ∃ a point $k \in X such that\ Fk = \sigma$. we put $x = j \wedge y = k$ in inequality (iv)

$$M_{IVFMS}(Kj, Lk, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(Dj, Fk, t_{norm}), M_{IVFMS}(Kj, Dj, t_{norm}), \\ M_{IVFMS}(Lk, Fk, t_{norm}), M_{IVFMS}(Kj, Fk, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(\sigma, Lk, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(\sigma, \sigma, t_{norm}), M_{IVFMS}(\sigma, \sigma, t_{norm}), \\ M_{IVFMS}(Lk, \sigma, t_{norm}), M_{IVFMS}(\sigma, \sigma, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(\sigma, Lk, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} \overline{1}, \overline{1}, \\ M_{IVFMS}(Lk, \sigma, t_{norm}), \overline{1} \end{Bmatrix} \right]$$

$$M_{IVFMS}(\sigma, Lk, t_{norm}) \leq \xi M_{IVFMS}(Lk, \sigma, t_{norm})$$

This is contradiction. So that $Lk = \sigma \Rightarrow Lk = \sigma = Fk$.

Since $Kj = Dj = \sigma$ and pair $\{K, D\} is\ D - intimate$ then by Preposition (2.11) we have

$M_{IVFMS}(D\sigma, \sigma, t_{norm}) \leq M_{IVFMS}(K\sigma, \sigma, t_{norm})$.

Suppose $K\sigma \neq \sigma$ then we put $x = \sigma \wedge y = k$ in inequality (iv)

$$M_{IVFMS}(K\sigma, Lk, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(D\sigma, Fk, t_{norm}), M_{IVFMS}(K\sigma, D\sigma, t_{norm}), \\ M_{IVFMS}(Lk, Fk, t_{norm}), M_{IVFMS}(K\sigma, Fk, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(K\sigma, \sigma, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(D\sigma, \sigma, t_{norm}), M_{IVFMS}(K\sigma, D\sigma, t_{norm}), \\ M_{IVFMS}(\sigma, \sigma, t_{norm}), M_{IVFMS}(K\sigma, \sigma, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(K\sigma, \sigma, t_{norm}) \\ \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(D\sigma, \sigma, t_{norm}), M_{IVFMS}\left(K\sigma, \sigma, \frac{t_{norm}}{2}\right), M_{IVFMS}\left(\sigma, D\sigma, \frac{t_{norm}}{2}\right) \\ M_{IVFMS}(\sigma, \sigma, t_{norm}), M_{IVFMS}(K\sigma, \sigma, t_{norm}) \end{Bmatrix} \right]$$

(By definition 2.6 (4)), since $a * b = min\{a, b\}$

$$M_{IVFMS}(K\sigma, \sigma, t_{norm}) \\ \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(K\sigma, \sigma, t_{norm}), M_{IVFMS}\left(K\sigma, \sigma, \frac{t_{norm}}{2}_{norm}\right), M_{IVFMS}\left(\sigma, K\sigma, \frac{t_{norm}}{2}\right) \\ M_{IVFMS}(\sigma, \sigma, t_{norm}), M_{IVFMS}(K\sigma, \sigma, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(K\sigma, \sigma, t_{norm}) \leq \xi M_{IVFMS}(K\sigma, \sigma, t_{norm})$$

This is contradiction. So that $K\sigma = \sigma$.

Then $M_{IVFMS}(D\sigma, \sigma, t_{norm}) \leq M_{IVFMS}(K\sigma, \sigma, t_{norm}) \leq M_{IVFMS}(\sigma, \sigma, t_{norm})$

$M_{IVFMS}(D\sigma, \sigma, t_{norm}) \leq \overline{1} \Rightarrow D\sigma = \sigma$ (By definition 2.6(2))
Therefore $K\sigma = \sigma = D\sigma$.

Also $Fk = Lk = \sigma$ and pair $\{L, F\} isF - intimate$then by Preposition (2.11) we have

$M_{IVFMS}(F\sigma, \sigma, t_{norm}) \leq M_{IVFMS}(L\sigma, \sigma, t_{norm})$.

Suppose $L\sigma \neq \sigma$ then we put $x = k \wedge y = \sigma$ in inequality (iv)

$$M_{IVFMS}(Kk, L\sigma, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(Dk, F\sigma, t_{norm}), M_{IVFMS}(Kk, Dk, t_{norm}), \\ M_{IVFMS}(L\sigma, F\sigma, t_{norm}), M_{IVFMS}(Kk, F\sigma, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(\sigma, L\sigma, t_{norm}) \leq \xi \left[Max \begin{Bmatrix} M_{IVFMS}(Dk, \sigma, t_{norm}), M_{IVFMS}(\sigma, Dk, t_{norm}), \\ M_{IVFMS}(L\sigma, F\sigma, t_{norm}), M_{IVFMS}(\sigma, F\sigma, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(\sigma, L\sigma, t_{norm}) \leq \\ \xi \left[Max \begin{Bmatrix} M_{IVFMS}(Dk, \sigma, t_{norm}), M_{IVFMS}(\sigma, Dk, t_{norm}), \\ M_{IVFMS}\left(L\sigma, \sigma, \frac{t_{norm}}{2}\right), M_{IVFMS}\left(\sigma, F\sigma, \frac{t_{norm}}{2}\right), M_{IVFMS}(\sigma, F\sigma, t_{norm}) \end{Bmatrix} \right]$$

$$M_{IVFMS}(\sigma, L\sigma, t_{norm}) \leq \xi \left[Max \left\{ \begin{matrix} M_{IVFMS}(\sigma, \sigma, t_{norm}), M_{IVFMS}(\sigma, \sigma, t_{norm}), \\ M_{IVFMS}\left(L\sigma, \sigma, \frac{t_{norm}}{2}\right), M_{IVFMS}\left(\sigma, F\sigma, \frac{t_{norm}}{2}\right), M_{IVFMS}(\sigma, L\sigma, t_{norm}) \end{matrix} \right\} \right]$$

$$M_{IVFMS}(\sigma, L\sigma, t_{norm}) \leq \xi M_{IVFMS}(\sigma, L\sigma, t_{norm})$$

This is contradiction. So that $L\sigma = \sigma$.

Then $M_{IVFMS}(F\sigma, \sigma, t_{norm}) \leq M_{IVFMS}(L\sigma, \sigma, t_{norm}) \leq M_{IVFMS}(\sigma, \sigma, t_{norm})$

$M_{IVFMS}(F\sigma, \sigma, t_{norm}) \leq \overline{1} \Rightarrow F\sigma = \sigma$ (By definition 2.6(2))

Therefore $L\sigma = \sigma = F\sigma$.

Hence $K\sigma = D\sigma = L\sigma = F\sigma = \sigma$.

Uniqueness: let us consider ϱ is another common fixed point of $K, D, L, \wedge F$ such that $\sigma \neq \varrho$.

$$M_{IVFMS}(K\sigma, L\varrho, t_{norm}) \leq \xi \left[Max \left\{ \begin{matrix} M_{IVFMS}(D\sigma, F\varrho, t_{norm}), M_{IVFMS}(K\sigma, D\sigma, t_{norm}), \\ M_{IVFMS}(L\varrho, F\varrho, t_{norm}), M_{IVFMS}(K\sigma, F\varrho, t_{norm}) \end{matrix} \right\} \right]$$

$$M_{IVFMS}(\sigma, \varrho, t_{norm}) \leq \xi \left[Max \left\{ \begin{matrix} M_{IVFMS}(\sigma, \varrho, t_{norm}), M_{IVFMS}(\sigma, \sigma, t_{norm}), \\ M_{IVFMS}(\varrho, \varrho, t_{norm}), M_{IVFMS}(\sigma, \varrho, t_{norm}) \end{matrix} \right\} \right]$$

$$M_{IVFMS}(\sigma, \varrho, t_{norm}) \leq \xi \left[Max \left\{ \begin{matrix} M_{IVFMS}(\sigma, \varrho, t_{norm}), \overline{1},, \\ \overline{1},, M_{IVFMS}(\sigma, \varrho, t_{norm}) \end{matrix} \right\} \right]$$

$$M_{IVFMS}(\sigma, \varrho, t_{norm}) \leq \xi M_{IVFMS}(\sigma, \varrho, t_{norm})$$

This is contradiction. So that $\sigma = \varrho$.

CONCLUSION

The findings of this paper are particularly valuable for researchers in theoretical mathematics. The extended theorems provide a stronger foundation for further research in metric spaces and fixed point theory. Additionally, the broader applicability of these results can accelerate progress in areas that rely on fixed point theorems, including sustainable development, optimization, and systems analysis.

In summary, this paper makes significant contributions by verifying and expanding fixed point results within the framework of interval-valued fuzzy metric spaces using intimate mapping. These advancements not only deepen the theoretical understanding of fixed point theorems but also enhance their practical applications in various fields.

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