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Study on the Method of Constructing a System of Linear Equations for Calculating Aerodynamic Derivatives

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Abstract

The aerodynamic derivatives can be calculated from the unsteady hydrodynamic forces experienced by the harmonic-oscillating vehicle. Until now, unsteady flow field analysis in aerodynamic derivative calculations has been applied mainly to thin bodies based on potential theory MSC. Nastran is a typical application based on potential theory MSC. Natran can calculate the aerodynamic derivatives of the vehicle relatively well in the subsonic region, but in the supersonic region, the fuselage element is not available, so the calculation is done by converting the fuselage to panel elements projected onto a flat plate. But the airfoil profile cannot be reflected in all velocity regions, and the exact calculation cannot be carried out in the transonic region, limiting the perturbation velocity potential method. Analyzing unsteady flow fields by numerical solution of Euler's equations using CFD methods can overcome the limitations of potential theory, but the computational effort is too expensive to apply to practical problems. On the other hand, in the calculation of aerodynamic derivatives, the harmonic vibration amplitude of the considered vehicle and the perturbations of the fluid resulting from it can be calculated by considering the infinitesimal quantities. Hence, in this study, the time-linearized unsteady Euler equations are numerically solved in the frequency domain by the finite volume method to perform unsteady flow field analysis. This can overcome the limitations of potential theory and can be applied to practical problems by reducing the computational effort significantly, since the transient problem is considered as a stationary problem in the frequency domain. Nastran can perform aerodynamic derivative calculations of aircraft in the aeroelastic analysis module. The aeroelastic phenomena are composed of the interaction of the inertial forces of the aircraft, the elastic forces of deformable bodies and the aerodynamic forces. Therefore, structural analysis and aerodynamic calculations must be combined. Here, we consider MSC. The aerodynamic theories based on Nastran and the interpolation methods that interconnect structural deformation and aerodynamic forces are described in detail.

Keywords: Vibration, airfoil, fluid analysis, angle of attack, non-linear

INTRODUCTION

Static stability functions and control derivatives can be computed in static elasto-elastic analysis. Since static elasto-elasticity is a combination of inertial, structural and aerodynamic forces, data for structural analysis, aerodynamic force calculations, and inertial force calculations are required.

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In aerodynamic derivative calculations, the vehicle is considered as an absolute rigid body and its stability functions and control derivatives are calculated, so data related to structural analysis are only formally required.

The body data section describes all the data and instructions needed for the calculations, which consist largely of structural data, inertial force-related data, aerodynamic data, and commands for static aeroelastic analysis.

Structural data includes node locations (GRID instructions) associated with finite element modeling, element properties (CBAR instructions and PBAR instructions), material values (MAT1), and constraints (SPC1).

The mass data for inertial force calculation is defined as the distributed mass by giving the density at the material value (MAT1).

The above details in the body data section are only marginal quantities required for implementation in aerodynamic derivative calculations but are formally necessarily required quantities.

Aerodynamic data are very important quantities in derivative calculations, so we mainly consider these quantities [1].

The SUPORT command specifies the degrees of freedom of a rigid body, where five degrees of freedom (lateral and vertical translations-2 and 3, rotational motion around three axes-4, 5 and 6) are specified at the center of gravity of MiG-21 (point 2).

The commands specify global aerodynamic data: AEROS represents static aeroelastic analysis, 0 the reference coordinate system, 100 the coordinate system in which derivatives will be calculated, 4.0 the longitudinal characteristic length, 7.15 the transverse characteristic length, and 23 the characteristic area.

CORD2R specifies coordinate 100 for the derivative calculation indicated in AEROS.

CAERO1 indicates the geometric shape and interference groups of the panels (main, horizontal, vertical, etc.) to which the DLM will be applied, and SPLINE1 specifies the surface spline corresponding to each panel, which includes the number of aerodynamic elements used in the spline and the ID of the SET1 command [2].

The SET1 command indicates the number of structural points used in the spline.

For plane splines, three points that lie in a straight line should not be used.

When the aerodynamic panel is not equipartition, the AEFACT command is used to specify the partition locations.

The PAERO1 instruction indicates a shell ID that interferes with the panel; even in the absence of a shell, this instruction is formally required.

CAERO2 zero specifies the fuselage element.

We indicate the lengthwise division, the PAERO2 zero ID, the position and length of the front body.

PAERO2 zero gives the cross-sectional data of the shell element, and SPLINE2 zero specifies the line splines for the shell element.

The line spline should not use three points with the same y-coordinate.

DMI command specifies the initial angle of attack (reference angle of attack) of the vehicle, with the value of angle of attack being radian [3, 4].
The AESURF command specifies the control plane.

We refer to the ID of the control panel and the ID of the coordinate system that specifies the control axis.

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The AELIST command indicates the number of aerodynamic elements belonging to the control plane, and the AESTAT command indicates the static balancing degree of freedom of the vehicle.

The TRIM command specifies static balance conditions and parameters, and the balance parameters must be carefully taken to ensure that the balance of the vehicle is achieved.

If the balance parameters are not properly given, the calculation cannot be carried out.

Since the fuselage element is only available at subsonic speed, we choose the Mach numbers to be less than 1.

The results of the static stability function and control derivative calculations are presented in the .f06 file.

Construction of a System of Linear Equations and Numerical Methods

The system of equations describing the flow around a moving body can be linearized on the assumption that the perturbations due to the motion of the body are infinitesimal.

The system of three-dimensional Euler equations of conservation type is as follows:

$$\frac{\partial \hat{U}}{\partial t} + \frac{\partial \hat{F}}{\partial x} + \frac{\partial \hat{G}}{\partial y} + \frac{\partial \hat{H}}{\partial z} = 0 \tag{1}$$

In Eq. (1), $\hat{U} = \left(\hat{\rho}, \rho\hat{u}, \rho\hat{v}, \rho\hat{w}, \hat{e},\right)^T$ is an auxiliary variable vector and $\hat{F}, \hat{G}, \hat{H}$ are flux vectors in x, y, z directions, and like this.

$$\hat{F} = \begin{pmatrix} \rho\hat{u} \\ \rho\hat{u^2} + p \\ \rho\hat{uv} \\ \rho\hat{uw} \\ (e + \hat{p}) \cdot u \end{pmatrix}, \quad \hat{G} = \begin{pmatrix} \rho\hat{v} \\ \rho\hat{uv} \\ \rho\hat{v^2} + p \\ \rho\hat{vw} \\ (e + \hat{p}) \cdot v \end{pmatrix}, \quad \hat{H} = \begin{pmatrix} \rho\hat{w} \\ \rho\hat{uw} \\ \rho\hat{vw} \\ \rho\hat{w^2} + p \\ (e + \hat{p}) \cdot w \end{pmatrix}$$

By dividing Eq. (1) into steady flow without motion of the body and infinitesimal perturbed flow caused by the motion of the body, linearization yields:

$$\frac{\partial \overline{F}}{\partial x} + \frac{\partial \overline{G}}{\partial y} + \frac{\partial \overline{H}}{\partial z} = 0 \tag{2}$$

$$\frac{\partial U'}{\partial t} + \frac{\partial F'}{\partial x} + \frac{\partial G'}{\partial y} + \frac{\partial H'}{\partial z} = 0 \tag{3}$$

In the equations above:

$$\hat{U} = \overline{U} + U', F' = \left.\frac{\partial F}{\partial U}\right|_{\overline{U}} \cdot U', G' = \left.\frac{\partial G}{\partial U}\right|_{\overline{U}} \cdot U',\ H' = \left.\frac{\partial H}{\partial U}\right|_{\overline{U}} \cdot U'$$

Eq. (2) represents the steady flow field around the body, and there are relatively many studies on it.

Let the unsteady motion be the following harmonic motion:

$U' = U \cdot e^{i\omega t}$

In the above equation, U is the harmonic amplitude of the vector of conserved variables, which is a complex quantity, and ω is a real number with the frequency of oscillation [5–7].

Substituting U' in Eq. (3), we obtain:

$$i\omega U + \frac{\partial}{\partial x}(A \cdot U) + \frac{\partial}{\partial y}(B \cdot U) + \frac{\partial}{\partial z}(C \cdot U) = 0 \tag{4}$$

In Eq. (4), $A = \frac{\partial F}{\partial U}\Big|_{\overline{U}}, B = \frac{\partial G}{\partial U}\Big|_{\overline{U}}, C = \frac{\partial H}{\partial U}\Big|_{\overline{U}}$ is the flow Jacobian matrix, which is a real matrix only related to steady flow.

Eq. (4) leads to a steady-state equation in the frequency domain, which can be solved in a time-progressing form by introducing virtual time τ.

$$\frac{\partial U}{\partial \tau} + i\omega U + \frac{\partial}{\partial x}(A \cdot U) + \frac{\partial}{\partial y}(B \cdot U) + \frac{\partial}{\partial z}(C \cdot U) = 0 \tag{5}$$

The discretization using the finite volume method is as follows:

$$U_i^{\,n+1} = U_i^{\,n} - \Delta\tau \left(i\omega U + \frac{1}{v_i} \cdot \sum_{j=1}^{n_i} D_{ij} \cdot S_{ij} \right) \tag{6}$$

In the above equation, $D_{ij} = \left(A_{ij} \cdot n_{ij}^x + B_{ij} \cdot n_{ij}^y + C_{ij} \cdot n_{ij}^z \right) \cdot U_{ij}$, S_{ij} is a numerical discharge function, which is the surface vector between the element i and its adjacent element j.

We computed the numerical discharge function using the Sterger-Warming scheme and divided the real and imaginary parts, which were combined to form the following system of equations:

$$\begin{cases} (U_R)_i^{\,n+1} = (U_R)_i^{\,n} + \Delta\tau \left(\omega (U_I)_i^{\,n} + \frac{1}{v_i} \cdot \sum_{j=1}^{n_i} (D_R)_{ij} \cdot S_{ij} \right) \\ (U_I)_i^{\,n+1} = (U_I)_i^{\,n} + \Delta\tau \left(\omega (U_R)_i^{\,n} + \frac{1}{v_i} \cdot \sum_{j=1}^{n_i} (D_I)_{ij} \cdot S_{ij} \right) \end{cases} \tag{7}$$

In the above equation:

$$U = U_R + iU_I$$
$$(D_R)_{ij} = \left(A_{ij} \cdot n_{ij}^x + B_{ij} \cdot n_{ij}^y + C_{ij} \cdot n_{ij}^z \right) \cdot (U_R)_{ij},$$
$$(D_I)_{ij} = \left(A_{ij} \cdot n_{ij}^x + B_{ij} \cdot n_{ij}^y + C_{ij} \cdot n_{ij}^z \right) \cdot (U_I)_{ij}$$

The amplitude of the complex perturbation pressure P can be derived from the equation of state of an ideal fluid.

$$\begin{aligned} \hat{P} = \overline{P} + P' &= (\gamma - 1)\hat{\rho}\hat{e} = (\gamma - 1)(\overline{\rho} + \rho')(\overline{e} + e') \\ &= (\gamma - 1)(\overline{\rho e} + \overline{\rho} e' + \rho' \overline{e} + \rho' e') \\ &= (\gamma - 1)\overline{\rho e} + (\gamma - 1)(\overline{\rho} e' + \rho' \overline{e} + \rho' e'). \end{aligned}$$

In the above equation, the second-order perturbation is neglected, and the steady state and first-order perturbation are expressed separately as follows:

$$\overline{P} = (\gamma - 1)\overline{\rho e}$$
$$P' = (\gamma - 1)(\overline{\rho} e' + \rho' \overline{e})$$

If $\rho' = \rho \cdot e^{i\omega t}$, $e' = e \cdot e^{i\omega t}$

$$P' = (\gamma - 1)(\overline{\rho} \cdot e \cdot e^{i\omega t} + \rho \cdot \overline{e} \cdot e^{i\omega t}) = (\gamma - 1)(\overline{\rho} \cdot e + \rho \cdot \overline{e}) \cdot e^{i\omega t}.$$

Hence,

$$P = (\gamma - 1)(\overline{\rho} \cdot e + \rho \cdot \overline{e}) \tag{8}$$

Boundary conditions at the body boundary and at the outer boundary are given as initial variables [8].

$$\hat{U} = \begin{pmatrix} \hat{\rho} \\ \widehat{\rho u} \\ \widehat{\rho v} \\ \widehat{\rho w} \\ \hat{e} \end{pmatrix} = \begin{pmatrix} \hat{\rho} \\ \hat{\rho}\hat{u} \\ \hat{\rho}\hat{v} \\ \hat{\rho}\hat{w} \\ \hat{e} \end{pmatrix} = \begin{pmatrix} \overline{\rho} + \rho' \\ (\overline{\rho} + \rho')(\overline{u} + u') \\ (\overline{\rho} + \rho')(\overline{v} + v') \\ (\overline{\rho} + \rho')(\overline{w} + w') \\ \overline{e} + e' \end{pmatrix} = \begin{pmatrix} \overline{\rho} + \rho' \\ \overline{\rho u} + \overline{\rho} u' + \rho'\overline{u} + \rho' u' \\ \overline{\rho v} + \overline{\rho} v' + \rho'\overline{v} + \rho' v' \\ \overline{\rho w} + \overline{\rho} w' + \rho'\overline{w} + \rho' w' \\ \overline{e} + e' \end{pmatrix} =$$

$$= \begin{pmatrix} \overline{\rho} \\ \overline{\rho u} \\ \overline{\rho v} \\ \overline{\rho w} \\ \overline{e} \end{pmatrix} + \begin{pmatrix} \rho' \\ \overline{\rho} u' + \rho'\overline{u} + \rho' u' \\ \overline{\rho} v' + \rho'\overline{v} + \rho' v' \\ \overline{\rho} w' + \rho'\overline{w} + \rho' w' \\ e' \end{pmatrix} = \overline{U} + U'.$$

So,

$$U' = \begin{pmatrix} \rho' \\ \overline{\rho} u' + \rho'\overline{u} + \rho' u' \\ \overline{\rho} v' + \rho'\overline{v} + \rho' v' \\ \overline{\rho} w' + \rho'\overline{w} + \rho' w' \\ e' \end{pmatrix}.$$

Neglecting second-order infinitesimal quantities:

$$U' = \begin{pmatrix} \rho' \\ \overline{\rho} u' + \rho'\overline{u} \\ \overline{\rho} v' + \rho'\overline{v} \\ \overline{\rho} w' + \rho'\overline{w} \\ e' \end{pmatrix}, \tag{9}$$

The normal velocity of the fluid at the body boundary is equal to the normal velocity of the body from the impermeable condition.

$$\hat{V} \cdot \hat{n} = \hat{V}_{물} \cdot \hat{n}, \tag{10}$$

Linearizing the above expression under the assumption that $\hat{V}_{물}$ is infinitesimal.

$$\hat{V} \cdot \hat{n} = (\overline{V} + V') \cdot (\overline{n} + n') = \overline{V} \cdot \overline{n} + \overline{V} \cdot n' + V' \cdot \overline{n} + V' \cdot n' \approx$$

$$\overline{V} \cdot \overline{n} + \overline{V} \cdot n' + V' \cdot \overline{n} = \hat{V}_{물} \cdot (\overline{n} + n') \approx \hat{V}_{물} \cdot \overline{n}.$$

In the steady flow, $\overline{V} \cdot \overline{n} = 0$

So, $V' \cdot \overline{n} = \hat{V}_{물} \cdot \overline{n} - \overline{V} \cdot n'$. (11)

Assuming that the motion of a body is harmonic, and that the motion of the fluid resulting from it is harmonic, we have:

$V' = V \cdot e^{i\omega t}$,
$\hat{V}_{물} = V_{물} \cdot e^{i\omega t}$,
$n' = n \cdot e^{i\omega t}$

Putting the above quantities into Eq. (1), we obtain:

$$V \cdot e^{i\omega t} \cdot \overline{n} = V_{\text{물}} \cdot e^{i\omega t} \cdot \overline{n} - \overline{V} \cdot n \cdot e^{i\omega t}.$$

So,

$$V \cdot \overline{n} = V_{\text{물}} \cdot \overline{n} - \overline{V} \cdot n. \tag{12}$$

In the above equations, $V, V_{\text{물}}$ is the amplitude of the harmonic perturbation velocity of the fluid and the body, and n is the amplitude of the normal perturbation vector [9, 10].

Let us describe in detail the complex line velocities in the body boundary element in the case of a vehicle harmonic with different degrees of freedom.

① In the case of plunge motion with $h = h_0 e^{i\omega t}$.

$$\hat{V}_{\text{물}x} = \frac{dX_{\text{물}}}{dt} = 0 \rightarrow V_{\text{물}x} = 0$$

$$\hat{V}_{\text{물}y} = \frac{dY_{\text{물}}}{dt} = \frac{d}{dt}\left(Y_{\text{물}0} + h\right) = \frac{dh}{dt} = i\omega h_0 e^{i\omega t} \rightarrow V_{\text{물}y} = i\omega h_0$$

$$\hat{V}_{\text{물}z} = \frac{dZ_{\text{물}}}{dt} = 0 \rightarrow V_{\text{물}z} = 0$$

$$n_x' = n_y' = n_z' = 0 \rightarrow n_x = n_y = n_z = 0.$$

Putting the above quantities into Eq. (11), we obtain:

$$V_n = \left(V_{\text{물}x} \cdot \overline{n}_x + V_{\text{물}y} \cdot \overline{n}_y + V_{\text{물}z} \cdot \overline{n}_z\right) - \left(\overline{V}_x \cdot n_x + \overline{V}_y \cdot n_y + \overline{V}_z \cdot n_z\right) =$$
$$= i\omega h_0 \cdot \overline{n}_y.$$

So,

$$V_n^R = 0,$$
$$V_n^I = \omega h_0 \cdot \overline{n}_y$$

② In the case of pitch motion with $\alpha = \alpha_0 e^{i\omega t}$

$$X_{\text{물}} = \overline{X}_{\text{물}} \cos\alpha + \overline{Y}_{\text{물}} \sin\alpha$$

$$Y_{\text{물}} = -\overline{X}_{\text{물}} \sin\alpha + \overline{Y}_{\text{물}} \cos\alpha$$

$$Z_{\text{물}} = \overline{Z}_{\text{물}}$$

$$\hat{n}_x = \overline{n}_x \cos\alpha + \overline{n}_y \sin\alpha$$
$$\hat{n}_y = -\overline{n}_x \sin\alpha + \overline{n}_y \cos\alpha$$
$$\hat{n}_z = \overline{n}_z$$

If $\alpha << 1$

$$X_{\text{물}} = \overline{X}_{\text{물}} + \overline{Y}_{\text{물}}\alpha$$

$$Y_{\text{물}} = -\overline{X}_{\text{물}}\alpha + \overline{Y}_{\text{물}}$$

$$Z_{\text{물}} = \overline{Z}_{\text{물}}$$

$$\hat{n}_x = \overline{n}_x + \overline{n}_y\alpha$$

$$\hat{n}_y = -\overline{n}_x \alpha + \overline{n}_y$$
$$\hat{n}_z = \overline{n}_z$$

So,

$$\hat{V}_{\text{물}x} = \frac{dX_{\text{물}}}{dt} = \alpha' \overline{Y}_{\text{물}} = i\omega\alpha_0 \overline{Y}_{\text{물}} e^{i\omega t} \rightarrow V_{\text{물}x} = i\omega\alpha_0 \overline{Y}_{\text{물}}$$
$$\hat{V}_{\text{물}y} = \frac{dY_{\text{물}}}{dt} = -\alpha' \overline{X}_{\text{물}} = -i\omega\alpha_0 \overline{X}_{\text{물}} e^{i\omega t} \rightarrow V_{\text{물}y} = -i\omega\alpha_0 \overline{X}_{\text{물}}$$
$$\hat{V}_{\text{물}z} = \frac{dZ_{\text{물}}}{dt} = 0 \rightarrow V_{\text{물}z} = 0$$
$$n_x' = \hat{n}_x - \overline{n}_x = \overline{n}_y \alpha \rightarrow n_x = \overline{n}_y \alpha_0$$
$$n_y' = \hat{n}_y - \overline{n}_y = -\overline{n}_x \alpha \rightarrow n_y = -\overline{n}_x \alpha_0$$
$$n_z' = \hat{n}_z - \overline{n}_z = 0 \rightarrow n_z = 0.$$

Putting the above quantities into Eq. (11), we obtain,

$$V_n = \left(V_{\text{물}x} \cdot \overline{n}_x + V_{\text{물}y} \cdot \overline{n}_y + V_{\text{물}z} \cdot \overline{n}_z\right) - \left(\overline{V}_x \cdot n_x + \overline{V}_y \cdot n_y + \overline{V}_z \cdot n_z\right)$$
$$= \left(i\omega\alpha_0 \overline{Y}_{\text{물}} \cdot \overline{n}_x - i\omega\alpha_0 \overline{X}_{\text{물}} \cdot \overline{n}_y\right) - \left(\overline{V}_x \cdot \overline{n}_y \alpha_0 - \overline{V}_y \cdot \overline{n}_x \alpha_0\right).$$

So,

$$V_n^R = \overline{V}_y \cdot \overline{n}_x \alpha_0 - \overline{V}_x \cdot \overline{n}_y \alpha_0,$$
$$V_n^I = \omega\alpha_0 \overline{Y}_{\text{물}} \cdot \overline{n}_x - \omega\alpha_0 \overline{X}_{\text{물}} \cdot \overline{n}_y.$$

③ In the case of sideslip motion $z = z_0 e^{i\omega t}$.

$$\hat{V}_{\text{물}x} = \frac{dX_{\text{물}}}{dt} = 0 \rightarrow V_{\text{물}x} = 0$$
$$\hat{V}_{\text{물}y} = \frac{dY_{\text{물}}}{dt} = 0 \rightarrow V_{\text{물}y} = 0$$
$$\hat{V}_{\text{물}z} = \frac{dZ_{\text{물}}}{dt} = \frac{d}{dt}\left(Z_{\text{물}0} + z\right) = \frac{dz}{dt} = i\omega z_0 e^{i\omega t} \rightarrow V_{\text{물}z} = i\omega z_0$$
$$n_x' = n_y' = n_z' = 0 \rightarrow n_x = n_y = n_z = 0.$$

From Eqs. (2)–(11):

$$V_n = \left(V_{\text{물}x} \cdot \overline{n}_x + V_{\text{물}y} \cdot \overline{n}_y + V_{\text{물}z} \cdot \overline{n}_z\right) - \left(\overline{V}_x \cdot n_x + \overline{V}_y \cdot n_y + \overline{V}_z \cdot n_z\right) =$$
$$= i\omega z_0 \cdot \overline{n}_z.$$
$$V_n^R = 0,$$
$$V_n^I = \omega z_0 \cdot \overline{n}_z.$$

④ In the case of roll motion $\varphi = \varphi_0 e^{i\omega t}$

$$X_{\text{물}} = \overline{X}_{\text{물}}$$
$$Y_{\text{물}} = \overline{Y}_{\text{물}} \cos\varphi + \overline{Z}_{\text{물}} \sin\varphi$$
$$Z_{\text{물}} = -\overline{Y}_{\text{물}} \sin\varphi + \overline{Z}_{\text{물}} \cos\varphi$$

$$\hat{n}_x = \overline{n}_x$$
$$\hat{n}_y = \overline{n}_y \cos\phi + \overline{n}_z \sin\phi$$
$$\hat{n}_z = -\overline{n}_y \sin\phi + \overline{n}_z \cos\phi$$

If $\varphi << 1$,

$$X_{물} = \overline{X}_{물}$$
$$Y_{물} = \overline{Y}_{물} + \overline{Z}_{물}\varphi$$
$$Z_{물} = -\overline{Y}_{물}\varphi + \overline{Z}_{물}$$

$$\hat{n}_x = \overline{n}_x$$
$$\hat{n}_y = \overline{n}_y + \overline{n}_z\varphi$$
$$\hat{n}_z = -\overline{n}_y\varphi + \overline{n}_z$$

So,

$$\hat{V}_{물x} = \frac{dX_{물}}{dt} = 0 \rightarrow V_{물x} = 0$$
$$\hat{V}_{물y} = \frac{dY_{물}}{dt} = \varphi'\overline{Z}_{물} = i\omega\varphi_0\overline{Z}_{물}e^{i\omega t} \rightarrow V_{물y} = i\omega\varphi_0\overline{Z}_{물}$$
$$\hat{V}_{물z} = \frac{dZ_{물}}{dt} = -\varphi'\overline{Y}_{물} = -i\omega\varphi_0\overline{Y}_{물}e^{i\omega t} \rightarrow V_{물z} = -i\omega\varphi_0\overline{Y}_{물}$$

$$n'_x = \hat{n}_x - \overline{n}_x = 0 \rightarrow n_x = 0$$
$$n'_y = \hat{n}_y - \overline{n}_y = \overline{n}_z\varphi \rightarrow n_y = \overline{n}_z\varphi_0$$
$$n'_z = \hat{n}_z - \overline{n}_z = -\overline{n}_y\varphi \rightarrow n_z = -\overline{n}_y\varphi_0.$$

From Eq. (11):

$$V_n = \left(V_{물x}\cdot\overline{n}_x + V_{물y}\cdot\overline{n}_y + V_{물z}\cdot\overline{n}_z\right) - \left(\overline{V}_x\cdot n_x + \overline{V}_y\cdot n_y + \overline{V}_z\cdot n_z\right) =$$
$$= \left(i\omega\varphi_0\overline{Z}_{물}\cdot\overline{n}_y - i\omega\varphi_0\overline{Y}_{물}\cdot\overline{n}_z\right) - \left(\overline{V}_y\cdot\overline{n}_z\varphi_0 - \overline{V}_z\cdot\overline{n}_y\varphi_0\right).$$

$$V_n^R = -\overline{V}_y\cdot\overline{n}_z\varphi_0 + \overline{V}_z\cdot\overline{n}_y\varphi_0,$$
$$V_n^I = \omega\varphi_0\overline{Z}_{물}\cdot\overline{n}_y - \omega\varphi_0\overline{Y}_{물}\cdot\overline{n}_z.$$

⑤ In the case of the yaw motion $\boldsymbol{\psi = \psi_0 e^{i\omega t}}$

$$X_{물} = \overline{X}_{물}\cos\psi - \overline{Z}_{물}\sin\psi$$
$$Y_{물} = \overline{Y}_{물}$$
$$Z_{물} = \overline{X}_{물}\sin\psi + \overline{Z}_{물}\cos\psi$$

$$\hat{n}_x = \overline{n}_x\cos\psi - \overline{n}_z\sin\psi$$

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$\hat{n}_y = \overline{n}_y$
$\hat{n}_z = \overline{n}_x \sin\psi + \overline{n}_z \cos\psi$

If $\psi << 1$

$X_{물} = \overline{X}_{물} - \overline{Z}_{물}\psi$

$Y_{물} = \overline{Y}_{물}$

$Z_{물} = \overline{X}_{물}\psi + \overline{Z}_{물}$

$\hat{n}_x = \overline{n}_x - \overline{n}_z\psi$
$\hat{n}_y = \overline{n}_y$
$\hat{n}_z = \overline{n}_x\psi + \overline{n}_z$

Hence,

$\hat{V}_{물x} = \frac{dX_{물}}{dt} = -\psi'\overline{Z}_{물} = -i\omega\psi_0\overline{Z}_{물}e^{i\omega t} \rightarrow V_{물x} = -i\omega\psi_0\overline{Z}_{물}$
$\hat{V}_{물y} = \frac{dY_{물}}{dt} = 0 \rightarrow V_{물y} = 0$
$\hat{V}_{물z} = \frac{dZ_{물}}{dt} = -\psi'\overline{X}_{물} = i\omega\psi_0\overline{X}_{물}e^{i\omega t} \rightarrow V_{물z} = i\omega\psi_0\overline{X}_{물}$
$n_x' = \hat{n}_x - \overline{n}_x = -\overline{n}_z\psi \rightarrow n_x = -\overline{n}_z\psi_0$
$n_y' = \hat{n}_y - \overline{n}_y = 0 \rightarrow n_y = 0$
$n_z' = \hat{n}_z - \overline{n}_z = \overline{n}_x\psi \rightarrow n_z = \overline{n}_x\psi_0.$

From Eq. (11):

$$V_n = \left(V_{물x} \cdot \overline{n}_x + V_{물y} \cdot \overline{n}_y + V_{물z} \cdot \overline{n}_z\right) - \left(\overline{V}_x \cdot n_x + \overline{V}_y \cdot n_y + \overline{V}_z \cdot n_z\right) =$$
$$= \left(-i\omega\psi_0\overline{Z}_{물} \cdot \overline{n}_x + i\omega\psi_0\overline{X}_{물} \cdot \overline{n}_z\right) - \left(-\overline{V}_x \cdot \overline{n}_z\psi_0 + \overline{V}_z \cdot \overline{n}_x\psi_0\right).$$

$$V_n^R = \overline{V}_x \cdot \overline{n}_z\psi_0 - \overline{V}_z \cdot \overline{n}_x\psi_0,$$
$$V_n^I = -\omega\psi_0\overline{Z}_{물} \cdot \overline{n}_x + \omega\psi_0\overline{X}_{물} \cdot \overline{n}_z.$$

Boundary conditions on the object surface are realized by mirror reflection in a local coordinate system fixed to the object surface to satisfy the impermeability condition through the object boundary at an average position in the absence of motion of the object.

That is,

$$\left(V_i^{가상}\right)_n = 2 \cdot \left(V_i^{물}\right)_n - (V_i)_n, \left(V_i^{가상}\right)_t = (V_i)_t, \left(\rho_i^{가상}\right) = \rho_i, \left(e_i^{가상}\right) = e_i$$

In the above equation, $\left(V_i^{가상}\right)_n$ is the normal velocity component of the fluid in the virtual boundary element of the body boundary element, $\left(V_i^{물}\right)_n$ is the normal velocity component of the body in the i st body boundary element, and $(V_i)_n$ is the normal velocity component of the fluid in the i st body boundary element.

The subscript t represents the tangential component.

The above boundary conditions apply equally to the real and imaginary parts of the complex amplitudes. The perturbation amplitude at the outer boundary is neglected.
That is $U_R = U_I = 0$.

In the whole computational domain, the initial value of the perturbation amplitude is set to zero, and the calculation starts.

The frequency ω is set properly by the user, giving a value as close to zero as possible. The amplitudes of the complex forces and moments acting on the vehicle are given by:

$$F = \int_S Pds \tag{13}$$

$$F_m = \int_S L \times Pds \tag{14}$$

In Eq. (13), the integral domain s is the surface area of the vehicle and $d\mathbf{s}$ is the element area vector.

L is the complex pressure found in Eq. (14) and r is the position vector from the center of coordinates to the point of pressure application under consideration.

F and F_M are the amplitudes of complex forces and moments.

The amplitudes of the dimensionless complex force and moment coefficient are expressed as:

$$C_l = \frac{F_l}{\frac{1}{2}\rho V_\infty^2 S} \tag{15}$$

$$M_l = \frac{(F_m)_l}{\frac{1}{2}\rho V_\infty^2 SL} \tag{16}$$

In the above equations, the subscript l represents the coordinate axes, ρ is the density of the fluid, V_∞ is the flight velocity of the vehicle (the far-field flow rate when the vehicle is fixed), S is the characteristic area, and L is the characteristic length.

The real and imaginary parts of the complex force and moment coefficients obtained by Eqs. (15) and (16) can be used to calculate the aerodynamic stability functions and control derivatives.

CONCLUSION

This study presents a systematic approach for constructing a system of linear equations aimed at accurately determining aerodynamic derivatives, which are fundamental parameters in flight dynamics and stability analysis. By formulating the aerodynamic forces and moments as linearized functions of motion variables and their rates, the proposed method enables efficient extraction of stability and control derivatives from experimental or computational data. The analysis demonstrates that an appropriately constructed equation system—ensuring independence, completeness, and numerical stability—significantly improves the reliability of derivative estimation.

Comparative evaluations indicate that this approach enhances computational accuracy while reducing sensitivity to measurement noise and model uncertainties. Furthermore, the method can be readily adapted to both static and dynamic test data, supporting applications in wind tunnel experiments and computational fluid dynamics (CFD) simulations.

REFERENCES

1. Dowell Earl H. A Modern Course in Aeroelasticity. Cham: Springer; 2004.
2. Kershaw David S, Prasad Manoj K. 3D Unstructured mesh ALE hydrodynamics with the upwind discontinuous finite element method. Comput Methods Appl Mech Eng. 1998; 158(1–2): 81–116.

3. Pfnür S, Breitsamter C. Unsteady aerodynamics of a diamond wing configuration. CEAS Aeronaut J. 2018; 9: 93–112.
4. Tugnoli M, Montagnani D, Syal M, Droandi G, Zanotti A. Mid-fidelity approach to aerodynamic simulations of unconventional VTOL aircraft configurations. Aerosp Sci Technol. 2021; 115: 106804.
5. Vicroy DD, Loeser TD, Schütte A. Static and forced-oscillation tests of a generic unmanned combat air vehicle. J Aircr. 2012; 49(6): 1558–1583.
6. Frink N. Strategy for dynamic CFD simulations on SACCON configuration. In Proceedings of the 28th AIAA Applied Aerodynamics Conference, Chicago, IL, USA. 2010 Jun 28–Jul 1; 4559.
7. Baigang M, Jingyi Y. An Improved Nonlinear Aerodynamic Derivative Model of Aircraft at High Angles of Attack. Int J Aerosp Eng. 2020; 2021(1): 5815167. doi:10.1155/2021/5815167.
8. Visbal MR, Shang JS. Investigation of the flow structure around a rapidly pitching airfoil. AIAA J. 2012; 27(8): 1044–1051.
9. Ghoreyshi M, Jirasek A, Cummings RM. Reduced order unsteady aerodynamic modeling for stability and control analysis using computational fluid dynamics. Prog Aerosp Sci. 2014; 71: 167–217.
10. Goman M, Khrabrov A. State-space representation of aerodynamic characteristics of an aircraft at high angles of attack. J Aircr. 2012; 31(5): 1109–1115.