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Comparative Buckling Load Analysis of Thin Functionally Graded Material Plates with Variable Distributions: Power Law vs. Sigmoidal vs. Exponential

Ravi Pant[1]*, Vikas Kumar[2]

Abstract

A comprehensive comparative analysis of buckling load behavior in thin Functionally Graded Material (FGM) plates with variable material distributions is presented in this paper. FGMs, characterized by gradient composition, possess unique mechanical properties suitable for diverse engineering applications. The study focuses on three distinct distribution functions: Power Law, Sigmoidal, and Exponential, each exerting different influences on the material gradient. Finite Element Analysis is utilized in exploring the buckling response for these plates under varying loading conditions, including uniaxial and biaxial compressive loads. The comparative examination explains the correlation of the distribution function selection and the critical buckling load of FGM plates, while concurrently exploring the influence of length-to-width ratio on the variability of critical buckling load. The results reveal how each distribution influences the critical buckling load, providing invaluable insights for designing FGM structures customized to particular needs. By enhancing our understanding of FGM mechanics, this study facilitates material selection and design optimization processes.

Keywords: Functionally graded materials, Buckling load analysis, Power law, Sigmoidal law, Exponential law, Navier solution

INTRODUCTION

Functionally graded materials (FGMs) have attracted considerable interest in engineering and materials science because of their distinct properties and possible uses in different structural elements. Unlike conventional homogeneous materials, FGMs exhibit spatially varying compositions, resulting in tailored mechanical and thermal properties. This versatility makes them highly desirable for designing structures subjected to diverse loading conditions and environmental factors. The exploration of functionally graded materials began in Japan in 1984, focusing initially on thermal barrier materials [1].

Significant research endeavors explore the performance of functionally graded materials under various loading conditions [2]. Researchers have utilized various analytical and computational methods to investigate the buckling traits exhibited by FGM plates with spatially varying material properties. Zenkour [3] concentrated on buckling and natural vibration assessment of graded sandwich plates, employing a distribution following power law for thickness variation. Chi and Chung [4] discussed the response of FGM plates to lateral loading, employing conventional plate theory and techniques involving Fourier series analysis. Reddy et al. [5] addressed buckling

***Author for Correspondence**
Ravi Pant

[1]Student, Department of Mechanical Engineering, NIT Kurukshetra, India
[2]Assistant Professor, Department of Mechanical Engineering, NIT Kurukshetra, India

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analysis employing advanced shear deformation theory with material property distribution following a power law. Singh and Harsha [6] investigated the impact of loading inequality on buckling behavior under uniform in-plane loading. Helal and Shi [7] explored optimal material gradients for mechanical parameters, while Yang et al. [8] explored the effect of porosity on FGM plates. Together, these studies contribute to a comprehensive understanding of FGM behavior and offer valuable insights for their application in various engineering fields.

Thin plates play a crucial role in aerospace, automotive, civil, and mechanical engineering, given their widespread use in building structures [9]. Ensuring the structural integrity and performance of thin Functionally Graded Material (FGM) plates under compressive loads requires a thorough understanding of their buckling behavior. Buckling, a critical failure mode in thin plates, involves abrupt deformation and eventual collapse as the applied load exceeds a threshold. Lately, researchers have investigated different distribution functions to precisely represent the spatial variability of material properties in FGM plates. Among these functions, power law, sigmoidal, and exponential distributions are prominent, each offering distinct advantages and challenges in capturing material property gradients across the plate's thickness.

This research aims to conduct a thorough comparison of the buckling behavior exhibited by thin FGM plates. These plates have variable material distributions, which are modeled through the utilization of power law, sigmoidal law, and exponential law. By systematically investigating the effects of these distribution functions on both the critical buckling load and their corresponding mode shapes, this study seeks to provide critical buckling load values for power law, sigmoidal law, exponential law, and compare them with cases for pure metal and ceramics. Such findings will assist engineers and designers in making informed decisions regarding the most suitable material distribution for specific applications, thereby enhancing the structural efficiency and reliability of thin FGM plates.

MATERIALS AND METHODS

Geometrical Configuration

A thin elastic functionally graded material plate, having dimensions a, b, h representing length, width and thickness respectively, typically comprises a base plate fabricated from metal and a top plate composed of ceramic material, as illustrated in Figure 1. The thin FGM plate is a square plate with a length to height ratio of 100.

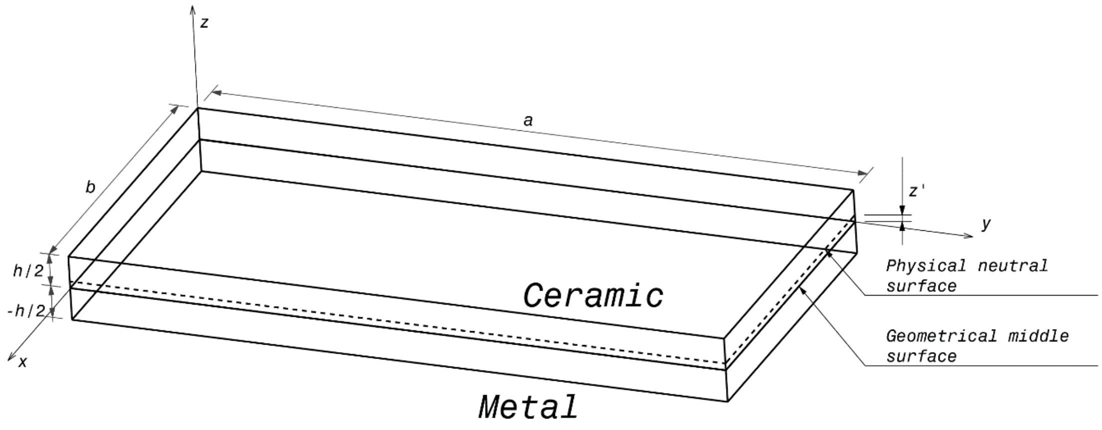

Figure 1. Schematic illustration of a thin FGM plate in cartesian coordinate system

Material Properties

While analyzing FGM plates, the chosen metal-ceramic combination is Al-ZrO2, and its properties are outlined in Table 1 [10]. Within FGM plates, Modulus of Elasticity, density, and Poisson's ratio change continuously solely through the thickness. Delale and Erdogan [11] underscored that Poisson's ratio has a notably smaller impact on deformation compared to Modulus of Elasticity and density. Consequently, Poisson's ratio is typically kept constant. Meanwhile, the Modulus of Elasticity and density changes through the thickness according to power law (pfgm), sigmoidal law (sfgm), and exponential law (efgm), thereby offering a range of mechanical properties tailored to specific applications.

Table 1. Material Properties of Metal and Ceramic [10].

Material	**Modulus of Elasticity (E)**	**Poisson Ratio (ν)**	**Density(ρ)**
Metal (Al)	68.9 GPa	0.33	2700 kg/m^3
Ceramic (ZrO_2)	211 GPa	0.33	4500 kg/m^3

Power law FGM

The proportion of material volume of the power-function graded material (pfgm) is considered to follow a function that adhere to power law. [12,13]:

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n$$

$$V_m(z) = 1 - V_c(z) \text{ with } (0 \leq n \leq \infty),$$

where n represents the material gradient whereas ‘c’ and ‘m’ subscript are used to denote ceramic and metal respectively. Once the proportion of material volume $V_c(z)$ is specified, the material characteristics of a pfgm can be obtained using the mixture law (Bao and Wang, 1995):

$$P(z) = P_c V_c(z) + P_m V_m(z)$$

where P_c and P_m are the material property values of ceramic and metal components, respectively. The Modulus of Elasticity and density in the thickness of the pfgm plate is illustrated in Figure 2.

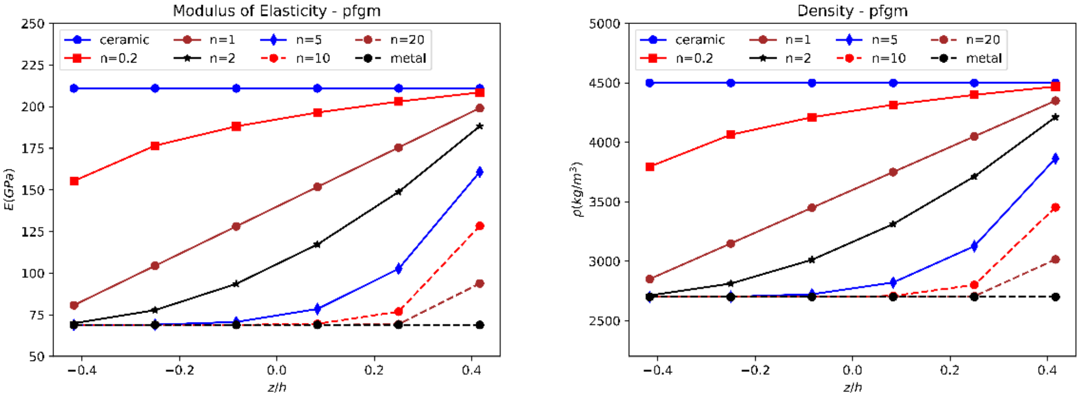

Figure 2. Modulus of Elasticity and density distribution across pfgm with respect to dimensionless thickness coordinate

Sigmoidal law FGM

When integrating a power law function graded material into a composite with multi layers, stress concentrations arise at interfaces where the material undergoes rapid changes but remains continuous. To address this issue, Chung and Chi [4] introduced a method defining the proportion of material volume specifying different power laws for each half of the plate to ensure a uniform stress across interfaces.

The proportion of material volume are defined as follows:

$$V_{c1}(z) = 1 - \frac{1}{2}\left(\frac{\frac{h}{2}-z}{\frac{h}{2}}\right)^{n} \text{ for } (0 \leq z \leq h/2)$$

$$V_{m1}(z) = 1 - V_{c1}(z) \text{ for } (0 \leq n \leq \infty),$$

$$V_{c2}(z) = \frac{1}{2}\left(\frac{\frac{h}{2}-z}{\frac{h}{2}}\right)^{n} \text{ for } (-h/2 \leq z \leq 0)$$

$$V_{m2}(z) = 1 - V_{c2}(z) \text{ for } (0 \leq n \leq \infty),$$

Following the principle of mixture, the material characteristics can be determined by:

$$P(z) = P_c V_{c1}(z) + P_m V_{m1}(z) \text{ within } (0 \leq z \leq h/2)$$
$$P(z) = P_c V_{c2}(z) + P_m V_{m2}(z) \text{ within } (-h/2 \leq z \leq 0)$$

Figure 3 illustrates the variation of material properties for sigmoid distributions, leading to the designation of this FGM plate as a sigmoid FGM (sfgm) plate.

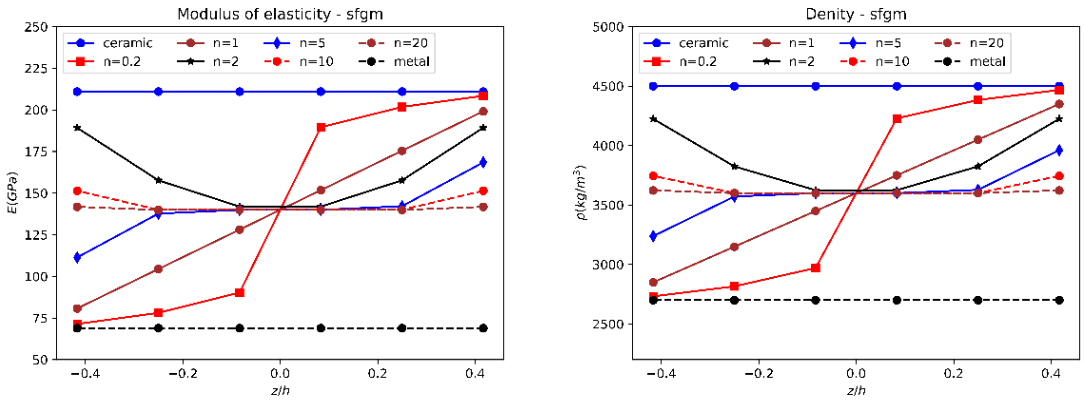

Figure 3. Modulus of Elasticity and density distribution across sfgm with respect to dimensionless thickness coordinate

Exponential law FGM

The exponential law is commonly used to characterize the material properties of FGMs, as indicated by [14]

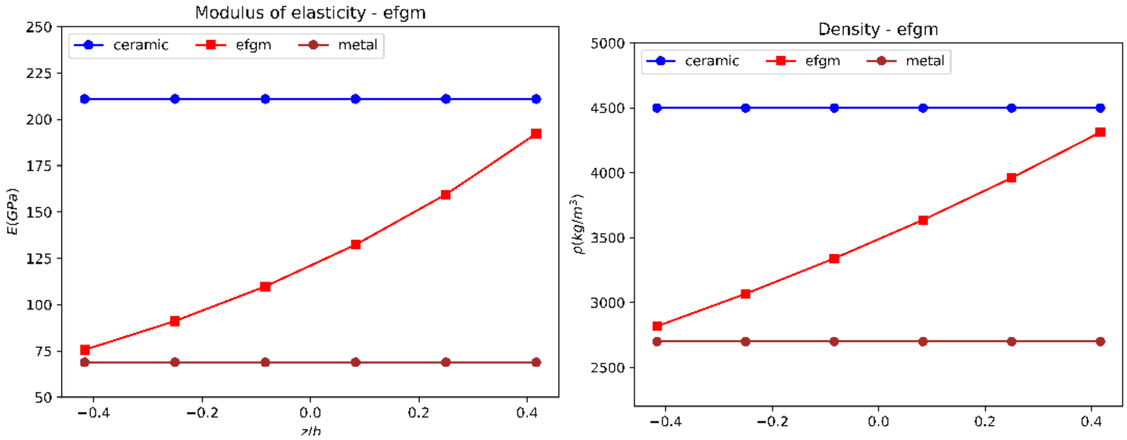

Figure 4. Modulus of Elasticity and density distribution across efgm with respect to dimensionless thickness coordinate

$P(z) = A\boldsymbol{e}^{\boldsymbol{B}(z+\frac{h}{2})}$ where

$A = P_m$ and $B = \frac{\mathbf{1}}{\boldsymbol{h}}\ \mathbf{ln}\,\frac{\mathbf{Pc}}{\mathbf{Pm}}$

The distribution of material characteristics of the efgm plates is depicted in Figure 4.

Methodology

Modeled in SOLIDWORKS with 6 layers of equal thickness, the plate undergoes eigenvalue buckling analysis in ANSYS Workbench. Material properties are assigned to each layer during preprocessing, and a mesh with 18018 nodes and 2400 elements is generated. The model is then solved for eigenvalue buckling analysis under uniaxial and biaxial compressive loading. Post-processing yields critical buckling loading factors and their corresponding buckling mode shapes. The analysis employs Navier-type boundary conditions where all four edges are simply supported.

RESULTS AND DISCUSSION

The critical buckling load is impacted by factors such as length-to-width ratio, boundary conditions, and material properties. Throughout the analysis, a consistent set of boundary conditions, specifically of the Navier type, is applied at the edges. Material properties are adjusted according to gradient laws, while length to width ratio varies at 0.5, 1, and 2 respectively. For convenience, the buckling load factor derived from the buckling analysis is presented in a dimensionless form.

$$\overline{N} = N_{cr}\,\frac{a^2}{\mathrm{Em}h^3}$$

where N_{cr} = the critical buckling load
a = plate length
E_m = Modulus of Elasticity of the metal

Buckling analysis for square plate

Under both uniaxial and biaxial compressive loading conditions, the highest dimensionless critical buckling load is observed with ceramics, followed by pfgm, sfgm, efgm, and metals, respectively. In pfgm, the critical buckling load occurs at n = 0.2, whereas in sfgm, it is attained at n = 2 (see Figure 5 and Figure 6). This discrepancy arises from the varying stiffness of the plate, which is maximum for ceramics and minimum for metals, with the three FGMs falling in between. Among the FGMs, pfgm with a material gradient of n equals 0.2 exhibits the critical load for buckling. Figure 7 illustrates the shapes of buckling modes under uniaxial and biaxial loading conditions for pfgm with n equals 0.2.

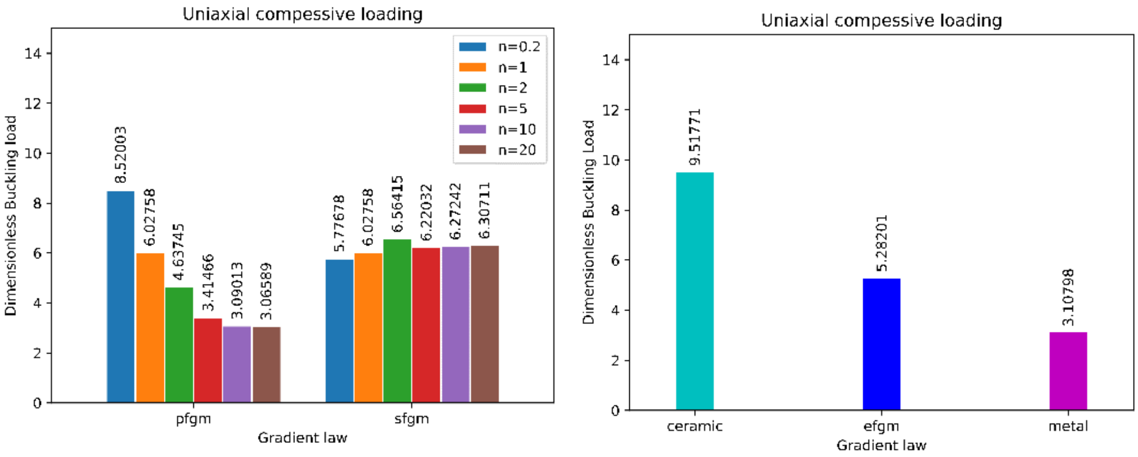

Figure 5. Variation in Dimensionless Critical Buckling load under uniaxial loading conditions

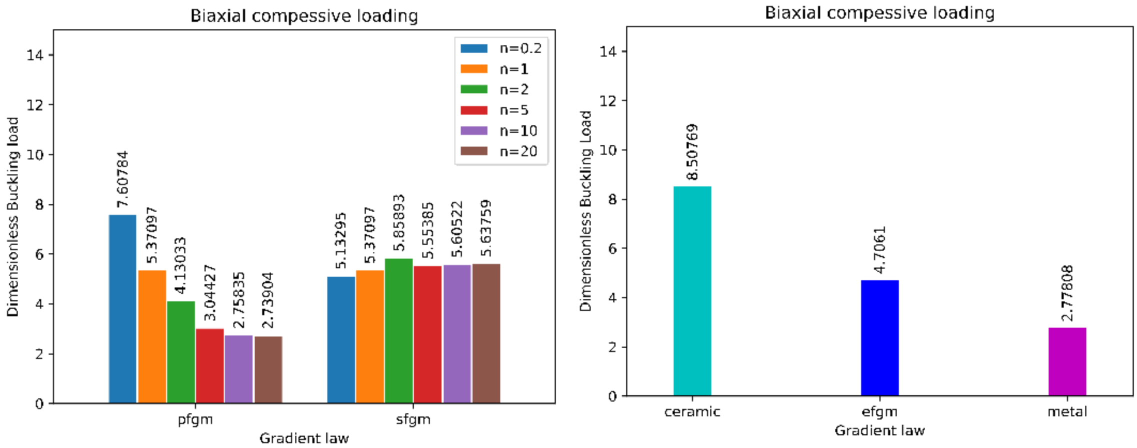

Figure 6. Variation in Dimensionless Critical Buckling load under biaxial loading conditions

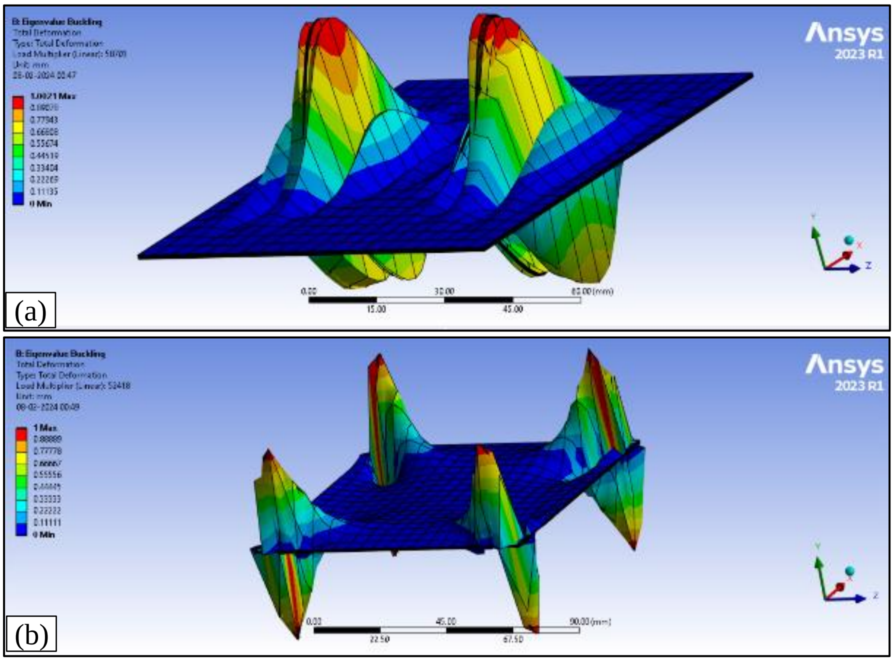

Figure 7. Buckling mode shape of a pfgm plate with material gradient n = 0.2 (a) Uniaxial loading condition (b) Biaxial loading condition

Influence of Length to Width Ratio (a/b) on Critical Buckling Threshold

(a) Uniaxial compressive loading – As the length-to-width ratio (a/b) increases, with 'a' held constant under uniaxial loading conditions, the plates become more vulnerable to buckling under compressive loads. This is because longer plates are less able to resist buckling due to increased tendency for deflection and deformation. The a/b ratio influences the distribution of stresses and deformation under compressive load. Plates with higher length to width ratio experiences greater lateral deflections and are more prone to buckling. Figure 8(a) illustrates the correlation between the dimensionless critical buckling load and the a/b ratio.

(b) Biaxial compressive loading – Under biaxial loading, the buckling behavior is influenced by the interaction of multiple factors. At lower length to width ratio (a/b = 0.5) the plate tends to be more stable initially because it is relatively short and wide. This initial stability resists buckling and the critical buckling loads are relatively lower. As the a/b ratio increases from 0.5 to 1, the plate becomes more elongated, but its buckling resistance improves due to increased stability. This is due to the fact that biaxial compressive loads introduce interactions between the two principal axes of the plate. At lower a/b ratio, the compressive loads along both the axis may reinforce each other, enhancing the plate's resistance to buckling. This reinforcement contributes to the observed increase in critical buckling load as the a/b ratio increases from 0.5 to 1. As the a/b ratio increases beyond 1 and approaches 2, the buckling mode may transition from a primary global buckling mode to a combination of global and local buckling modes. Localized buckling modes can become dominant as the plate becomes longer and narrower, leading to decrease in critical buckling load. At a/b ratio below 1 slenderness ratio is relatively low and plate is less prone to buckling and as a/b ratio becomes greater than 1 slenderness ratio is higher and plate is more prone to buckling. Figure 8(b) illustrates the relationship between critical buckling load and a/b ratio.

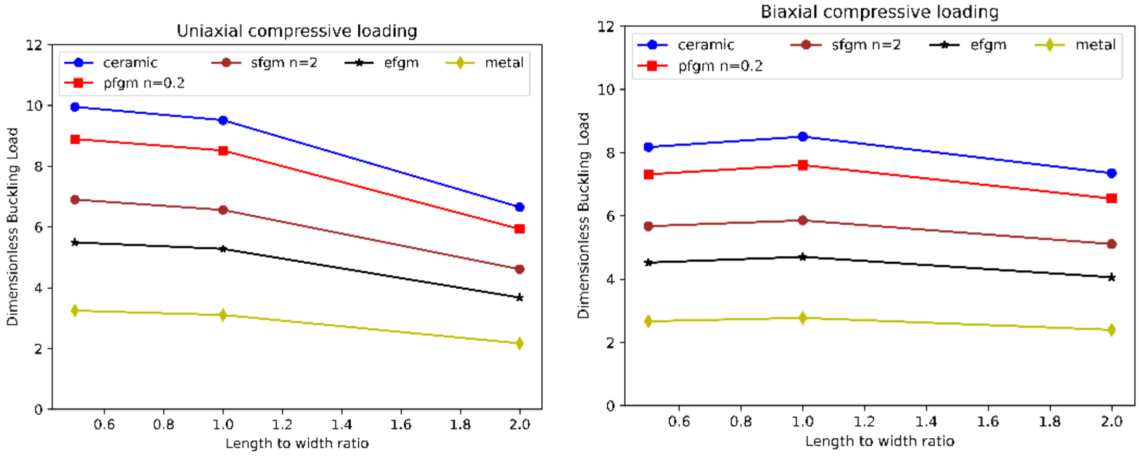

Figure 8. Relationship between dimensionless critical buckling load and length-to-width ratio across different gradient laws.

CONCLUSION

This study conducts a comparative analysis of buckling behaviour of thin FGM plates with variable material distribution, specifically pfgm, sfgm, and efgm. The focus is to comprehend how the buckling characteristics of these plates is affected by changes in material properties along the thickness direction under various loading conditions. The key findings are outlined below:

1. The critical buckling load, which indicates the load threshold at which buckling occurs, is found to be highest for ceramic materials and lowest for metals, with FGMs exhibiting values in between. Notably, among FGMs, plates featuring a pfgm (power function graded material) with a material gradient n=0.2 demonstrate the highest threshold for buckling.
2. In case of sfgm (sigmoid function graded material), the critical load peaks when the material gradient n is set to 2.
3. When subjected to uniaxial loading, where the load is applied in one direction, the buckling susceptibility increases with an increase in the length to width ratio of the plate. This suggests that as the plate becomes longer relative to its width, it becomes more prone to buckling under uniaxial loading conditions.
4. Conversely, in the case of biaxial loading, where load is applied in two perpendicular directions, a more intricate behavior is noted. Initially, with the increase of material n from 0.5 to 1, the buckling load rises. However, with further increase of n from 1 to 2, the buckling load begins to decrease. Additionally, it is observed that the lowest buckling load occurs when the length to width ratio of the plate is set to 2.

These findings have significant implications for structural design, indicating the importance of considering material properties, loading conditions, and plate dimensions when designing thin FGM plates to withstand buckling.

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