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Time and Frequency Response of Non-uniform Overhead Lines Under Corona

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Abstract

This study presents an efficient and direct technique for determining both the time and frequency responses of non-uniform overhead power transmission lines operating under corona conditions. The assumed-line's non-uniformity is due to the conductors' sag. Expressions will be presented for the location-dependent line surge impedance and the unevenly distributed lines' electrical parameters. The analysis starts with solving the relevant system of differential and algebraic equations, subject to the boundary conditions. Their numerical solution is possible through the application of the software program Mathematica, leading to Laplace-domain expressions for the line's voltage and current distributions, in terms of the location and the Laplace operator location's. The line's input impedance can then be derived from the voltage and current at the sending end. The line's resonance frequencies can also be easily determined. A numerical Laplace inversion using the Hosono algorithm will lead to the corresponding voltage and current equations in the time domain. The effect of the line length, the number of towers, the intensity of the corona discharge as well as the loading condition is taken into consideration. The suggested approach is applied to a typical multi-span overhead high voltage transmission line. A special test case study is investigated to validate the suggested approach and computer code.

Keywords: Electromagnetic, transients, simulation, high voltage lines, differential equations, Parametricndsolve, Mathematica, numerical solutions, frequency response, Laplace transform, Hosono algorithm, Inverse Laplace transform

INTRODUCTION

The linearity and uniformity of the circuit parameters are two basic assumptions in the classical analysis of power network components. Examples of nonlinearities include iron-cored coils, surge arresters, the saturation phenomena as well as the corona discharges [1–4]. There are numerous publications dealing with analyzing power line transients in the presence of corona [5–10]. Various studies, for instance, assume distributed nonlinear shunt resistances (or currents for that matter) to simulate the line's transient performance under corona discharge [6–11]. Their parameters can be determined by curve fittings applied to the line's corona loss measurements [8, 11]. In time domain analyses, the lines are usually divided into adequate, usually large number of cascaded sections, for which a system of differential/algebraic simultaneous differential and algebraic equations are derived and then solved numerically to get the voltage and current transient responses [6–11]. The computation burden increases rapidly with the assumed number of sections. This study suggests another more practical and promising approach

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based on incorporating the nonlinear expressions of the corona current in terms of the local voltage into the simulating differential equations, as shown in Figure 1 [9, 10]. River crossings, line towers and sagging line conductors are examples for possible nonuniformities encountered in the analysis of high voltage overhead power lines as shown in Figure 2 [11].

The Method of Analysis

This study presents a new technique for line analysis in the simultaneous presence of non-uniformity and non-linearity. Special emphasis will be devoted to multi-span overhead lines with many supporting towers subject to corona discharge. Saied gives a corona model based on the following Eq. (1) expressing the corona current per unit length in terms of the voltage *v* at the considered location along the line [11]:

$$i_c = a.v^k \tag{1}$$

For the sample high voltage power line considered in the above equation 1, the two constants a and k could be determined as a=5 and k=10^{-32}, respectively. They were derived using curve fitting of available power loss data versus voltage. Saied derives the following expression for the dependence of the surge impedance on the coordinate x along an overhead transmission line [10]:

$$Z_0(x) = 145.(2 + 0.88[cos(\pi.x/span)^4]), \text{Ohms} \tag{2}$$

Where both the *span* and the coordinate *x* are substituted in meters. It has an average value of approximately 360 Ω. The nonuniform capacitance c(x) and inductance l(x) can then be derived as:

$$c(x) = 1/[Zo(x).speed]$$
$$l(x) = Zo(x)/speed \tag{3}$$

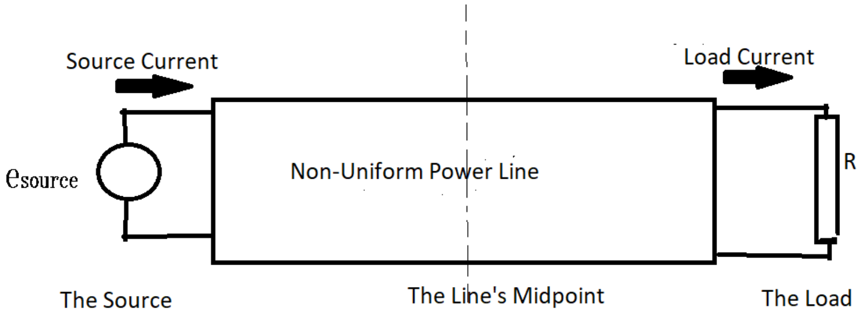


Figure 1. The Sample Radial Power Network.

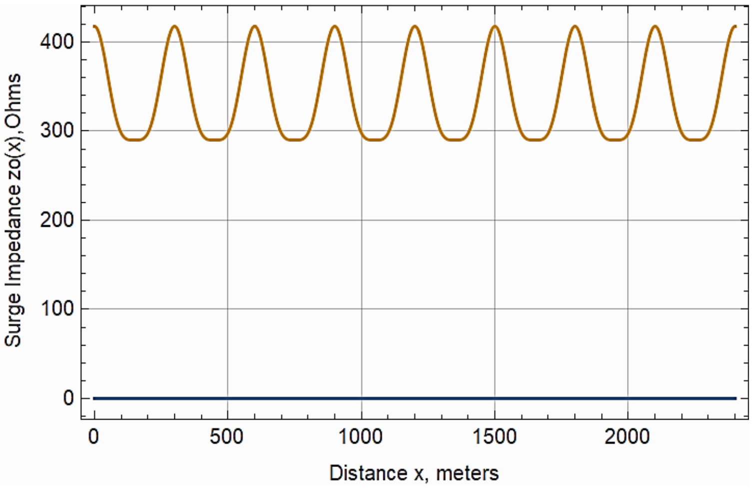


Figure 2. The assumed dependence of the surge impedance on the location x, measured from the source.

The governing partial differential equations for the voltage v(x, t) and the current *i(x, t)* are :

$$\frac{\partial v(x,t)}{\partial x} = -l\frac{\partial i(x,t)}{\partial t} \tag{4}$$

and

$$\frac{\partial i(x,t)}{\partial x} = -c\frac{\partial v(x,t)}{\partial t} - a.[v(x,t)]^k \tag{5}$$

They can be solved using the *Mathematica* statement (ParametricNDSolve). The two boundary conditions are that the voltage at the source (x=0) is equal to the source voltage e_{source} substituted in the Laplace domain, and that the voltage and current at the receiving end (i.e. x=line length), are satisfying Ohm's Law. The line length is equal to the product of the span and the number of tower spans n [12, 13]. The output voltage and current expressions should then be numerically Laplace-inverted. In this study, the Hosono algorithm has been used.

SAMPLE RESULTS

Case Study (A)

- Line Length: 2400 m,
- Series resistance m=0.1 mΩ
- Number of Tower Spans: 8; 300 m each.

The solution is done over the time range from t=0 to 0.4 msec. During this time the source voltage increases linearly from 0 to about 400 kV. Source Voltage $esource(t) = 10^8 t$ V, or $(100/s^2)MV$ in the Laplace domain. A high intensity corona conditions are assumed with the following previously defined constants:

a=5 and k=10^{-32}

The line receiving end is loaded with a 100 Ω resistance. The plots of the time response are depicted on logarithmic scales. The time-axes start from about 0.667 sec. The final values of the voltages and currents are about 200 mV and 2 kA, respectively.

The dependence of the magnitude of the line's input impedance shows several minima and maxima at parallel and series resonances. A large impedance occurs at 30 and 90 kHz, etc., exhibiting an approximate value of 1100 Ω. The least value of the impedance (100 Ω) appears at approximately 63 kHz. It is approximately equal to the initially assumed load resistance.

This agrees well with the plot giving the line, s impedance angle. The impedance locus is shown in the last plot of Figure 3. The real part of the impedance changes between the two extreme values 100 and 1100 Ω, whereas the imaginary part varies between +500 and –500 Ω.

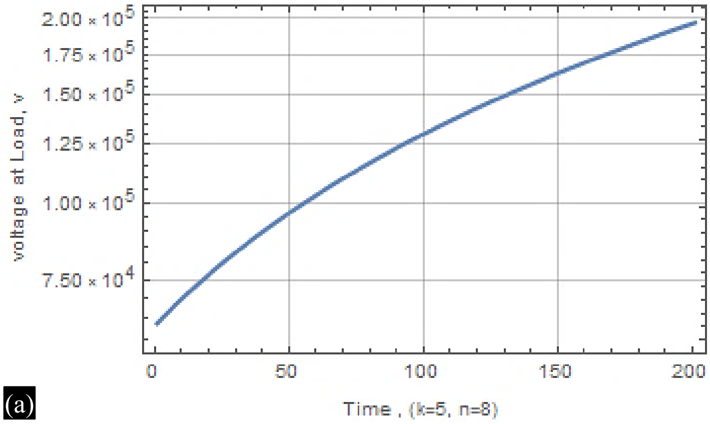
For convenience, the plots of the time response are depicted on logarithmic scales. The time-axes plots start from about 0.667 sec. The final values of the voltages and currents are about 200 kV and 2 kA, respectively.

Case Study (B)

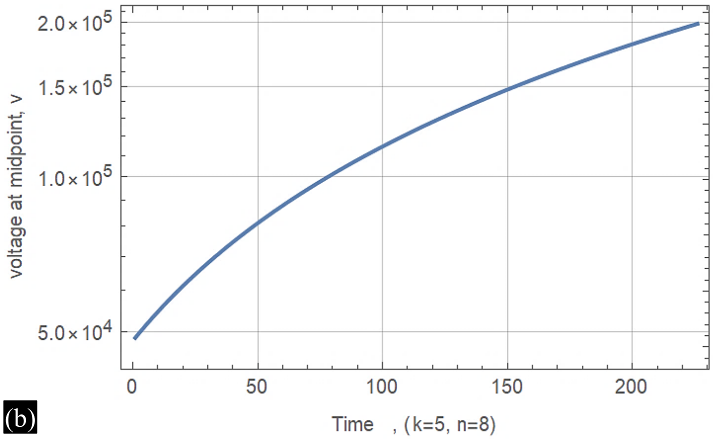
- Line Length: 2400 m,
- Number of Tower Spans: 8; 300 m each,
- Source Voltage $esource(t) = 10^8 t$ V.

No Corona

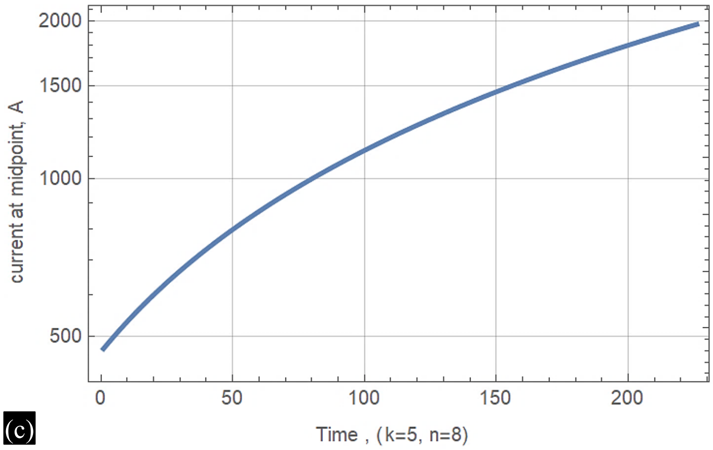
Represented by the value a=0.


2.00 × 10^5
1.75 × 10^5
1.50 × 10^5
1.25 × 10^5
1.00 × 10^5
7.50 × 10^4
voltage at Load, v
0
50
100
150
200
Time , (k=5, n=8)

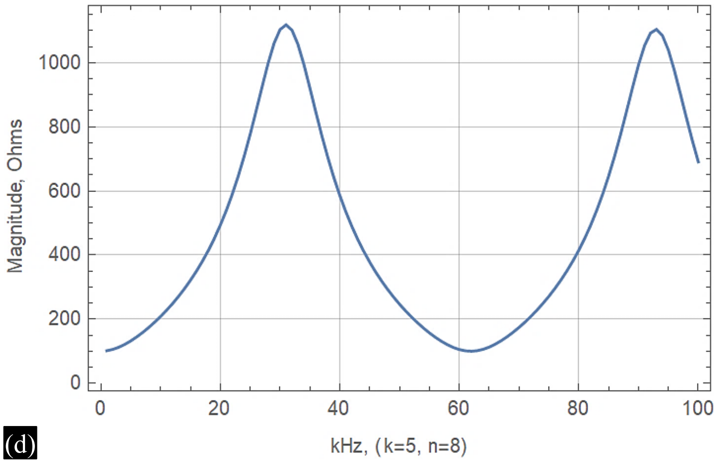
(a)


2.0 × 10^5
1.5 × 10^5
1.0 × 10^5
5.0 × 10^4
voltage at midpoint, v
0
50
100
150
200
Time , (k=5, n=8)

(b)


2000
1500
1000
500
current at midpoint, A
0
50
100
150
200
Time , (k=5, n=8)

(c)


1000
800
600
400
200
0
Magnitude, Ohms
0
20
40
60
80
100
kHz, (k=5, n=8)

(d)

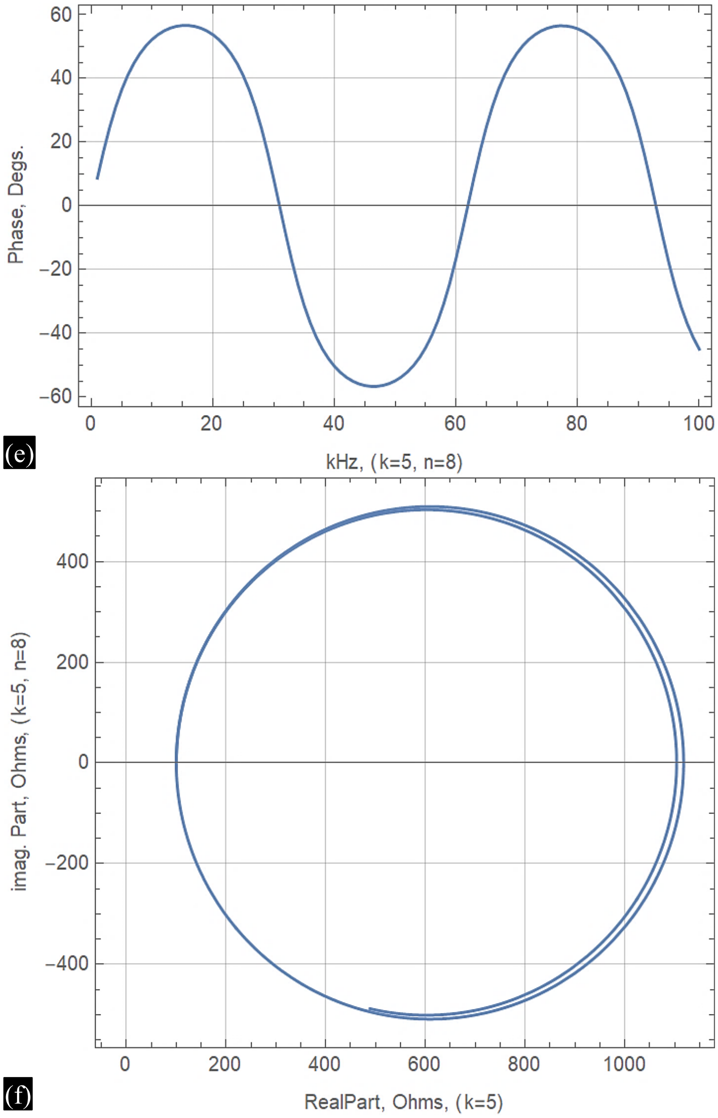


Figure 3. (a–f) The computed results of the voltages and current transients (1 division =10 µsec) and the frequency characteristics of the line's input impedance over the frequency range from 0 to 100 kHz, (1division=1 kHz).

This section deals with the computed results of the same line treated in the previous Case Study (B), but in the absence of corona discharge. Both the time domain voltage and current transients as well as the frequency characteristics of the line's input impedance are illustrated.

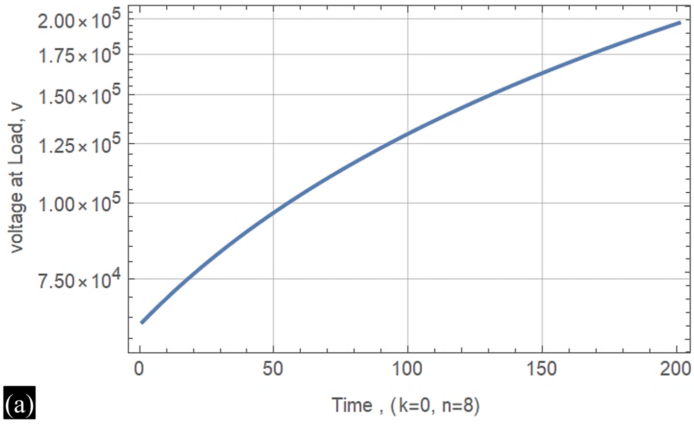
The corresponding plots in Figure 4 indicate the changes in the time- and frequency-domain results due to the presence of corona. They should be also investigated if the line load is not pure resistive, i.e. if it comprises series- or parallel, capacitive and/or inductive circuit elements.

Case Study (C)

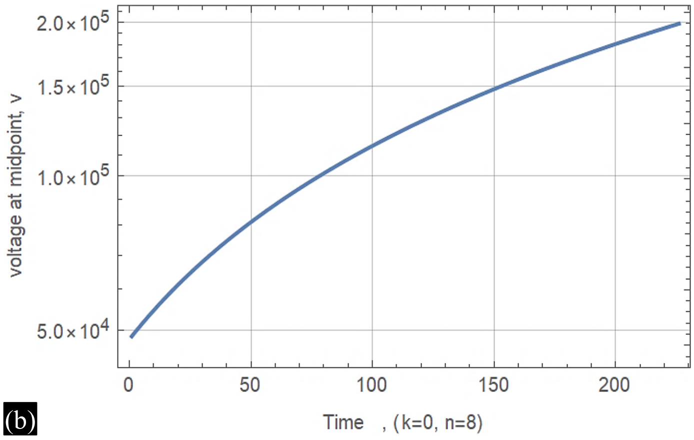
The Test Case,
Line Length 2400 m,
Line_Assumed Uniform:

- Constant Surge Impedance=360 Ω
- Number of Tower Spans: 8; 300 m each,
- Source Voltage $esource(t) = 10^8 t$ V.

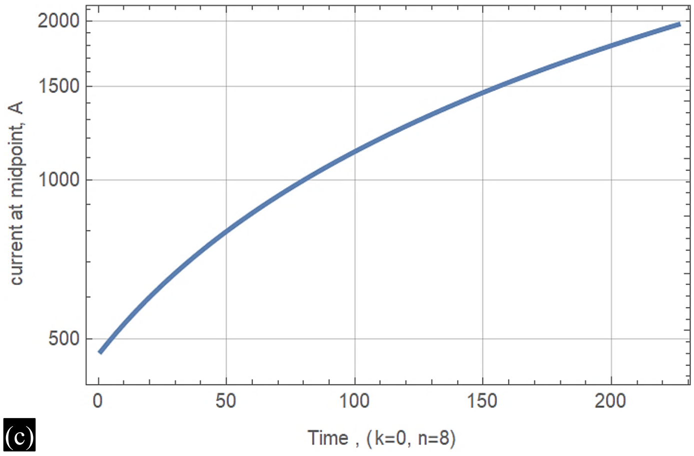
No Corona: i.e. a=0


2.00 × 10^5
1.75 × 10^5
1.50 × 10^5
1.25 × 10^5
1.00 × 10^5
7.50 × 10^4
voltage at Load, v
0
50
100
150
200
Time , (k=0, n=8)

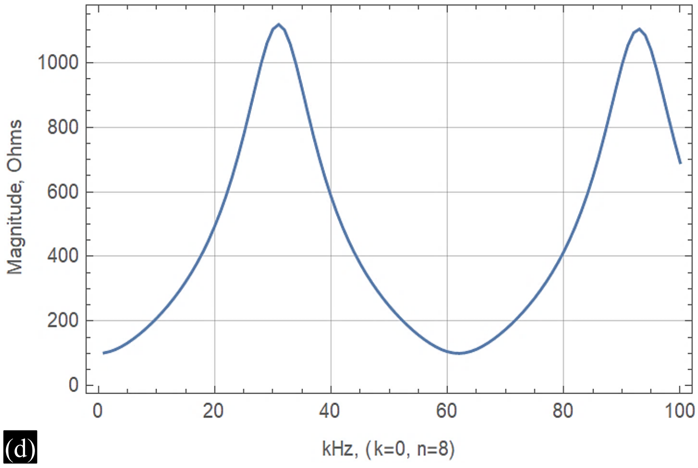
(a)


2.0 × 10^5
1.5 × 10^5
1.0 × 10^5
5.0 × 10^4
voltage at midpoint, v
0
50
100
150
200
Time , (k=0, n=8)

(b)


2000
1500
1000
500
current at midpoint, A
0
50
100
150
200
Time , (k=0, n=8)

(c)


1000
800
600
400
200
0
Magnitude, Ohms
0
20
40
60
80
100
kHz, (k=0, n=8)

(d)

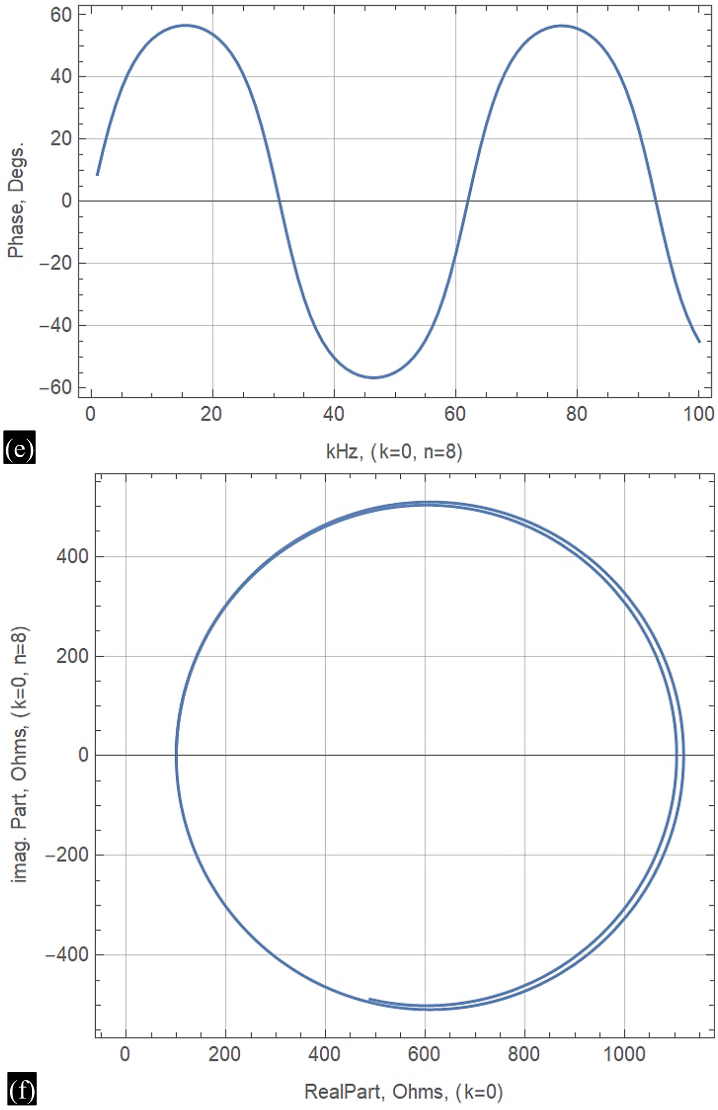


Figure 4. (a–f) The computed results of the voltages and current transients (1 division =10 μsec) and the frequency characteristics of the line's input impedance over the frequency range from 0 to 100 kHz, (1 division =1 kHz). Line extends over 8 tower spans. No Corona.

In this test case study, the source current is computed using both the *Mathematica* code as well as the exact solution based on the inverse Laplace transform of the following expression derived from the long line theory:

$$I_{source} = \frac{SourceVoltage}{ZInput} \tag{6}$$

Where the assumed source voltage is 100 MV/(s2),

And the input impedance is $\frac{(100Cosh\tau s+360Sinh\tau s)}{(100Sinh\tau s/360+Cosh\tau s)}$ (7)

The plot of this current is given in Figure 5. It is identical to the result obtained from the developed *Mathematica* code.

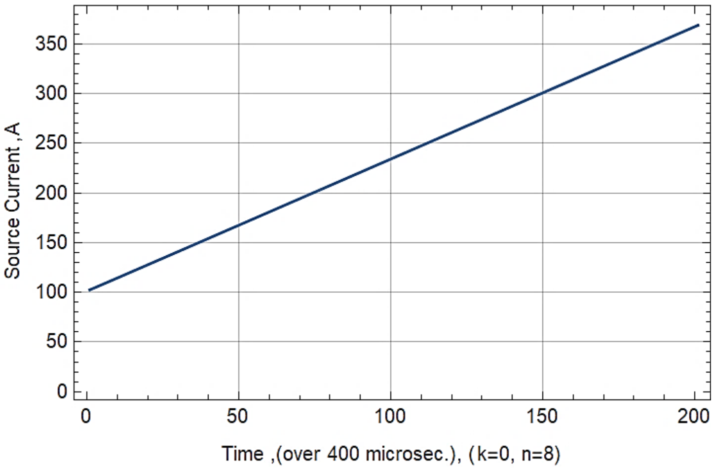


Figure 5. The Computed Source Current, based on analytical analysis, versus time in the test case.

CONCLUSION

The study gives a detailed derivation of a mathematical model and the associated computer program capable of analyzing the time and frequency response of long nonuniform overhead high voltage power lines subject to corona discharge. The line is composed of several cascade-connected tower spans. In principle, any number of supporting towers, any value of the conductor sag and any waveform of the voltage source initiating the transients can be dealt with. The analysis is based on the classical line theory as well as the efficient application of numerical inversion of Laplace Transform using the software *Mathematica*. The results of several case studies are presented and discussed. The results show how to determine the impact of eventually existing corona discharge on the transient voltages and currents and on the frequency characteristics of the line's input impedance. This includes the identification of the resonance frequencies. The investigation also includes the results pertinent to a special test case study aiming at validating the suggested approach and computer code.

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