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Evaluation of T-stress using FEM in an Unsymmetrical Cracks Emanating from a Hole Placed Centrally in a Thin Plate

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Abstract

Studying the impact of higher order terms of the William's series expansion on fracture mechanics problems has received more attention in recent years. Research has demonstrated that the constant term T-stress has a significant impact on the stress and strain fields around the crack tip. In response to mixed mode (I/II) loading, a number of fracture criteria had been developed to characterize failure in linear elastic bodies. When evaluating the residual life in structures with micro-cracks, stress intensity factor (SIF) and T-stress are crucial factors to be considered in fracture mechanics. In this research, SIF and T-stress are computed using finite element method. At crack tip, higher order elements were utilized to depict the behavior of displacement. There are various technique to obtain T-stress. Here finite element method is applied in a new test specimen and examined. It is possible to evolve fracture parameters like T-stress of an unsymmetrical cracks originating from the hole edges under tension. T-stress at the crack tip had drawn attention; nonetheless, majority of the theoretical studies had been seen for single-crack problems, with relatively less information available regarding closed form solutions for T-stress at the tips under biaxial loading. A summary of T-stress is provided in this research study to better understand how T-stress varies with respect to the crack inclination angle, hole diameter, and biaxial load factor. The objective is to propose a finite element technique for numerical analysis in ANSYS APDL software in 2D. The current study uses correlation counterplots between T-stress and stress intensity factor to better understand the structural integrity of the specimen.

Keywords: Fracture Mechanics, T-stress, Unsymmetrical cracks, Finite element method

INTRODUCTION

Material will fail only when it contains any micro- cracks and further it results in growth of voids and hence becomes localized. As a result, the discontinuities will develop and macroscopic crack occur leading to global failure. So, failure of material is due to development of cracks. The cracks may occur due to dynamic and static loading within the materials. There are three modes of failure, mode I (opening mode), mode II (shearing mode) and mode III (tearing mode). Sometimes it is not possible to find an analytical solution due to the complexity of the configuration of unsymmetrical cracks originating from holes. Numerical methods like FEM, BEM, and others have been used to solve these kinds of difficult problems. FEM is the most useful approach for resolving problems of this nature. The fracture parameters can be calculated using a variety of methods. T-stress is an important parameter in fracture mechanics. Stresses in the plastic zone are influenced by the T-stress.

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Toughness in Mode I as well as Mode II, T-stress has the most significant effect. **Cotterell and Rice (1980)** proposed the higher orders terms (non-singular terms) in the William series expansion as the T-stress to predict the crack path under pure Mode I in metals. This became important to predict the path stability. [1] According to them for T< 0 the crack path is more stable. T-stress can be positive (tensile) or negative (compressive). Positive (tensile) T-stress increases the stress of a material but reduces the materials strength while Negative (compressive) T-stress decreases the stress within a crack tip and increases the material strength. Magnitude of T-stress depends upon the size, loading mode and crack size. Through many studies less attention had been paid to the higher order terms [2-3] showed that magnitude and sign of T-stress vary the shape and size of plane strain crack tip zone. As compared with no T-stress situation there is increase in the plastic zone size of positive and negative T-stress. For T>0, in plane strain condition the plastic zone oriented along crack extension but for T<0 in opposite zone. [4-6] showed that when negative T increases fracture toughness increases.

Evaluation of T-stress from finite element analysis (FEA) for mode I and mode II loading was done by [7]. According to them T-stress can be obtained directly with the use of displacement u_x along the cracks. Displacement method provides more feasible results as compared with the stress method. Without calculating stress intensity factor (K), T-stress can be calculated directly using stress or displacement method along two faces of crack in mixed mode problems. [8] determined the mode I cracks in a rectangular bar when subjected to large T-stress. They theoretically studied the brittle fracture for mode I cracks and used the modified maximum tangential stress (MTS) criterion for mixed mode loading. It was observed that for large value of positive T-stress the crack initiation in brittle fracture for mode I cracks occurs at an angle rather than directly ahead of crack. [9] in Paris law regime proposed when T increases fatigue crack growth rate decreases. T-stress is used as a measure of constraint ahead of crack tip when analytically and experimentally studied. According to [10] value of T-stress increases from higher negative value to lower negative value whenever there is variation of specimen loading mode and change in geometry from tension to bending. [11] for various specimen configurations large range values of T-stress was investigated. Both positive and negative values of T-stress was obtained for CT specimen made of steel. T-stress value is sensitive to specimen, geometry, loading mode and size of cracks. Investigation upon the methods of T-stress calculation in fracture mechanics was done by [12]. Major objective of research was the numerical study and numerical calculation of T- stress in 2-D using finite element method (FEM) by ANSYS 15.0 software. In the interval a/w = 0.2 to 0.6, the values of T-stress were positive for CT specimen under tension stress. Observation also showed that for a CT specimen along ligament (θ=0°) and crack mouth (θ=180°) T-stress is not constant ahead of a crack tip. As they increases the distance from the crack tip for the cracks, the non- singularity terms were not negligible. This showed that for the crack depth ratio variation the value of T-stress was not constant and with this type of distribution theory can't be developed whereas Maleski method provides good results for the T-stress along ligament (θ=0°) and crack mouth (θ=180°). [13] evolved the convergence, accuracy and the domain-independence of the integral connection numerically. [14] determined stress intensity factor (SIF) and T-stress around the Hypocycloidal-shaped holes in sheet structures. The findings indicated that when plate size increased, T-stress decreases. There was a noticeable difference between finite and infinite plates for the T-stress.

MATERIALS AND METHOD

Around 8% of the earth's crust is made up of aluminum, the third most frequent element and most abundant metal in the world. It is the most extensively used metal after steel due to its versatility. The metal is white and silvery. It comes from the mineral bauxite. Through the Bayer process, aluminum oxide (alumina) is produced from bauxite. Aluminum metal is obtained by converting alumina into an electrolytic cell and using the Hall-Heroult process. Properties of aluminium includes poison's ratio is 0.33, Young's modulus is 71 GPa and density is 2710 kg/m^3. Pure aluminum has excellent electrical conductivity, is soft, ductile, recyclable, and resistant to corrosion. You must alloy with other elements to get greater strength. It is the lightest metal available and has a higher strength to weight ratio than steel. Foils, conductor cables, electronic casings, electrical transmission lines, and other applications use it. Zinc, copper, silicon, lithium, and manganese are alloyed with it. Aluminum is the lightest metal since its density is only one-third that of steel or copper.

Numerical Approach

T-stress was determined using FEM software (ANSYS 2020) in APDL for a thin plate with unsymmetrical cracks originating from a circular hole. Analysis was conducted on different crack orientations under various conditions.

Finite Element Method (FEM)

A numerical method of approximate solution to partial differential equation problems is the finite element method The problem are boundary value problem. The domain is divided into smaller components known as sub-domains or finite elements, which are joined to one another at joints, and are known as nodes. Every node in the continuum has fixed attributes and a finite degree of freedom. We call a group of elements a mesh. Discreteization is the process of describing a component as a collection of elements. Three steps are involved in FEM, and they are as follows: Processing, post-processing, and preprocessing.

SPECIMEN GEOMETRY AND ASSUMPTIONS

Figure 1 shows the geometry of the specimen. L (length of specimen), W (width of the specimen), α is crack inclination angle (α= 0° to 60°), σ is load applied in y-direction**,** βσ is load applied in x-direction A, B are crack tips, a_1 and a_2 are crack lengths and β is biaxial load factor. A uniformly distributed load and plane stress condition with the following dimensions were suggested for the current aluminum plate. T-stress values were computed for various crack tip A and B configurations. Understanding the impact of the geometrical parameters crack inclination angle (α), biaxility factor (β), and hole diameter (d) would be made easier with the help of geometry.

Element Type used

For analysis Plane183 is used as shown in Figure 2. It is a higher order 2-D, 8 node or 6 node element. It provides quadratic displacement and is most suitable for modelling irregular meshes. The element is having two degree of freedom at each nodes that is translation in x and y directions. The element can be used as plane element (plane stress, generalized plane strain, plain strain or as an axi-symmetric element). The element is having hyper-elasticity, plasticity, creep, stress stiffening, large strain and large deflection capacities. It also have mixed formulation capability for stimulating deformation of nearly incompressible elasto-plastic materials and fully incompressible hyper-elastic materials. It also supports the initial stress import.

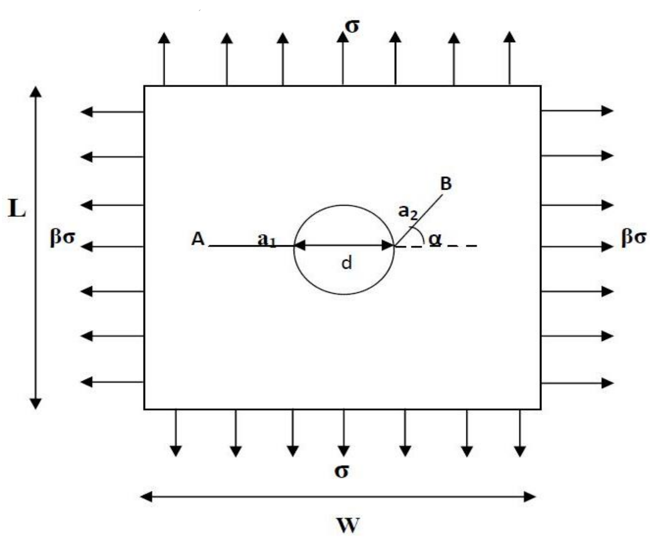

Figure 1. Specimen geometry.

The data can be obtained for all nodes of elements in contour plots or tabular form. To obtain stress intensity factor, path is manually defined by selecting five nodes at the crack face. After defining the path stress intensity factor (SIF) were obtained.

Discretization of Model

It is an important process in FEM. In this, the model is divided into number of elements or sub-elements. The elements are of different shape and sizes and they do not overlap each other. Each elements are connected to each other through nodes. The discretization of continuum is known as mesh generation. For perfect solution the division of body into small elements is necessary.

Modelling a crack tip is a big issue in fracture analysis. So, quarter point (1/4) singular element should be used for the analysis of fracture parameter at the crack tip as shown in figure 3. Plane 183 element is used for 2-D fracture mechanics problems.

Boundary Condition

In mechanics problem, one should specify the boundary conditions of model to obtain the solution. It is the last phase of analysis. It is to set the known value of displacement or an associated load. Boundary conditions for the analysis are pressure, force, temperature. Here, in the analysis biaxial tensile load is applied.

Solution Phase

In solution phase, the set of equations of finite element method were solved in finite element program. Primary solution is obtained by solving nodal degree of freedom that is rotation and displacement. Then the derived results were obtained by calculating the stress and strain at the integral points. In ANSYS software the primary were called nodal solution and the derived results are called element solution. Various methods are there for solving simultaneous system of equations and some of them are for larger modes, some are faster for non-linear analysis, some allows to distribute the solution by parallel computation. Commercial finite element program solve such type of equations in batch modes. The direct frontal solution method is used commonly because of more efficient finite element analysis.

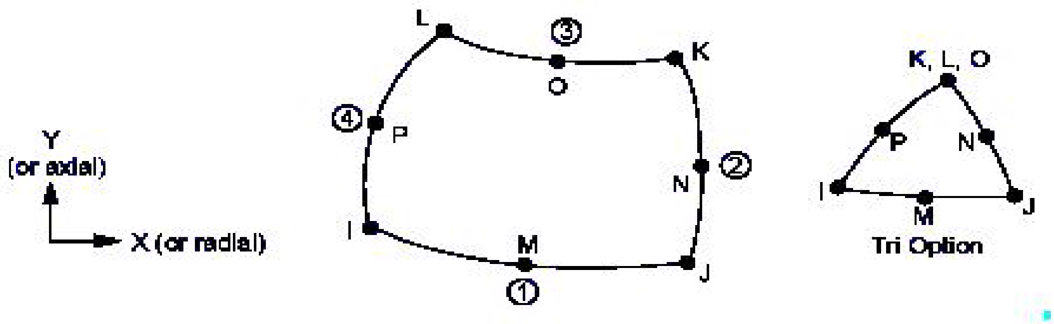

Figure 2. Plane 183 with 8-node element.

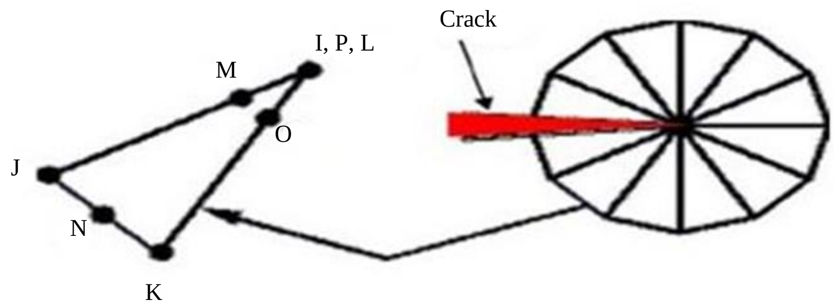

Figure 3. Singular element.

For non-linear analysis, the equation should be solved repeatedly thus significantly increasing the computational time. Some of the methods for solving system of equation are sparse direct solution, Incomplete Cholesky conjugate gradient solution, Jacobi conjugate gradient solution, an automatic iterative solver option and preconditioned conjugate gradient solution.

NUMERICAL APPROACH FOR ANALYSIS OF T-STRESS

Displacement method for evaluating T-stress from **Ayatollahi (1998)** are used which is as:

For mode I,

$$T = \frac{E'[U_x(x) - U_x(0)]}{x} \tag{1}$$

For mode II,

$$T = \frac{E'[U_x(x,-\pi) - U_x(x,\pi)]}{2x} \tag{2}$$

Here,

E′= E - Young's modulus for stress condition
U_x - Displacement in x-direction
x - distance of the first row of the element from the crack tip.

CRACK ANALYSIS

For a linear elastic fracture mechanics analysis the stress intensity factor (SIF) at a crack may be computed (using the KCALC command). The analysis in the vicinity of the crack uses a fit of the nodal displacements. The actual displacements for linear elastic materials at and near a crack are:

$$u = \frac{K_I}{4G}\left(\sqrt{\frac{r}{2\pi}}\right)\left[(2k-1)cos\frac{\theta}{2} - cos\frac{3\theta}{2}\right) - \frac{K_{II}}{4G}\left(\frac{\sqrt{r}}{2\pi}\right)\left[(2k+3)sin\frac{\theta}{2} + sin\frac{3\theta}{2}\right] + O(r) \tag{3}$$

$$v = \frac{K_I}{4G}\left(\sqrt{\frac{r}{2\pi}}\right)\left[(2k-1)sin\frac{\theta}{2} - sin\frac{3\theta}{2}\right) - \frac{K_{II}}{4G}\left(\frac{\sqrt{r}}{2\pi}\right)\left[(2k+3)cos\frac{\theta}{2} + cos\frac{3\theta}{2}\right] + O(r) \tag{4}$$

$$w = \frac{2K_{III}}{G}\left(\sqrt{\frac{r}{2\pi}}\right)\left[(sin\frac{\theta}{2})\right] + O(r) \tag{5}$$

Where,

u, v,w = displacements in a local Cartesian coordinate system as shown in Figure 4
r, θ = coordinates in a local cylindrical coordinate system also shown in Figure 4
G = shear modulus
K_I, K_{II} = Mode I and Mode II stress intensity factor
ν = Poisson's ratio
0(r) = terms of order r or higher

Evaluating equations at $\theta = \pm 180^0$ and neglecting the higher order terms

$$u = \frac{K_{II}}{2G}\left(\sqrt{\frac{r}{2\pi}}\right)(1+k) \tag{6}$$

$$v = \frac{K_I}{2G}\left(\sqrt{\frac{r}{2\pi}}\right)(k) \tag{7}$$

$$w = \frac{2K_{III}}{G}\left(\sqrt{\frac{r}{2\pi}}\right) \tag{8}$$

Equations for full crack model are as follows-

$$K_I = \left(\sqrt{2\pi}\right)\left(\frac{G}{k}\right)\left(\frac{|\Delta v|}{\sqrt{r}}\right) \tag{9}$$

$$K_{II} = \left(\sqrt{2\pi}\right)\left(\frac{G}{1+k}\right)\left(\frac{|\Delta u|}{\sqrt{r}}\right) \tag{10}$$

$$K_{III} = \left(\sqrt{2\pi}\right)G\left(\frac{|\Delta w|}{\sqrt{r}}\right) \tag{11}$$

Where Δv, Δu, and Δw are the motions of one crack face with respect to the other.

On the nodal displacements and locations k =3-4*v* if plane strain or axisymmetric; (3-*v*) if plane stress; where *v* is Poisson's ratio the final factor $\frac{\Delta v}{\sqrt{r}}$ is calculated.

$$\left(\frac{|\Delta v|}{\sqrt{r}}\right) = A + Br \tag{12}$$

At points J and K. R approach 0

$$lim_{r\to 0}\left(\frac{|\Delta v|}{\sqrt{r}}\right) = A \tag{13}$$

The equation becomes

$$K_I = \left(\sqrt{2\pi}\right)\left(\frac{2GA}{k}\right) \tag{14}$$

Similarly,

$$\left(\frac{|\Delta u|}{\sqrt{r}}\right) = C + Dr \tag{15}$$

$lim_{r\to 0}\left(\frac{|\Delta u|}{\sqrt{r}}\right) = C$ ..... (16)

The equation becomes

$$K_{II} = \left(\sqrt{2\pi}\right)\left(\frac{GC}{1+k}\right) \tag{17}$$

and,

$$\left(\frac{|w|}{\sqrt{r}}\right) = E + Fr \tag{18}$$

$$lim_{r\to 0}\left(\frac{|\Delta w|}{\sqrt{r}}\right) = E \tag{19}$$

The equation becomes

$$K_{III} = \left(\sqrt{2\pi}\right)G\left(\frac{|\Delta w|}{\sqrt{r}}\right) \tag{20}$$

RESULTS AND DISCUSSION

Validation for T-stress

The specimen used for validation of T-stress is shown in Figure 5. The different dimensions of the specimen are a/W= 0.2, W/L= 1, a= 2mm. Tensile stress of 100 MPa is applied to the specimen and the fracture parameter T-stress is obtained from the FEM. The existing results of **Ayatollahi (1998)** were used to compare with the results obtained by FEM. The Displacement method is used in the point load to calculate T/σ stress in 2D analysis.

Here, displacement method for calculation of T-stress has been used. The normalized T/σ stress obtained by Ayatollahi was 0.55 whereas the result obtained for normalized T/σ stress in the present study is 0.57. The percentage error is 3.50%. Hence, the results obtained by FEM can be well used for any type of specimen geometry. The results are presented graphically in Figure 6.

Effect of Crack Inclination Angle (α) on T-stress

Correlation between Stress Intensity Factor (SIF) and T-stress for Crack Tip A

From the Figure 7 it can be observed that the T-stress and SIF are strongly correlated. It is observed that normalized T-stress decreases as the mode-mixity increases. However, this decrease is only upto K_{IIA}/ K_{IA}=0.041 beyond which there is a gradual increase. The graph in the correlation is parabolic in nature.

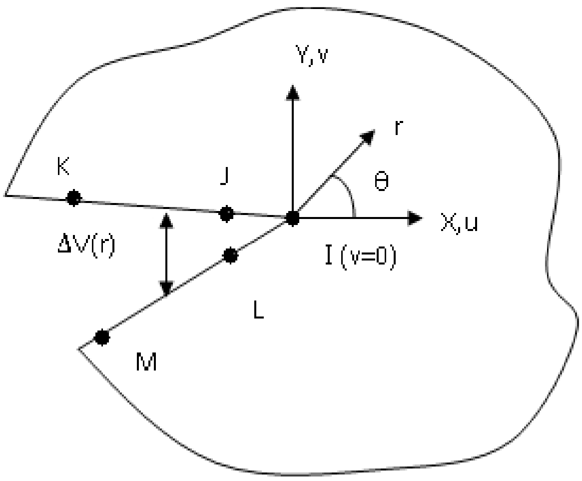

Figure 4. Representation of displacements in analysis.

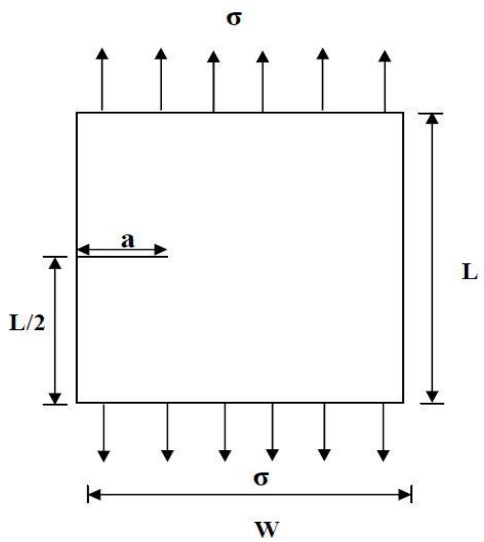

Figure 5. Side edge crack on a square plate.

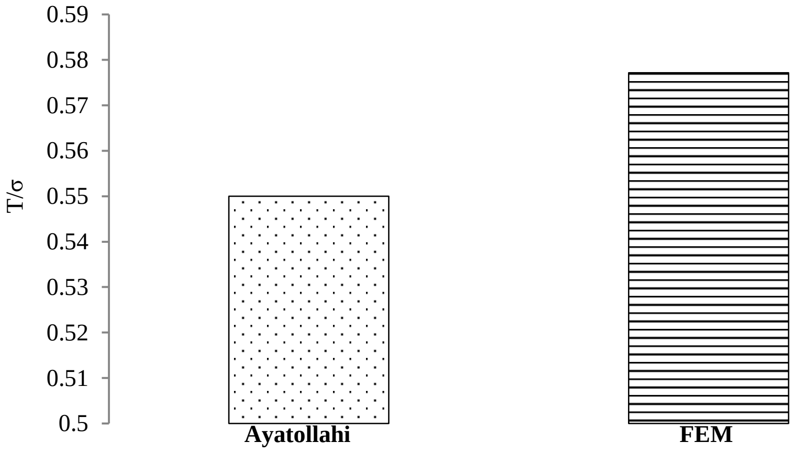

Figure 6. Comparison between the results obtained by Ayatollahi (1998) and FEM.

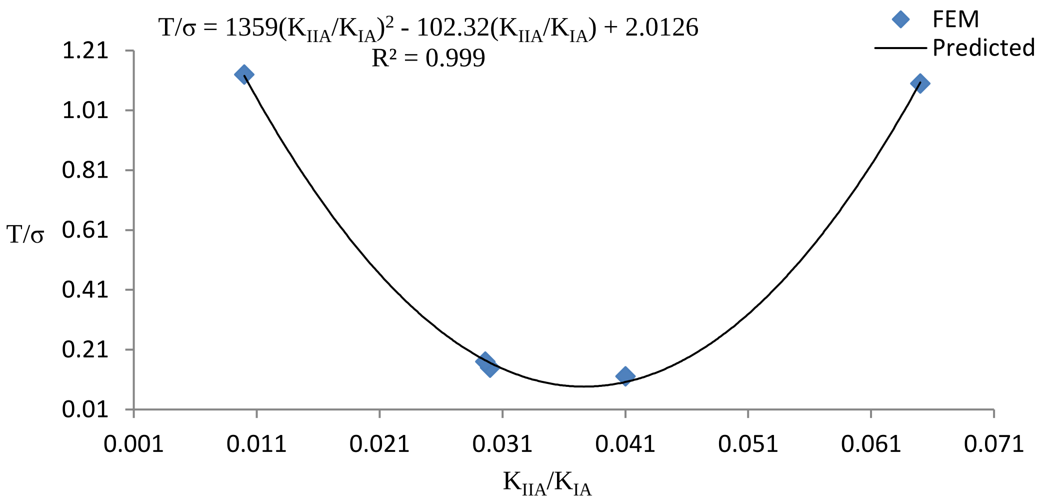

Figure 7. Correlation between SIF and T-stress for crack tip A.

Further it can be seen that T-stress is positive for all the ranges of K_{II}/K_I signify that the higher order terms cannot be neglected when studying these kind of cracked structure. However, T-stress can be neglected for predominately mode II type of loading.

Correlation Between SIF and T-stress for Crack Tip B

The correlation between mode-mixity and T-stress is studied for crack tip B. From the Figure 8, it can be concluded that as mode-mixity increases there is decrease in the T-stress significantly for crack tip B. It is also observed that correlation is parabolic in nature. The decrease is about 25%. Hence, T-stress cannot be neglected for failure of material.

The variation of crack inclination angle will affect the T-stress (crack path stability)**.** For crack tip A, T/σ value increases with increase crack inclination angle to 45° and this increment then it around 63.8% then it start to decrease. For crack tip B, T/σ value is decreased to α=60° which is 13%.

Effect of Biaxial Load Factor (β) on T-stress

Correlation Between SIF and T-stress for Crack Tip A

From the Figure 9 it can be observed that the T-stress and SIF are strongly correlated. It is observed that the normalized T-stress increases as the mode-mixity increases. However, this increase is only K_{IIA}/K_{IA}=0.71 beyond which it gradually decreases. It can be concluded that T-stress cannot be neglected for these kind of studies. For higher mode-mixity the T-stress becomes very high. It becomes important that T-stress is taken into account for these kind of studies hence, showing parabolic behavior.

Correlation Between SIF and T-stress for Crack TIP B

The correlation between stress intensity factor and T-stress is shown in the Figure 10. It also shows their correlation with each other. Results shows that with increase in the biaxial load factor (β) from 0 to 2 the T-stress decreases linearly.

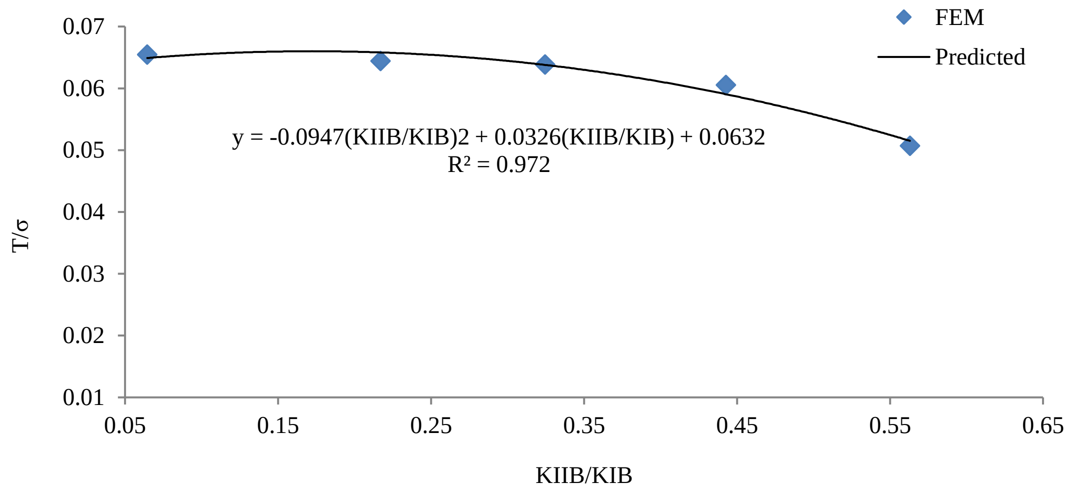

Figure 8. Correlation between SIF and T-stress for crack tip B.

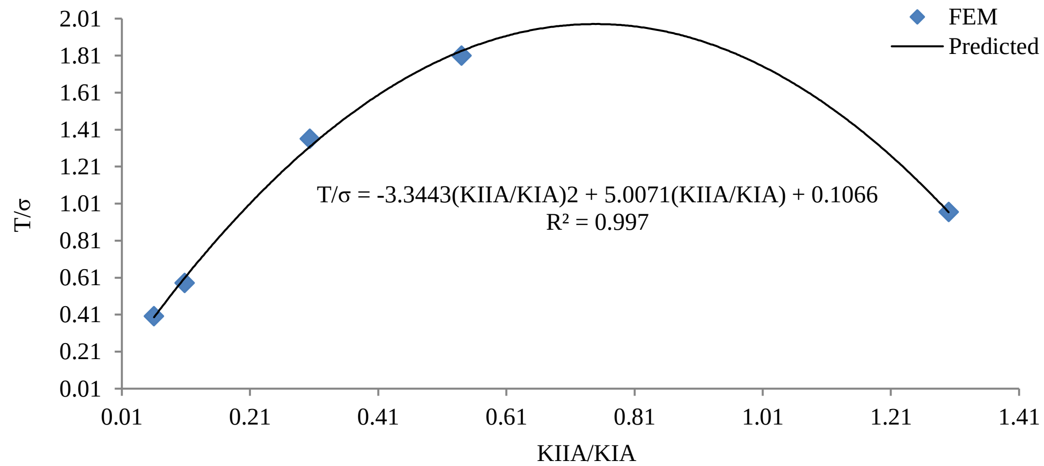

Figure 9. Correlation between stress intensity factor and T-stress for crack tip A.

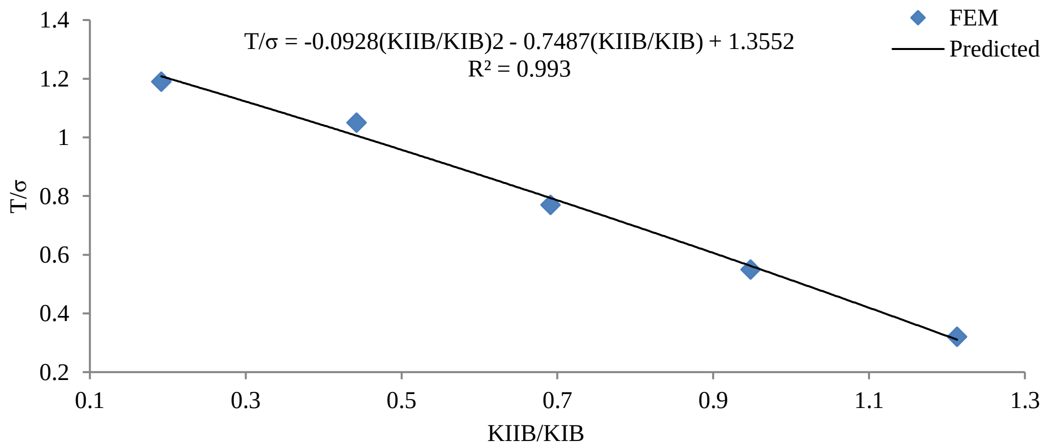

Figure 10. Correlation between SIF and T-stress for crack tip B.

It can be seen that for β>2, T-stress would become negative and as such it would be beneficial for structural integrity. Higher β results in higher mode-mixity which is shown in Figure 4.10 and 4.11 as well.

The variation of biaxial load factor will affect T/σ at fixed inclination and crack length ratio. The value T/σ is increases as the biaxial load factor increases from 0 to 1 which is about 77% for crack tip A, the value of normalized T-stress is decreases. Whereas for crack tip B, T/σ increases upto β=1, afterwards it decreases. Hence, it cannot be neglected.

Effect of Hole Diameter (d) on the SIF and T-stress

Correlation Between SIF and T-stress for Crack Tip A

From the Figure 11, it can be observed that the T-stress and SIF are strongly correlated. Normalized T-stress decreases as the mode-mixity increases. However, this decrease is only upto K_{IIA}/K_{IA}=1.4 beyond which it gradually increases.

It can be concluded that in this case also T-stress cannot be neglected. Moreover, as per the T-stress calculations the structure is unsafe and the cracks will propagate more vigorously. The nature of the trend is parabolic.

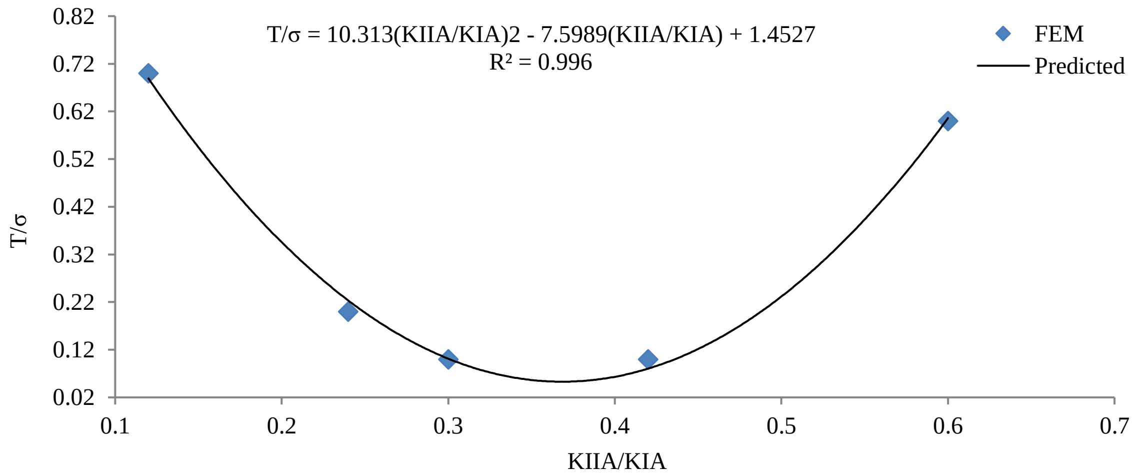

Figure 11. Correlation between SIF and T-stress for crack tip A.

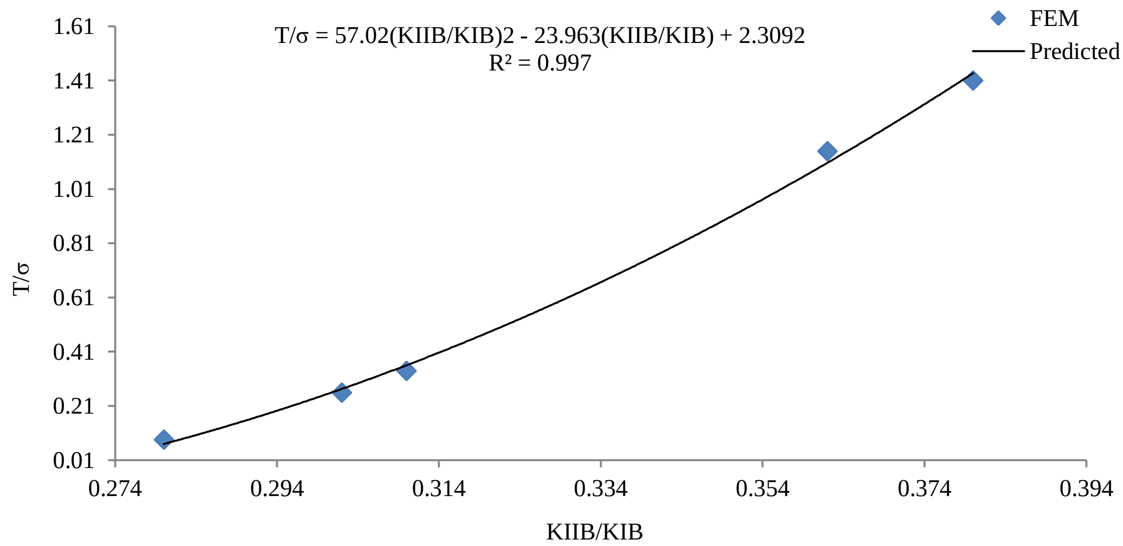

Figure 12. Correlation between SIF and T-stress for crack tip B.

Correlation Between SIF and T-stress for Crack Tip B

The correlation between stress intensity factor and T-stress is shown in the Figure 12. Result at different diameter of the hole (d) stress intensity factor increases with the T-stress significantly. The results obtained for crack tip B shows the increase in mode-mixity with increase in the T-stress. For crack tip B also it is evident that it cannot be neglected. Moreover, the structure is unsafe and is parabolic in nature.

The variation of hole diameter will affect T/σ at fixed inclination angle, biaxial load factor and cracks length. For crack tip A, the value of T/σ decreases with increase in the hole diameter. However, for crack tip B, T/σ increases. T/σ is 0.04 for d=1mm and T/σ is 0.008 for d=3mm. Hence, it cannot be neglected for failure of any specimen.

In a nutshell it can be concluded that to access the fracture behavior of cracks emanating from holes various factors viz. d, α, β play a crucial role. It is seen that the loading changes from predominately mode I to mixed mode and predominately mode II. If these types of cracks emerge in structures it becomes essential that these various factors are taken into account to access the service life of structure.

CONCLUSION

Crack inclination Angle (α)

It can be observed that normalized T-stress increases as the mode-mixity increases. Further it can be seen that T-stress is positive for all the ranges of K_{II}/K_{I}. It signify that the higher order terms cannot be neglected when studying these kind of cracked structure. However, T-stress can be neglected for predominately mode I or predominately mode II type of loading for both crack tip A and B.

Crack biaxial Factor (β)

Observation shows that the normalized T-stress increases as the mode-mixity increases. However, this increase is only to K_{IIA}/ K_{IA}=0.71 beyond which it gradually decreases. Moreover, it is seen that K_{II} dominates over K_I for crack tip A. With increase in the biaxial load factor (β) from 0 to 2, the T-stress decreases for crack tip B.

Hole Diameter (d)

Correlation between T-stress and SIF are strongly correlated. Normalized T-stress decreases as the mode-mixity increases. However, this decrease is only to K_{IIA}/ K_{IA}=1.4 beyond which it gradually increases for crack tip A. With the variation in hole diameter (d), stress intensity factor increases significantly with the T-stress for crack tip B. The results obtained for crack tip B shows the increase in mode-mixity with increase in the T-stress.

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