STM JOURNALS

Research & Reviews: Journal of
Statistics

https://journals.stmjournals.com/rrjost

ISSN: 2278-2273 (Online)
ISSN: 2348-7909 (Print)
Volume 13, Issue 3, 2024
September—December
DOI (Journal): 10.37591/RRJoST

Modeling Dispersed Count Data: Evaluating the Conway–Maxwell–Poisson Regression with COVID-19 Mortality Data

Nazmin Akter[1], Md. Rezaul Karim[2,]*, Sultana Begum[3]

Abstract

Count data are prevalent in diverse fields such as biology, healthcare, psychology, and marketing, characterized by non-negativity and inherent heteroskedasticity, often exhibiting overdispersion or underdispersion. Traditional Poisson regression, which assumes equal mean and variance, is inadequate for such dispersed data. To address this, various generalized linear models (GLMs) and their extensions, including negative binomial (NB) and Conway–Maxwell–Poisson (CMP) regressions, are utilized. This study evaluates the performance of CMP regression compared to Poisson, NB, and generalized Poisson models using COVID-19 death data from Bangladesh. The CMP distribution, a flexible two-parameter generalization of the Poisson distribution, accommodates both overdispersion and underdispersion, enhancing model accuracy. Model comparisons based on the Akaike Information Criterion (AIC) indicate that the CMP model, influenced by temperature and humidity, provides the best fit with the lowest AIC value. The next-best models, NB, and generalized Poisson show significantly higher AIC values, underscoring CMP's superiority. Results indicate that while NB and CMP regressions provide superior accuracy over the Poisson model, CMP regression demonstrates the best fit in terms of log-likelihood and AIC. This research underscores the CMP distribution's efficacy and highlights the importance of using appropriate models for dispersed count data. Advances in computational power have enabled the revival and application of CMP regression, offering new insights into discrete data modeling.

Keywords: Negative binomial regression, generalized Poisson regression, Conway–Maxwell–Poisson regression, COVID-19, Bangladesh

INTRODUCTION

Count data can be found in a variety of sectors, including biology, healthcare, psychology, and marketing. The distribution of count data is non-negative and naturally heteroskedastic, with a right-skewed variance that increases with the mean [1]. Classical Poisson regression is the most widely used technique for modeling count data; however, as it relies on the premise that the variance and mean are equally distributed, it cannot be used in many real-world situations where the data are dispersed (i.e., the variance is greater than or less than the mean). Dispersion frequently occurs for a variety of reasons, such as systems that produce an excessive number of zero counts or censoring. This excess variation can lead to inaccurate conclusions regarding parameter estimates, confidence intervals, standard errors, and tests. Generalized linear models (GLMs) and expansions, which measure the impact of predictor variables on anticipated counts, are commonly utilized to assess

***Author for Correspondence**
Md. Rezaul Karim
E-mail: rezaul@juniv.edu

[1]Lecturer, Department of Social Relations, East West University Dhaka, Bangladesh
[2]Professor, Department of Statistics and Data Science, Jahangirnagar University, Bangladesh
[3]Assistant Professor, Department of Statistics and Data Science, Jahangir Nagar, Bangladesh

Received Date: January 06, 2025
Accepted Date: January 18, 2025
Published Date: May 29, 2025

Citation: Nazmin Akter1, Md. Rezaul Karim, Sultana Begum. Modeling Dispersed Count Data: Evaluating the Conway–Maxwell–Poisson Regression with COVID-19 Mortality Data. Research & Reviews: Journal of Statistics. 2024; 13(3): 18–26p.

these counts [2]. These types of count data are frequently modeled with fundamental statistical models, such as Poisson or negative binomial distributions, utilizing Generalized linear models (GLMs) and GLMMs. When the variance of a Poisson model exceeds its mean, the model is said to be overdispersed (mean<variance) [3, 4]. However, it is inappropriate for the majority of count data analyses. There are numerous methods to account for Poisson overdispersion. A popular technique is negative binomial (NB) regression, which has been effectively used to understand overdispersed counts in statistics. Another lesser-known regression can be modeled using the Conway–Maxwell–Poisson distribution (CMP) [5, 6] when data have either overdispersion or underdispersion. In addition to including Bernoulli and Geometric distributions as special instances, the Conway–Maxwell–Poisson distribution is a two-parameter generalization of the Poisson distribution that relies on the dispersion value. If statistical models do not account for over- and underdispersion, they may result in some bias in the calculation of the variance of parameter estimates, goodness-of-fit, and information criteria (IC) [7].

We investigate the performance of the COM-Poisson regression against a few other regression models: Poisson, negative binomial, and generalized Poisson, using COVID-19 death data to demonstrate its utility in real-world applications. However, a recent study [8] examined the impact of some meteorological variables on COVID-19 mortality using only negative binomial and quasi-Poisson regression analyses, and both results were significant. A new R package, **glmmTMB** (glmmTMB is an R package that fits generalized linear mixed models), was introduced [9] which can swiftly estimate a wide range of models such as GLMs, GLMMs, hurdle models, and extensions. The most appealing feature of GLMMs is the combination of fastness and flexibility with other GLMMs. Another distinct characteristic of **glmmTMB** is its ability to calculate the mean-parameterized Conway–Maxwell–Poisson distribution [9].

The goal of this research is to introduce proper regression modeling for dispersed count data using several regression models. We conducted a case study utilizing the daily COVID-19 death number in Bangladesh. The remainder of this paper is organized as follows. Section 2 provides a brief overview of the negative binomial, generalized Poisson, and Maxwell–Poisson regressions. Section 3 contains the regression modeling for the dataset of all the regression models with comparisons and a general discussion, and Section 4 concludes the paper.

MATERIALS AND METHODS

Negative Binomial Regression

A negative binomial regression model was built based on the Poisson-gamma mixed distribution. The Poisson distribution can be generalized by incorporating a gamma noise variable, where the scale parameter is ν, and the mean is set to 1. This results in a negative binomial distribution with a probability mass function.

$$P(Y_i = y_i \vee \lambda_i, \alpha) = \frac{\Gamma(y_i+\vartheta)}{\Gamma(y_i+1)(\Gamma\vartheta)}\left(\frac{\vartheta}{\vartheta+\lambda_i}\right)^{\vartheta}\left(\frac{\lambda_i}{\vartheta+\lambda_i}\right)^{y_i} \tag{1}$$

Where, $\lambda_i = t_i\lambda$ and the dispersion parameter $\alpha = \vartheta^{-1}$

Parameter λ represents the mean incidence rate of the response variable y, and it can be used to illustrate the possibility of a repeat of the incident during a specific exposure period, t. The mean is

$E(Y_i) = \lambda_i$and $V(Y_i) = \lambda_i(1 + \vartheta^{-1}\lambda_i)$

NB regressions transform to Poisson regression in the limit as $\alpha \rightarrow 0$ and indicate overdispersion when $\alpha > 0$. The negative binomial regression model can be expressed as follows:

$$lo\ g\left(\frac{\lambda_i}{t_i}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p$$

The regression coefficients $\beta_0, \beta_1, \dots, \beta_p$ are unknown parameters of a set of p repressors estimated from a set of data.

Generalized Poisson Regression

The Poisson model is effective for representing discrete event counts that occur within a fixed time or space interval. This is particularly valuable when the counts are right-skewed, making them unsuitable for approximation by a Gaussian distribution. The generalized Poisson model is appropriate when the observation is overdispersed [10, 11]. The probability mass function (pmf) of the GP distribution can be defined as:

$$P(Y_i = y_i) = \lambda \frac{(\lambda+\vartheta y_i)^{(y|i-1)} exp\,(-\lambda-\vartheta y_i)}{y_i!} \tag{2}$$

where, $Y_i = 0, 1, 2, \dots$ is the random variable, y is count; ϑ is dispersion parameter, $0 \leq \vartheta < 1$; λ is the rate parameter, $\lambda > 0$ [12]. The mean of the GP distribution is $\lambda/(1-\vartheta)$, and the variance is $\lambda/(1-\vartheta)^2$. When $\vartheta = 0$ the GP distribution is reduced to the standard Poisson distribution with mean λ. GP regression reduces to Poisson regression when $\vartheta = 0$, indicating overdispersion when $\vartheta > 0$ ($\alpha > 0$) and underdispersion when $\vartheta < 0$ ($\alpha < 0$) [13]. The log-likelihood function (LF) of the GP regression is given by [14]:

$$logL = \sum_{i=1}^{n} \; [loglog\lambda(\mu_i) + (y_i - 1)loglog(\lambda(\mu_i) + \vartheta y_i) - (\lambda(\mu_i) + \vartheta y_i) - loglog(y_i!)]$$

Where, $\mu_i = (1-\vartheta)$ exp $(x_i\beta)$ and $\lambda(\mu)$ is the solution of the preceding equation for the mean. The maximum likelihood estimates can be obtained by maximizing the log-likelihood[12] established a generalized Poisson distribution that is more flexible in modeling overdispersion than the Poisson distribution. However, it does not belong to the exponential family and sometimes makes analysis more difficult.

Conway–Maxwell–Poisson Regression

The Conway–Maxwell–Poisson distribution, introduced by Conway and Maxwell in the context of queuing systems, is a two-parameter extension of the Poisson distribution. It generalizes Poisson, Binomial, and negative binomial discrete distributions [15]. Its useful statistical and probabilistic properties have been elegantly derived [7, 16, 17]. Its probability function is defined as

$$P(Y_i = y_i) = \frac{\lambda_i^y}{(y|i!)^{\vartheta} \cdot \frac{1}{Z(\lambda,\vartheta)}}; y = 0,1,2, \tag{3}$$

where,

$$Z(\lambda, \vartheta) = \sum_{j=0}^{\alpha} \frac{\lambda^j}{(j!)^{\vartheta}}$$

for $\lambda > 0$ and $\vartheta \geq 0$. The addition of the scale parameter ϑ enables the ratio $\frac{P(Z=j-1)}{P(Z=j)}$ to increase either sub- or super-linearly and allows Z to have a variance that is either less than or larger than its mean [16] (the mean of $Z \sim CMP\ (\lambda, \vartheta)$. With parameter $n\lambda$, CMP approaches an ordinary Poisson distribution, as $\vartheta = 1$ (thus $Z(\lambda,\vartheta) = \exp(\lambda)$). Less than one value of ϑ corresponds to successive ratios that are flatter than the Poisson distribution; hence, tails that are too long or overdispersion[7].

The mean was used to parameterize the Conway–Maxwell–Poisson distribution [18]. To estimate the parameter of CMP, three methods were used, including the maximum likelihood estimator using iteration (more computationally intensive) and the Bayesian method using conjugate prior, the posterior density of the parameters. For $\vartheta \leq 1$ or $\lambda > 10^{\vartheta}$, the mean value and variance of CMP distribution are [17].

$$E(Y_i) = \lambda_i \frac{\delta[log\,(Z(\lambda_i,\vartheta))]}{\delta\lambda_i} \approx \lambda_i{}^{\vartheta} - \frac{\vartheta-1}{2\vartheta}$$

$$V(Y_i) = \frac{\delta E(Y_i)}{\delta log\,(\lambda_i)}$$

It is noteworthy that the useful result for this distribution is $E\ (Y^{\vartheta}) = \lambda$. The relationship between these two moments can be rewritten as:

$$V(Y) = \lambda \frac{\delta}{\delta\lambda} E(Y) \approx \frac{1}{\vartheta} \lambda^{\frac{1}{\vartheta}} \approx \frac{1}{\vartheta} E(Y)$$

For *n* independent and identically distributed observations $y_1 y_2,\ldots,y_n$ the log-likelihood is given by:

$$logL = \sum_{i=1}^{n}\ y_i log\,(\lambda_i) - \vartheta \sum_{i=1}^{n}\ log(y|i!) - \sum_{i=1}^{n}\ logZ(\lambda_i,\vartheta).$$

CMP distribution is a flexible and versatile model that can handle both overdispersion and underdispersion, which are frequently observed in count data. It is user-friendly, adaptable, and effective in various contexts. The benefits and practical applications of the COM-Poisson distribution, including its use in areas such as marketing and online auctions, have been demonstrated in several studies [19].

Case Study: COVID-19 Death Data

Data on COVID-19 cases were sourced from the daily reports provided by the Institute of Epidemiology Disease Control and Research (IEDCR) in Dhaka, Bangladesh, covering the period from March 8, 2020, to April 30, 2022. This information is available at *https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Bangladesh*. Daily temperature (°C) and humidity (%) data for Bangladesh were obtained from *https://www.timeanddate.com/weather/bangladesh/dhaka*.

Testing for Variable Dispersion

Sellers and Shmueli (2010) developed a hypothesis testing approach to detect whether there is considerable data dispersion, demonstrating the importance of a COM-Poisson regression model over a standard Poisson regression model [17]. This can be performed using the Likelihood Ratio Test (LRT), H_0: $\vartheta = 0$ vs. H_1: $\vartheta \neq 0$. The critical value from the chi-squared distribution at a significance level of α, the critical value to test the null hypothesis at a significance level of 2α used. If the LRT statistic exceeded the critical value of the chi-square distribution, the null hypothesis was rejected [13]. The LRT statistics are defined as:

$$LRT = 2(\ln L_1 - \ln L_0),$$

where $\ln L_1$ and $\ln L_0$ are the log-likelihoods of the models under their respective hypotheses.

Akaike Information Criteria

When comparing the performance of different models, one can use a variety of likelihood metrics proposed in the statistical literature. AIC is one of the most widely used metrics.

A model with additional parameters is penalized by the Akaike Information Criterion (AIC), which is given by the formula:

$$AIC = 2K - 2\ln L$$

Where, *K* represents the number of independent variables, and L is the log-likelihood estimate. A lower AIC value indicates a better-fitting model. To assess the difference in AIC scores between the best model and the comparison model, delta-AIC was used. Additionally, the Akaike Information Criterion corrected (AICc) weight reflects the proportion of the predictive power contributed by the full set of models being evaluated. The log-likelihood (LL) quantifies how likely the model is, given the observed data, and the AIC score is derived from both the LL and the number of parameters K.

NUMERICAL ILLUSTRATION

Descriptive Analysis

As of March 8, 2020, to April 30, 2022, a total of 27514 cases of deaths were officially reported in Bangladesh. These data indicated a positive link between mortality and daily peak temperatures (r= 0.228) and humidity (r= 0.295). Table 1 summarizes the descriptive statistics of the number of COVID-19 deaths and climatic parameters for the 764 days. We use a histogram of the observed count frequencies to obtain a preliminary understanding of the dependent variable.

While the humidity and temperature on average are 30.30°*C* and 63.67%, respectively, the average daily confirmed death rate from COVID-19 is approximately 36. The maximum temperature recorded during this pandemic was 37°*C*, the minimum temperature was 21%, and the highest humidity recorded was 100%.

Figure 1 illustrates the number of COVID-19-related deaths using both a histogram and kernel density plot. The data suggest that a bell-shaped distribution is a strong candidate for modeling this variable, as it indicates a roughly symmetrical distribution of deaths.

Table 1. Descriptive statistics of the number of daily COVID-19 deaths and meteorological factors (temperature and humidity).

Statistics	Number of deaths	Temperature (°*C*)	Humidity (%)
Mean SD	36.013 49.988	30.307 3.831	63.679 16.300
Median	23.00	31.00	65.00
Skewness	2.859	-0.834	-0.175
1Q	7.00	28.00	52.00
3Q	38,00	33.00	75.00
Min.	0	10.00	21.00
Max.	267	37.00	100.0

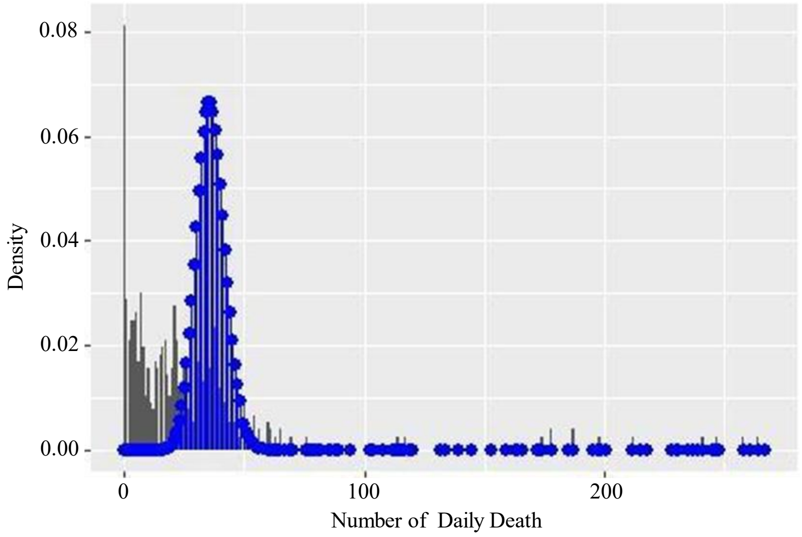

Figure 1. Distribution of the number daily of deaths due to COVID-19, during the period March 2020 to April 2022.

However, while the total number of deaths seems to follow a distribution that leans toward a skewed pattern, this suggests that an asymmetric distribution would be more appropriate for accurately predicting the values of this variable.

Figure 2 depicts the scatter plot of the daily number of COVID-19-related deaths against daily temperature, humidity, and time for the period from March 8, 2020, to April 30, 2022. The response and explanatory variables exhibit an obvious nonlinear relationship. These graphs also illustrate the relationship between the experimental variables and covariates.

Regression Model Fitting and Selection

We fitted GLMMs to the COVID-19 death data using Poisson, Conway–Maxwell–Poisson, and negative binomial distributions in the conditional model, selecting the best-fitting model. Among the models examined, the Conway–Maxwell–Poisson GLMM, which allowed counts to vary with temperature and humidity, proved to be the most efficient. To highlight the additional outputs from the dispersion models, we present a summary of the more complex models in Table 2.

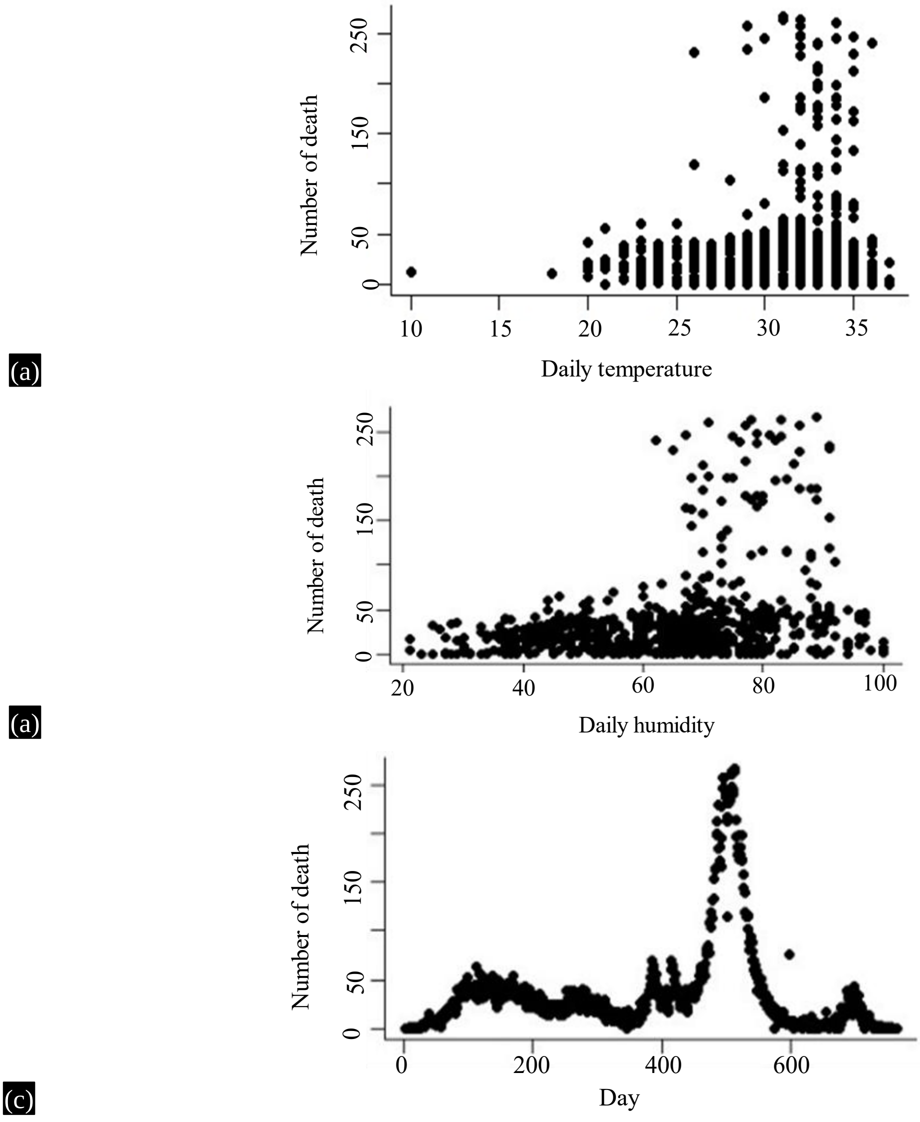

Figure 2. (a) Scatter plot showing the daily number of COVID-19 deaths versus daily temperature; (b) Scatter plot illustrating the daily number of COVID-19 deaths versus daily humidity; (c) Scatter plot depicting the daily number of COVID-19 deaths versus time from March 2020 to April 2022.

The interpretation of the coefficients is clearer in the CMP model. After dividing the COM-Poisson coefficients by v dispersion parameter ($0.025/3.53\times10^9 = 7.0821$), the results in Table 2 indicate that the regression parameters for all models have almost similar estimates in terms of the coefficient magnitudes. The estimated dispersion parameter for the COM-Poisson model is $\vartheta = 3.53\times10^9$, indicating severe overdispersion, so we can use the approximation.

$$E(\grave{Y}) = \grave{\lambda}^{\frac{1}{\vartheta}} - \frac{\grave{\vartheta}-1}{2\grave{\vartheta}} = \grave{\lambda}^{2.83\times10^{-10}} - 0.50,$$

Where $\grave{\lambda}$is given by:

$$\log(\grave{\lambda}) -0.654 + 0.080 \times temperature + 0.025 \times humidity$$

A hypothesis test developed by Sellers and Shmueli (2010) was used to determine whether the dispersion parameter is significant [17]. Because the p-value (= 0.002) is nearly zero, dispersion is present, necessitating a CMP regression as opposed to a Poisson regression.

Model Comparison Using Information Criteria

We can compare all GLMMs using their AIC values, which assess the model's relative information value by considering the highest likelihood estimate and the number of parameters (independent variables) in the model. Table 3 presents the working models. The most parsimonious model is the Conway–Maxwell–Poisson distribution, which incorporates the effects of temperature and humidity. As shown in Table 3, the CMP model has the lowest AIC value, indicating a better fit. The AIC score differences between the CMP model and other models were notable. The third-best model in Table 3 has a delta-AIC of 197.91 compared with the best model, while the next-best model has a delta-AIC of 100.46. Furthermore, the cumulative akaike information criterion corrected (AICc) weights of the top two models accounted for 100% of the total weights.

Table 2. Summary value for Poisson, NB, GP, and CMP regression models in **glmmTMB** for overdispersed counts of COVID-19 death data in Bangladesh.

		glmmTMB		
Coefficient	**Poisson**	**NB**	**GP**	**CMP**
Intercept	-1.948(0.073)	1.414(0.336)	1.856(0.326)	-0.654(0.304)
Temperature	0.113 (0.001)	0.044 (0.009)	0.036(0.008)	0.080(0.009)
Humidity	0.029 (0.001)	0.012(0.001)	0.009(0.001)	0.025(0.002)
Dispersion		- 46.80	82.40	3.53×10^9
Deviance	30027.5	6911.2	7008.7	6810.70
AIC	30033.5	6919.2	7016.7	6818.70
BIC	30047.4	6937.8	7035.2	6837.30

Note: The numbers in the parentheses are the standard errors.
glmmTMB is an R package that fits generalized linear mixed models.

Table 3. Model selection based on AICc.

Model	**k**	**AICc**	**dAIC**	**AICc Wt.**	**Cum. Wt.**	**LL**
CMP	4	6818.80	0.00	0	1	-3405.37
NB	4	6919.26	100.46	0	1	-3455.60
GP	4	7016.71	197.91	0	1	-3504.33
Poisson	3	30033.50	23214.70	0	1	-15013.7

CONCLUSIONS

In practice, the use of discrete distributions to fit count data is often limited. The Poisson distribution is the most popular; however, its equidispersion assumption limits its applicability. For overdispersed data, NB regression is commonly employed, while Conway–Maxwell–Poisson (CMP) regression provides a flexible alternative capable of addressing both overdispersion and underdispersion. This study compares the performance of CMP, Poisson, NB, and generalized Poisson regression models using COVID-19 death data from Bangladesh. The analysis demonstrated that while NB and CMP regressions outperformed the Poisson model, CMP regression provided the best fit according to the log-likelihood and AIC values. The CMP model, influenced by temperature and humidity, was the most parsimonious model with the smallest AIC value, significantly outperforming other models. The study underscores the efficacy of CMP distribution in handling dispersed count data and highlights the importance of selecting appropriate models for accurate data analysis. The revival of CMP distribution, facilitated by advances in computational power, offers new insights into discrete data modeling and emphasizes its utility in real-world applications.

Author Contributions

- *Nazmin Akter:* Conceptualization, data curation, formal analysis, investigation, methodology, software, visualization, writing the original draft, review, and editing.
- *Md. Rezaul Karim:* Methodology; supervision; validation; writing—review.
- *Sultana Begum:* supervision, review, and editing.

Conflict of Interest Statement

The authors have no conflicts of interest to disclose.

Ethics Statement

We conducted our study with integrity, honesty, and respect. We have not knowingly engaged in or contributed to the harm to individuals or animals.

Transparency Declaration

- *Funding:* This research was not supported by any specific grants from public, commercial, or not-for-profit funding agencies.
- *Consent to participate:* Not Applicable.
- *Consent for publication:* Not Applicable.
- *Availability of data and materials:* The data can be accessed on the website with the link provided in Section 2. This will also be made available upon request.
- *Code availability:* The R code is available and will be provided upon request.

REFERENCES

1. Barriga GDC, Louzada F. The zero-inflated Conway–Maxwell–Poisson distribution: Bayesian inference, regression modeling and influence diagnostic. Stat Methodol. 2014;21:23–34. doi: 10.1016/j.stamet.2013.11.003.
2. Boatwright P, Borle S. The timing of bid placement and extent of multiple bidding: An empirical investigation using eBay online auctions. Stat Sci. 2006;21(2):194–205.
3. Borle S, Boatwright P, Kadane JB, Nunes JC, Galit S. The effect of product assortment changes on customer retention. Mark Sci. 2005;24(4):616–22. doi: 10.1287/mksc.1050.0121.
4. Consul PC. Generalized Poisson distributions: properties and applications. 1989.
5. Coxe S, West SG, Aiken LS. The analysis of count data: A gentle introduction to Poisson regression and its alternatives. J Pers Assess. 2009;91(2):121–36. doi: 10.1080/00223890802634175. PubMed: 19205933.
6. Conway R. A queueing model with state dependent service rate. J Ind Eng. 1961;12:132.
7. Daly F, Gaunt RE. The Conway–Maxwell–Poisson distribution: distributional theory and approximation. arXiv Preprint arXiv:1503.07012. 2015.

8. Hilbe JM. Modeling count data. Cambridge University Press; 2014. doi: 10.1017/CBO9781139236065.
9. Harris T, Yang Z, Hardin JW. Modeling underdispersed count data with generalized Poisson regression. STATA J. 2012;12(4):736–47. doi: 10.1177/1536867X1201200412.
10. Huang A. Mean-parametrized Conway–Maxwell–Poisson regression models for dispersed counts. Stat Model. 2017;17(6):359–80. doi: 10.1177/1471082X17697749.
11. Ismail N, Zamani H. Estimation of claim count data using negative binomial, generalized Poisson, zero-inflated negative binomial, and zero-inflated generalized Poisson regression models. Cas Actuar Soc E-Forum. 2013;41(20):1–28.
12. Karim R, Akter N. Effects of climate variables on the COVID-19 mortality in Bangladesh. Theor Appl Climatol. 2022;150(3):1463–75. doi: 10.1007/s00704-022-04211-4. PubMed: 36276261.
13. Lynch HJ, Thorson JT, Shelton AO. Dealing with under- and over-dispersed count data in life history, spatial, and community ecology. Ecology. 2014;95:3173–80. doi: 10.1890/13-1912.1.
14. O'Hara R, Kotze J. Do not log-transform count data. Nat Prec. 2010. doi: 10.1038/npre.2010.4136.1.
15. Rodríguez G. Models for count data with overdispersion. Addendum to the WWS. Vol. 509; 2013.
16. Sellers KF, Shmueli G. A flexible regression model for count data. Ann Appl Stat. 2010;4(2):943–61. doi: 10.1214/09-AOAS306.
17. Shmueli G, Minka TP, Kadane JB, Borle S, Boatwright P. A useful distribution for fitting discrete data: revival of the Conway–Maxwell–Poisson distribution. J R Stat Soc C. 2005;54(1):127–42. doi: 10.1111/j.1467-9876.2005.00474.x.
18. Scollnik DPM. A damaged generalised Poisson model and its application to reported and unreported accident counts. ASTIN Bull. 2006;36(2):463–87. doi: 10.2143/AST.36.2.2017930.
19. Wang G. Bayesian regression models for ecological count data in PyMC3. Ecol Inform. 2021;63:101301. doi: 10.1016/j.ecoinf.2021.101301.