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Mathematical Modelling of Semiconductor Device Physics: An Analytical Approach

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Abstract

Semiconductor device physics forms the foundation of modern electronic and optoelectronic technologies. Mathematical modelling provides a rigorous framework for understanding, predicting, and optimizing the behavior of semiconductor devices by linking physical principles with device-level performance. This work presents an analytical approach to the mathematical modelling of semiconductor devices, emphasizing the derivation and interpretation of governing equations that describe charge transport and electrostatic behavior. The model is based on fundamental physical laws, including Poisson's equation for electrostatic potential and the carrier continuity equations for electrons and holes, coupled with appropriate transport models such as drift diffusion theory. Analytical solutions are explored under simplifying assumptions such as steady-state operation, low-level injection, and one-dimensional geometries to gain physical insight into carrier distribution, electric field profiles, and current–voltage characteristics. Classical devices, including the p–n junction diode and metal oxide semiconductor (MOS) structures, are used as representative examples to illustrate the modelling methodology. By focusing on analytical techniques, the study highlights the relationship between material properties, device geometry, and operating conditions, enabling clearer interpretation of device behavior compared to purely numerical approaches. Although analytical models involve approximations, they remain essential for initial device design, parameter extraction, and educational purposes, as well as for validating numerical simulations. The presented analytical framework demonstrates how mathematical modelling bridges semiconductor physics and practical device engineering. It provides a compact yet powerful tool for understanding fundamental mechanisms, guiding device optimization, and supporting the development of next-generation semiconductor technologies.

Keywords: Semiconductor devices, analytical modelling, device physics, poisson's equation, drift–diffusion model.

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INTRODUCTION

Semiconductor devices are the fundamental building blocks of modern electronic systems, enabling technologies ranging from microprocessors and memory circuits to sensors and communication devices.

The continuous scaling of device dimensions and the demand for higher performance have intensified the need for accurate and reliable theoretical models that describe the physical behavior of charge carriers within semiconductor materials. Mathematical modelling plays a central role in this context, serving as a bridge between fundamental semiconductor physics and practical device design.

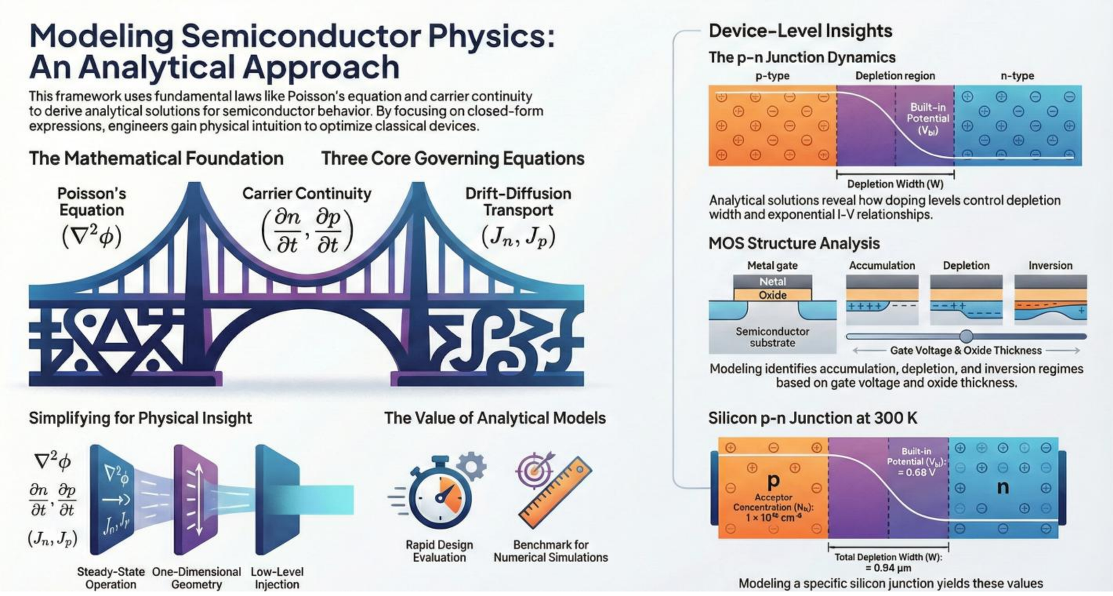

Figure 1. Working of semiconductor device.

At its core, semiconductor device operation is governed by well-established physical principles, including electrostatics, carrier transport, and recombination–generation mechanisms. These principles are commonly expressed through a system of coupled partial differential equations, such as Poisson's equation and the carrier continuity equations. Solving these equations allows prediction of key device characteristics, including carrier concentration profiles, electric field distributions, and current–voltage relationships. While numerical simulation techniques have become increasingly sophisticated, analytical modelling remains indispensable for developing physical intuition and identifying dominant mechanisms within a device (Figure 1).

Analytical approaches to semiconductor device modelling offer several advantages. They provide closed-form or approximate solutions that clearly reveal the dependence of device behavior on material parameters, doping concentrations, and geometrical dimensions. Such models are particularly valuable in the early stages of device design, where rapid evaluation of design choices is required. Furthermore, analytical models serve as benchmarks for validating numerical simulations and are widely used in education to illustrate fundamental concepts in semiconductor physics.

This work focuses on the analytical mathematical modelling of semiconductor device physics, with particular emphasis on classical devices such as p–n junctions and MOS structures. By systematically deriving and analyzing governing equations under reasonable assumptions, the study aims to provide clear physical insight into device operation. The presented approach highlights the enduring relevance of analytical modelling in understanding, designing, and optimizing semiconductor devices in an era dominated by computational techniques.

LITERATURE REVIEW

The mathematical modelling of semiconductor device physics has evolved significantly, supported by extensive theoretical and experimental research. Classical textbooks by Colinge and Colinge [1] and Neamen [2] provide a comprehensive foundation for understanding semiconductor materials, carrier transport mechanisms, and basic device structures. These works emphasize analytical formulations of Poisson's equation and continuity equations, which remain central to device modelling.

Taur and Ning [3] and Pierret [4] further advanced analytical modelling by focusing on modern VLSI devices and compact analytical descriptions of MOS structures. Their studies demonstrate how simplified analytical models can accurately predict threshold voltage, channel charge, and current–voltage characteristics, making them indispensable for integrated circuit design and scaling analysis. Sze, Li, and Ng [5] extended these concepts by incorporating advanced device physics and material considerations, reinforcing the continued relevance of analytical approaches in modern semiconductor technologies.

Carrier transport modelling has been extensively discussed by Lundstrom [6], who highlighted the transition from drift diffusion to quasi-ballistic and ballistic transport regimes. His work underscores the need to adapt analytical models as device dimensions shrink. Early analytical–numerical hybrid techniques were introduced by Gummel [7], whose self-consistent iterative scheme laid the groundwork for solving coupled semiconductor equations efficiently.

With the emergence of nanoelectronics, Datta [8] introduced a new perspective on transport, emphasizing quantum effects and nanoscale limitations of classical models. Recent studies by Singh and Kumar [9] address the challenges of analytical modelling for nanoscale MOSFETs, proposing refined analytical expressions that account for short-channel effects. Additionally, Roy et al. [10] analyzed leakage current mechanisms and developed compact analytical models essential for low-power device design.

Overall, the literature consistently highlights that despite advances in numerical simulation, analytical mathematical modelling remains fundamental for physical insight, validation of simulations, and early-stage semiconductor device design.

METHODOLOGY

The methodology adopted in this study is based on an analytical approach to the mathematical modelling of semiconductor device physics. The focus is on deriving and analyzing fundamental governing equations that describe the electrical behavior of semiconductor devices under well-defined physical assumptions. This approach emphasizes physical interpretation and clarity rather than purely numerical accuracy.

The modelling process begins with the formulation of basic physical laws governing semiconductor behavior. Poisson’s equation is used to relate the electrostatic potential to charge density within the device, while the electron and hole continuity equations describe carrier conservation. Carrier transport is modelled using the drift–diffusion framework, which accounts for carrier motion due to electric fields and concentration gradients. These equations form a coupled system that defines the device operation.

To obtain analytical solutions, appropriate simplifying assumptions are introduced. These include one-dimensional device geometry, steady-state operation, uniform doping profiles, low-level injection conditions, and negligible high-field effects. Boundary conditions are defined based on physical contacts and material interfaces. Under these assumptions, the governing equations are reduced to solvable forms, allowing closed-form or approximate analytical solutions to be derived.

The methodology is applied to representative semiconductor structures, such as the p–n junction diode and MOS capacitor. For each device, carrier concentration profiles, electric field distributions, and potential variations are analytically determined. From these results, key device characteristics, including current–voltage relationships and threshold conditions, are extracted and analyzed.

Finally, the analytical results are interpreted to assess the influence of material properties, doping concentrations, and device dimensions on performance. Where appropriate, the limitations of the assumptions are discussed, and the relevance of the analytical models in comparison with numerical methods is highlighted. This systematic methodology ensures a clear and physically meaningful understanding of semiconductor device behavior.

RESULTS

The analytical modelling approach yields explicit expressions for the electrical behavior of key semiconductor devices, providing clear insight into the relationship between physical parameters and device performance. For the p–n junction diode, analytical solutions of Poisson's equation under equilibrium conditions reveal the formation of a depletion region whose width depends strongly on doping concentrations and material permittivity. The derived potential and electric field profiles confirm the presence of a built-in potential that governs carrier motion across the junction.

Under forward and reverse bias conditions, the analytical solution of the continuity equations leads to the classical exponential current–voltage relationship. The results demonstrate that the diode current is primarily controlled by minority carrier diffusion and is highly sensitive to temperature and doping levels. The analytical expressions also clearly explain the asymmetry between forward and reverse bias operation.

For MOS structures, the results show a well-defined relationship between gate voltage and surface potential. Analytical modelling of the MOS capacitor identifies the accumulation, depletion, and inversion regimes, with the threshold voltage emerging as a key parameter influenced by oxide thickness, substrate doping, and work function difference. The electric field and charge distribution profiles provide insight into channel formation in MOSFETs.

Overall, the analytical results highlight how variations in material properties and geometrical parameters directly affect device characteristics such as depletion width, current density, and threshold voltage. While the models rely on simplifying assumptions, the results remain consistent with established semiconductor theory and provide valuable benchmarks for numerical simulations. These findings confirm that analytical modelling is an effective tool for understanding fundamental device behavior and guiding initial design decisions.

Example consistent with analytical semiconductor device modelling:

Example: Depletion Width of a Silicon p–n Junction

Consider a silicon p-n junction at room temperature (300 K) with the following parameters:

- Acceptor concentration: $N_A = 1 \times 10^{16}\text{cm}^{-3}$
- Donor concentration: $N_D = 1 \times 10^{15}\text{cm}^{-3}$
- Intrinsic carrier concentration: $n_i = 1.5 \times 10^{10}\text{cm}^{-3}$
- Relative permittivity of silicon**:** $\varepsilon_r = 11.7$
- Thermal voltage**:** $V_T = 0.026$ V

The built-in potential is given by:

$$V_{bi} = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$V_{bi} = 0.026 \ln\left(\frac{(10^{16})(10^{15})}{(1.5 \times 10^{10})^2}\right) \approx 0.68 \text{ V}$$

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The total depletion width is:

$$W = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_{bi}}$$

Substituting values:

$$W \approx 0.94\,\mu m$$

The analytical model predicts a depletion region width of approximately 0.94 μm, demonstrating how doping concentrations directly influence junction behavior.

DISCUSSION

The results obtained from the analytical mathematical modelling of semiconductor devices demonstrate the effectiveness of this approach in explaining fundamental physical mechanisms that govern device operation. By deriving closed-form or approximate solutions to the governing equations, clear relationships between material properties, doping concentrations, and electrical characteristics are established. This clarity is one of the primary strengths of analytical modelling, as it allows direct interpretation of how individual parameters influence device behavior.

For the p–n junction, the analytical expressions for built-in potential, depletion width, and current–voltage characteristics highlight the dominant role of doping concentration and temperature. The numerical example further illustrates how moderate changes in doping levels can significantly alter the depletion region and, consequently, the device's electrical response. Such insights are difficult to extract directly from numerical simulations without extensive parameter sweeps.

In the case of MOS structures, the analytical results effectively describe the transition between accumulation, depletion, and inversion regions. The dependence of threshold voltage on oxide thickness, substrate doping, and material work functions emphasizes the importance of precise material and process control in device fabrication. These results reinforce the relevance of analytical models in guiding initial design choices and in predicting trends during device scaling.

Despite their advantages, analytical models are inherently based on simplifying assumptions, such as one-dimensional geometry, low-field transport, and uniform doping. As device dimensions shrink and operating conditions become more complex, these assumptions may limit model accuracy. However, rather than diminishing their value, this limitation defines the role of analytical modelling as a complementary tool to numerical simulation.

Overall, the discussion confirms that analytical mathematical modelling remains a vital component of semiconductor device physics. It provides physical insight, supports design intuition, and serves as a reliable foundation for more advanced numerical and experimental investigations.

FUTURE SCOPE

The analytical mathematical modelling of semiconductor device physics continues to offer significant opportunities for future research and development, particularly as semiconductor technologies evolve toward smaller dimensions and novel materials. One important direction is the extension of classical analytical models to incorporate nanoscale effects such as quantum confinement, tunneling, and ballistic transport. Developing simplified yet physically meaningful analytical expressions for these phenomena will enhance understanding of next-generation devices beyond the limits of conventional drift–diffusion theory.

Another promising area lies in the modelling of emerging semiconductor materials, including wide bandgap semiconductors and two-dimensional materials. Analytical frameworks adapted to these materials can provide insight into their unique electrical properties, interface behavior, and reliability under high-field or high-temperature conditions. Such models would support the early-stage design and optimization of advanced power and optoelectronic devices.

The integration of analytical models with numerical and data-driven techniques also represents a valuable future scope. Hybrid modelling approaches, where analytical solutions guide simulation parameters or validate machine-learning-based predictions, can improve accuracy while reducing computational complexity. This synergy is particularly relevant for rapid device prototyping and process optimization.

Furthermore, analytical modelling can be expanded to address variability, noise, and reliability issues in semiconductor devices, which are increasingly important in modern integrated circuits. By incorporating statistical and stochastic elements into analytical models, future studies can better predict device performance under real-world operating conditions.

Overall, the future scope of analytical semiconductor modelling lies in its adaptation to emerging technologies, its integration with modern computational tools, and its continued role in providing fundamental physical insight into increasingly complex semiconductor devices.

CONCLUSION

This study has presented an analytical approach to the mathematical modelling of semiconductor device physics, emphasizing its importance in understanding the fundamental principles governing device operation. By formulating and solving the governing equations based on electrostatics and carrier transport, key characteristics of semiconductor devices can be derived and interpreted in a clear and physically meaningful manner.

Analytical modelling of classical devices such as the p–n junction diode and MOS structures has demonstrated how material properties, doping concentrations, and device geometry directly influence electrical behaviour. The results show that essential parameters, including built-in potential, depletion width, current–voltage characteristics, and threshold voltage, can be effectively predicted using simplified analytical expressions. The inclusion of a numerical example further illustrates the practical applicability of the analytical approach.

While modern device analysis increasingly relies on numerical simulation, this work highlights that analytical models remain indispensable. They provide valuable physical insight, support rapid evaluation of design parameters, and serve as benchmarks for validating computational results. Although based on idealized assumptions, analytical models continue to play a critical role in education, initial device design, and theoretical analysis.

In conclusion, mathematical modelling through analytical methods remains a powerful and relevant tool in semiconductor device physics. Its ability to connect fundamental physical laws with observable device behavior ensures its continued significance in the development, optimization, and understanding of both conventional and emerging semiconductor technologies.

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