

# Investigating the Dynamics of Quantum Plasmas: Current Advancements and Prospective Trajectories

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## Abstract

Plasma is regarded as quantum if its macroscopic properties are significantly affected by the quantum nature of its constituent particles. A proper description is necessary to comprehend the importance of collective quantum plasma effects. In this entry, the field of quantum plasmas, a generic exotic state of highly ionized matter where quantum effects are relevant, is discussed, for example, by dense plasmas arising from strong laser irradiation of solid targets in compact astrophysical objects such as white dwarfs or neutron stars, solid-state plasmas, and ultrasmall electronic devices. In addition, many condensed matter systems, including the electron gas in metals, metallic nanoparticles, and electron-hole systems in semiconductors and heterostructures, exhibit quantum plasma behavior. The early developments in this subject have been described. The most commonly used microscopic approaches to describe quantum plasma are discussed along with their related assumptions and restrictions. In particular, the quantum hydrodynamic (QHD) model for finite-temperature plasmas has been consistently described, and the range of applicability of the QHD is discussed.

**Keywords:** Quantum plasma, QHD model, kinetic models, nanoplasmas, nanoparticles

## INTRODUCTION

### Quantum Plasmas

Quantum plasmas are charged particle systems in which at least one component (typically electrons) exhibits quantum degeneracy. Such systems are ubiquitous in nature [1–4] and include matter in the interior of the Earth [5], giant planets [6–8], brown and white dwarf stars [9–11], and the outer crust of neutron stars [12, 13].

Quantum plasmas have potential applications in nanoscale systems [14] such as semiconductor quantum wells [15], quantum diodes [16], piezoelectric semiconductors [17], ultra-small electronic devices [18], thin metal films [19], nanowires [20], semiconductor quantum wells [21], ultracold plasmas [22], laser fusion plasmas [23], next-generation high-intensity light sources [24], and plasmonic devices [25]. Quantum plasma effects also appear in biophotonics [26], cool vibes [27], dense astrophysical environments (e.g., white dwarfs and neutron stars) [28], and laser-produced plasmas [29]. Other areas of relevance to quantum plasmas include metal clusters, spintronics, future generation–compression-based high-density plasma experiments, and quantum X-ray free-electron lasers. A particularly exciting application is inertial confinement fusion [30–32], in which the electronic quantum effects are important during the initial phase. Another example is the quark-gluon plasma, which is now routinely produced at the relativistic heavy-ion collider in Brookhaven and the Large Hadron Collider at CERN and exhibits interesting similarities with Coulomb plasmas [33, 34].

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The quantum effects of ions are relevant only at very high densities, particularly for the phase diagram of light elements such as hydrogen [35], ion crystals in white dwarfs, crystals of heavy holes in semiconductors [36, 37], and exotic matter in the interior of neutron stars [12]. The quantum effects of electrons are relevant at low temperatures and/or if matter is highly compressed such that the temperature is lower than the Fermi temperature. Quantum effects that must be considered include spatial delocalization (diffraction effects), exchange effects (antisymmetry of the wave function and Pauli blocking), electronic spin effects, bound states, and their many-particle renormalization (continuum lowering) [38, 39]. Quantum plasmas are characterized by correlations and finite-temperature effects. Thus, a theoretical description of quantum plasmas is challenging.

### BASIC PROPERTIES OF QUANTUM PLASMA

Degeneracy becomes important when the individual electron wave functions have a significant overlap such that the total quantum state cannot be expressed as a simple tensor product of isolated electron quantum states. Therefore, the fermionic character of the charge character is evident for sufficiently dense plasmas, satisfying:

$$\lambda_B > n_0^{-\frac{1}{3}} \quad (1)$$

where  $\lambda_B$  is the De Broglie wavelength. For a completely ionized gas,  $\lambda_B$  is a measure of the particle wave function spreading. In terms of the thermodynamic temperature,  $T$ , we define

$$mv_T^2 = \kappa_B T$$

where  $\kappa_B$  is Boltzmann's constant, and  $v_T$  is the thermal velocity. Moreover, in Eq. (1),  $n_0^{-1/3}$  is a measure of the inter-particle distance, valid for fully ionized plasma, where  $n_0$  is the equilibrium number density of the electron gases.

Condition 1 can be rewritten alternatively as,

$$E_F = \frac{\hbar^2}{2m} \quad (2)$$

where  $E_F$  is Fermi energy. According to Pauli's exclusion principle, even for a zero thermodynamic temperature, it is necessary to occupy higher energy levels, with the top level defining the Fermi surface. In terms of the Fermi temperature  $T_F = E_F/\kappa_B$  one then has the degeneracy condition

$$\chi = \frac{T_F}{T} > 1$$

where  $\chi$  denotes the degeneracy factor. Hence, dense low-temperature plasmas tend to exhibit quantum behavior, whereas dilute high-temperature plasmas tend to behave classically. Consequently, for classical plasmas, the Fermi-Dirac distribution of states can be approximated using the Maxwell–Boltzmann distribution.

In addition to the de Broglie wavelength, another notable characteristic spatial scale in dense quantum plasmas is provided by:

$$\lambda_F = \frac{v_F}{\sqrt{3}\omega_p}$$

which is the Thomas-Fermi length, where  $v_F$  is the Fermi velocity and,  $\omega_p$  is the plasma frequency given in terms of the elementary charge  $e$  and vacuum permittivity  $\epsilon_0$ . The quantity  $\lambda_F$  plays the role of screening length in degenerate plasmas, similar to the Debye length  $\lambda_D = v_T/\omega_p$  for classical plasmas. More detailed accounts of the basic properties of quantum plasmas can be found in the studies of Haas [40], Shukla and Eliasson [41, 42], Manfredi [43], Haas [44], Bonitz [45, 46], and Vladimirov and Tyshetskiy [47].

## HISTORICAL ANTECEDENTS

Using a collective variables approach, Bohm and Pines [48] applied a series of canonical transformations to express the N-body Hamiltonian of an electron gas in an ionic background as the sum of contributions from independent fields (each associated with plasmon excitation) plus a perturbation. In this manner, they derived a linear dispersion relation

$$1 = \omega_p^2 \sum_i \frac{1}{(\omega - k \cdot v_i)^2 - \frac{\hbar^2 k^4}{4m^2}}$$

with a sum of the electron velocities  $v_i$  inside the Fermi sphere, or  $v_i = v_i \leq v_F$ , we have

$$\omega^2 = \omega_p^2 + \frac{3}{5} k^2 v_F^2 + \frac{\hbar^2 k^4}{4m^2} + \dots$$

in the high-frequency limit, which yields the quantum Langmuir wave dispersion relation for a fully degenerate electron gas. In the above equation, we recognize the salient features of quantum plasmas, namely, the influence of Fermi statistics in the second term and of quantum diffraction in the third term on the right-hand side.

It should be noted that historically the first derivation of the dispersion relation was provided by Klimontovich and Silin [49]. They considered the quantum Vlasov equation satisfied by the Wigner function in the presence of a self-consistent electrostatic field. Similarly, Lindhard [50, 51] considered the response of fully degenerate quantum plasmas including quantum recoil for both longitudinal and transverse high-frequency waves. Although the longitudinal response is more widely known, in connection with solid-state plasma applications [52], the transverse response has recently been found to be interesting, for example, in the discussion of wake fields and energy-loss experiments in electron microscopy [53].

Pioneering work on quantum plasmas has been strongly influenced by field theoretical methods, which is in line with the consolidation of quantum electrodynamics at that time. In this context, Gell-Mann and Brueckner [54] estimated the correlation energy of a degenerate free-electron gas in an immobile homogeneous ionic background to lead order on the Wigner–Seitz radius [55]. They used renormalization techniques to sum the most divergent terms in the perturbation series for the self-energy of the electron gas. In addition, Kelly [56] derived the dielectric function for magnetized quantum plasmas using quantum kinetic theory under the assumption of Fermi–Dirac statistics. Melrose [57, 58] provided extensive references to previous developments in the context of quantum plasma dynamics, combining quantum electrodynamics and the kinetic theory of charged particle systems in a covariant description of relativistic quantum plasmas.

## OCCURRENCES OF QUANTUM PLASMAS

Let us now turn to the various occurrences of quantum plasmas in nature and in the laboratory. The phase diagram (Figure 1) contains a variety of important examples and indicates where they are located in the density/temperature plane. The Figure 1 includes dense plasmas in the core of the giant planets of the sun and dwarf stars, as well as the WDM and HEDP. Some examples are discussed below.

### Astrophysical Plasmas

There is an enormous variety of plasmas in space. The interstellar medium contains electrons and ions; however, its density is so low that it behaves in a classical manner. However, quantum effects are expected to occur in many dense astrophysical plasmas [59], particularly in the cores of giant planets such as Jupiter, Saturn, and Neptune, because of the high density caused by gravitation. Theoretical models predict that the plasma density there exceeds the density of solid matter and pressure reaches the multi-megabar range [60, 61].

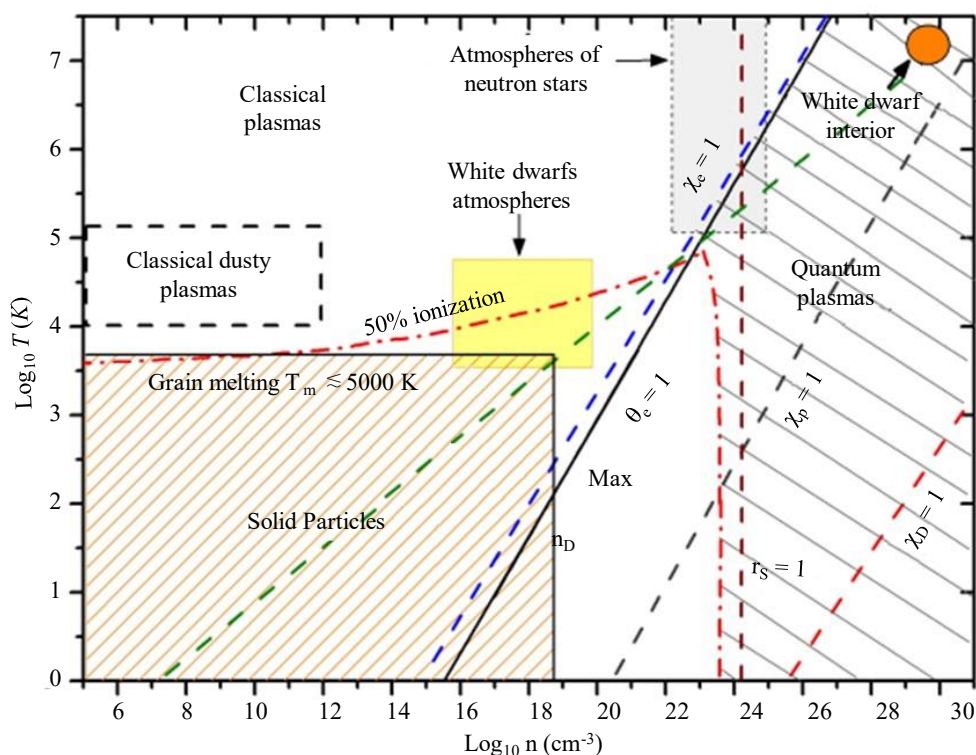
In stars like our Sun, matter is usually also fully ionized owing to strong heat production in the core. Again, gravity produces a radial density profile that peaks at the center. As a result, quantum degeneracy and nonideality also increase toward the center (Figure 1). One indicates that in the core of the Sun, electrons are expected to be quantum degenerate and weakly nonideal [62]. At the same time, ions, mostly protons and  $\alpha$ -particles (hydrogen and helium nuclei), are classical. Other representatives of quantum plasmas include compact stars in the late stages of their evolution, such as brown or white dwarf stars [63]. The last astrophysical object mentioned is a neutron star.

### Dense Laboratory Plasmas

Plasma compression has been achieved by many groups worldwide using various static and dynamic techniques, including diamond anvils [64], gas guns [65], pinches in high-current-carrying plasmas [66], and shockwaves [67]. While the first three have been the main methods employed in recent decades, the latter two are currently actively used, together with lasers and ion beams [68]. In recent years, the densities and pressures have increased steadily, and typical solid-state conditions of  $10^{23} \text{ cm}^{-3}$  have been achieved. At these densities, it is possible to produce degenerate electrons, as long as the temperature is moderate. Usually, this is the case in the initial stage of compression, whereas later, the temperature increases steadily, and the plasma becomes classical again. The corresponding density–temperature range is indicated by the area named “WDM” in Figure 1.

### Laser Plasmas

With the advent of high-power lasers [69], there is a new tool for compressing matter by exploiting the pressure produced by electromagnetic waves [70–72]. Laser beam ionization of metal or plastic foils is now routine, and ionized matter, a mixture of electrons and ions, is subsequently compressed well beyond the solid-state densities. The most spectacular application of this method is thermonuclear (inertial confinement) fusion, where a small capsule of deuterium and tritium is compressed and heated so strongly that it is hoped that the fusion of two hydrogen atoms into one helium atom will occur [73–75]. Here, quantum effects are not the main goal; the thrust is to achieve a high pressure and temperature for a sufficiently long period to fulfill the Lawson criterion for fusion.



**Figure 1.** Density-temperature plain with examples of plasmas and characteristic plasma parameters (Source: Physics of Plasmas 26(9), 090601 (2019)).

### Plasmas in Condensed Matter Systems

Different types of quantum plasma exist in semiconductors. At room temperature, these materials do not contain free mobile electrons. It requires external excitation by heat, voltage, or laser field to excite electrons from the valence band into the formerly empty conduction band, which gives rise to vacancies in the valence band, which act like free positive charges called *holes*. If the excitation is sufficiently strong, the number of electron–hole pairs can be large, with densities reaching  $10^{20}$  cm<sup>-3</sup>, behaving very much like a two-component plasma, which is commonly called an *electron–hole plasma*.

### Highly Compressed Two-Component Plasmas: Mott Effect

We now focus on the dense TCPs in the solid-state density range. As discussed, at low temperatures,  $\kappa_B T < E_R$ , tends to form bound states, so that the plasma behavior is dominated by neutral atoms and molecules. Although these bound states can exist only at low temperatures, we may question whether there is also a restriction with respect to density. Compress the material so strongly that the average distance between two atoms becomes comparable to the average extension of the electron wave function, which is given by the Bohr radius. Then, the electronic wave functions of two electrons of neighboring atoms overlap, allowing one electron to move away from its parent ion to the next one. Such electrons are no longer tightly bound inside the atom; that is, we observe tunnel ionization of the atom. This is also called *the pressure ionization* or *Mott effect*, and it is a purely quantum effect that occurs even at zero temperature.

### Ultra-Dense Plasmas in Nuclear Matter: Quark–Gluon Plasma and the Big Bang

At higher densities, the nuclei crystals exhibit quantum melting, giving rise to a quantum fluid of nuclei embedded in the Fermi gas of the electrons. Suppose we further compressed the plasma. Although nuclei are extremely small entities, we expect their internal structure (i.e., the composition of protons and neutrons) to become relevant at high densities. Indeed, if the distance between two nuclei becomes comparable to the quantum extension of a proton or neutron, we again have a Mott transition situation: a proton (neutron) can tunnel from one nucleus to the next. Consequently, nuclei break up, giving rise to a plasma of free electrons, protons, and neutrons. However, this is still not the end of the compression road, and it is easy to imagine what would happen if the density is so large that two neighboring nucleons overlap. This is expected to occur at a density of approximately  $10^{39}$  cm<sup>-3</sup> [76] particles that are bound inside one nucleon would become free to tunnel out. In fact, the standard model of elementary particles predicts that protons and neutrons are not elementary particles themselves, but that each is composed of three quarks. At this density, quarks would become free (the so-called *deconfinement transition*), and the nuclear matter undergoes another Mott effect and transforms into *QGP*. It is a special type of quantum plasma consisting of charged particles (electrons and quarks), their antiparticles, and their interaction quanta (photons and gluons).

### BASIC METHODS OF THE MICROSCOPIC DESCRIPTION OF QUANTUM PLASMAS

Similar to the case of a classical plasma, the complete microscopic description of a quantum plasma as a system of many interacting particles is a practically hopeless task, not only because it would be impossible to solve the Schrodinger equation for the N-particle wavefunction of the system, but also because of the principal absence of such a wavefunction for a macroscopic system that interacts with its environment, albeit weakly [77]. A convenient approach based on the so-called mixed representation of the density matrix suggested by Wigner [78], in which the quantum distribution function (also called the Wigner function) was introduced [79–81], allowed us to achieve the closest possible similarity to the description of classical plasmas in terms of the distribution function in the phase space of the particle coordinates and momenta. The Wigner function  $f_N(q, p, t)$  is defined from the density matrix  $\rho_N(q, q', t)$  of the system in the coordinate representation, as follows [82–85]:

$$f_N(q, q', t) = \frac{1}{(2\pi)^{3N}} \int d\tau \exp(-i\tau p) \rho_N(q, q' + \frac{1}{2}\hbar\tau, q - \frac{1}{2}\hbar\tau, t)$$

where  $N$  is the number of particles in the system and  $q$  and  $p$  are the  $3N$ -component vectors representing the set of coordinates and momenta of all particles in the system. In the limit  $\hbar \rightarrow 0$ , function  $f_N(q, q', t)$  becomes the classical  $N$ -particle distribution function. Hence, the plasma description in terms of the Wigner function covers both quantum and classical plasma.

The equation that governs the evolution of the Wigner function can be obtained from the evolution equation for the density matrix in the coordinate representation, which is given by [82, 83, 85]

$$\begin{aligned} \frac{\partial f_N(q, p, t)}{\partial t} = & \frac{1}{(2\pi)^{6N}} \frac{i}{\hbar} \int \dots \int d\tau dk d\eta \exp\{i[k(r - q) \\ & + \tau(\eta - p) \times f_N(r, \eta, t) \\ & \left[ H\left(r - \frac{1}{2}\hbar\tau, \eta + \frac{1}{2}\hbar k, t\right) - H\left(r + \frac{1}{2}\hbar\tau, \eta - \frac{1}{2}\hbar k, t\right) \right] \} \end{aligned}$$

where  $H(q, p, t)$  is the Hamiltonian of the system containing the exact (not averaged) fields through which the particles of the system interact.

This approximation, in which a quantum plasma is considered an ensemble of quantum particles interacting via average collective fields, is analogous to the mean-field approximation for classical plasmas suggested by Vlasov [86] and is called the Hartree mean-field approximation [87]. Taking into account particle correlations due to their identity (i.e., taking into account exchange interactions) leads to an additional term in the equation for  $f_1$ , the corresponding approximation is called the Hartree-Fock (mean-field) approximation [88].

One important parameter determining the plasma properties is the coupling parameter  $\Gamma_q$ , which is the ratio of the characteristic potential energy of plasma particle interactions to their characteristic kinetic energy. For quantum plasmas with degenerate electrons, the coupling parameter is given by

$$\Gamma_q = \frac{U_{\text{int}}}{E_F} = \frac{e^2 n^{1/3}}{E_F} \sim \left(\frac{\hbar\omega_p}{E_F}\right)^2 \ll 1,$$

where,  $E_F = (\hbar^2/2m)$  is the Fermi energy. For small values of the coupling parameter,  $\Gamma_q \ll 1$  (i.e., when a plasma can be considered as an almost ideal gas), the role of plasma particle collisions is small compared with the role of collective processes determined by self-consistent plasma fields; therefore, only in this case can the plasma be approximately considered as a collisionless one, at least for collective processes whose characteristic times are small compared with the characteristic time between collisions.

Thus, the collisionless plasma approximation (in which the collision integral is neglected in the equation for  $f_1$ ) is justified only for weakly coupled (ideal) plasmas:  $\Gamma_q \ll 1$ . We note here that in plasmas with  $\Gamma_q \geq 1$ , particle correlations are significant and cannot be neglected; such plasmas are called strongly coupled plasmas. Remarkably, in degenerate quantum plasmas, the coupling parameter  $\Gamma_q$  decreases with increasing density as  $\Gamma_q \propto n^{-1/3}$ , that is, such quantum plasmas become increasingly ideal and collisionless with increasing densities, in contrast to classical plasmas, which become increasingly strongly coupled with increasing densities.

### Collisionless Kinetic Models of Quantum Plasmas with Electrostatic and Electromagnetic Interactions of Particles

Collisionless kinetic models of quantum plasmas with electrostatic and electromagnetic interactions of particles are based on the time-evolution equation for the one-particle Wigner function  $f_1(q, p, t)$  in the mean-field approximation, in which the self-consistent electrostatic or electromagnetic fields are described by either Poisson's equation or Maxwell's equations, respectively. The Wigner function  $f_1(q, p, t)$  is normalized as  $n(q, t) = \int f_1(q, p, t) dp$  where  $n(q, t)$  is the number density of plasma particles. For a system of charged particles interacting via self-consistent electrostatic fields with potential  $\varphi(q)$ , the Hamiltonian is  $H(q, p, t) = p^2/2m + e\varphi(q, t)$ , where  $p$  is the kinetic momentum

of a particle (which in this case coincides with its canonical momentum  $P$ ), and the equation for  $f_1(q, p, t)$  (for simplicity, below we omit the subscript 1 for the Wigner function, so  $f$  connotes the one-particle distribution function) becomes

$$\frac{\partial f_N(q, p, t)}{\partial t} + \frac{p}{m} \frac{\partial f}{\partial q} = \frac{1}{(2\pi)^3} \frac{ie}{\hbar} \int d\tau d\eta \exp\{i\tau(\eta - p)\} f(q, \eta, t) \left[ \varphi\left(q - \frac{1}{2}\hbar\tau, t\right) - \varphi\left(q + \frac{1}{2}\hbar\tau, t\right) \right]$$

The quantum kinetic Wigner equation (5), together with the Poisson equation for electrostatic potential  $\varphi(q, t)$ , describes quantum plasmas with electrostatic interactions between particles. The set of coupled Wigner–Poisson equations is also called the Wigner-Poisson (model) set of equations.

For a system of spinless charged particles interacting via self-consistent electromagnetic fields, the Hamiltonian is:

$$H(q, P, t) = \frac{[P - (e/c)A(q, t)]^2}{2m} + e\varphi(q, t)$$

$H$  where  $P$  is the particle canonical momentum and  $\varphi(q, t)$  and  $A(q, t)$  are the scalar and vector electromagnetic field potentials, respectively.

The main assumptions for collisionless kinetic models based on the Wigner-Poisson or Wigner-Maxwell set of equations are as follows:

(1) The plasma is ideal,

$$\Gamma_q = \frac{U_{\text{int}}}{E_F} = \frac{e^2 n^{1/3}}{E_F} \sim \left(\frac{\hbar\omega_p}{E_F}\right)^2 \ll 1,$$

We note that this condition is not satisfied for electron gas in metals, where  $\Gamma_q \sim 1$ , therefore, collisionless kinetic models are not applicable to metals.

- (2) Plasma particles interact only via average classical collective fields that satisfy Poisson's or Maxwell's equations (self-consistent (classical) mean-field approximation).
- (3) Particle collisions are not considered (the models are collisionless); the same holds for particle correlations because of their identity (exchange interactions are ignored).
- (4) Particle spins are usually not considered.
- (5) A nonrelativistic approximation was used.

### Multistream Model

The multistream model is based on the Hartree mean-field approximation. Here, plasma is considered to be a collection of 'cold beams' formed by groups of particles with the same momenta. Using linearized equations of 'cold hydrodynamics' with self-consistent fields for each of these groups of particles (beams), their current density is calculated and thus the corresponding dielectric permittivity tensor is determined. Then, adding up the contributions of all groups of plasma particles with the corresponding 'weight functions,' i.e., averaging these contributions over plasma equilibrium distribution function  $f_0(p)$ , one obtains the dielectric permittivity tensor of the whole plasma.

### Quantum Hydrodynamic (QHD) Model

This model, described in Ref. [89], is constructed similar to the multistream model (note that it can also be derived from the kinetic equation for the one-particle Wigner function). The wavefunctions of plasma particles are represented as  $\Psi_\alpha(r, t) = a_\alpha(r, t) \exp(iS_\alpha(r, t)/\hbar)$  [90], where  $a_\alpha(r, t)$  and  $S_\alpha(r, t)$  are the real functions of space and time, and the number density  $n_\alpha$  and velocity  $v_\alpha$  of particle group  $\alpha$  are defined as  $n_\alpha = |\Psi(r, t)|^2 = a_\alpha^2(r, t)$  and  $v_\alpha = \nabla S_\alpha$ . Then, the macroscopic plasma density  $n(r, t) = \langle n_\alpha \rangle$  and velocity  $u(r, t) = \langle v_\alpha \rangle$  are introduced, where  $\langle \dots \rangle$  represents the average over an ensemble of plasma particles; the first two hydrodynamic equations are written for  $n(r, t)$  and  $u(r, t)$ , the continuity equation, and the equation of motion, with the latter containing two pressure-like

terms, namely [91] the classical pressure defined by  $P^{cl} = mn(\langle v_\alpha^2 \rangle - \langle v_\alpha \rangle^2)$ , and the quantum pressure is given by

$$P^q = \frac{\hbar^2}{2m} \langle (\nabla a_\alpha)^2 - a_\alpha (\nabla^2 a_\alpha) \rangle$$

To close the set of these two equations, the following two assumptions were made for  $P^q$  and  $P^{cl}$ .

- (1) The wavefunctions of all plasma electrons have equal amplitudes; nevertheless, they can vary in space and time while having different phases. This assumption agrees with the assumption of uncorrelated plasma particles; however, the spatial distribution of each quantum particle does not depend on the spatial distribution of other particles in the system. This assumption leads to the following relationship between  $P^q$  and  $n$ .

$$P^q = \frac{\hbar^2}{2m} n$$

- (2) An equation of state relates the 'classical' pressure  $P^{cl}$  of a quantum plasma to its macroscopic density  $n(r, t) \equiv n_\alpha >$ .

By assuming a particular equation of state for the classical pressure  $P^{cl}$ , we imposed corresponding limitations on the applicability of the closed hydrodynamic model. Of course, in addition to assumptions 1 and 2 stated above, the construction of the quantum plasma hydrodynamic model also includes all the main assumptions characteristic of the Wigner kinetic model.

To summarize, we list the full set of main assumptions for the quantum plasma hydrodynamic model as follows:

- (1) The plasma is ideal,

$$\Gamma_q = \frac{U_{int}}{E_F} = \frac{e^2 n^{1/3}}{E_F} \sim \left( \frac{\hbar \omega_p}{E_F} \right)^2 \ll 1,$$

- (2) Plasma particles interact only through the average classical collective fields described by Maxwell's equations.
- (3) Particle collisions are not considered (the models are collisionless).
- (4) Exchange interactions were ignored.
- (5) A non-relativistic approximation was used.
- (6) Wavefunctions  $\Psi_\alpha(r, t) = a_\alpha(r, t) \exp(iS_\alpha(r, t)/\hbar)$  have equal amplitudes  $a_\alpha(r, t) = a(r, t)$  (which can vary in space and time) for all plasma particles, while differing only in their phases  $S_\alpha(r, t)$ . This imposes a relationship between  $P^q$  and  $n$ .
- (7) Some equation of state is assumed, which relates the 'classical' gas pressure  $P^{cl} = mn(\langle v_\alpha^2 \rangle - \langle v_\alpha \rangle^2)$  to the macroscopic gas density  $n(r, t) \equiv n_\alpha >$ . Usually, the adiabatic equation of state is postulated as  $P^{cl} = P_0^{cl}$ , with  $P^{cl} = n_0 E_F$  for degenerate electrons. From this, it follows a restriction on the lengths of waves that can be correctly described within such a hydrodynamic model,  $k\lambda_F \ll 1$  for degenerate electrons, where  $\lambda_F = v_F/\sqrt{3}\omega_p$  is the Thomas-Fermi length,  $v_F$  is the Fermi velocity of electrons, or  $k\lambda_D \ll 1$  for nondegenerate electrons, where  $\lambda_D = \sqrt{T/2\pi e^2 n}$  is the electron Debye length.

The set of hydrodynamic equations becomes (in the one-dimensional case)

$$\begin{aligned} \left( \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \vec{v} &= -\frac{e}{m} \left[ \vec{E} + \frac{1}{c} (\vec{v} \times \vec{B}) \right] - \frac{v_F^2}{3n_0^2} \frac{\vec{\nabla} n^3}{n} \\ &+ \frac{\hbar^2}{2m^2} \vec{\nabla} \left( \frac{1}{\sqrt{n}} \vec{\nabla}^2 \sqrt{n} \right) + \frac{2\mu}{m\hbar} \vec{S} \cdot \vec{\nabla} B, \end{aligned}$$

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

where the spin magnetic moment  $S$  is governed by

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}\right) \vec{S} = -\left(\frac{2\mu}{\hbar}\right) (\vec{B} \times \vec{S})$$

Owing to its relative simplicity, the quantum hydrodynamics model has a significant advantage over the more exhaustive kinetic model - a smaller number of variables on which the plasma characteristics depend (i.e., four variables,  $r$ , and  $t$  in the hydrodynamic model, instead of seven variables,  $r$ ,  $p$ , and  $t$  in the kinetic model). This advantage allows the consideration of nonlinear plasma phenomena relatively easily; it is eventually the reason why the hydrodynamic approach is preferred for describing such phenomena in quantum plasmas [92, 93].

## CONCLUSIONS

In this section, we briefly review the salient characteristics of quantum plasmas, including remarks on pioneering works, basic models, and future perspectives in the field. Quantum plasmas are a traditional subject that has attracted attention over the years, at least because of some natural questions: when and how will a large system of charged particles with collective behavior (a plasma) exhibit quantum behavior? Recently, the accrued urge of interest around quantum plasmas has been facilitated by the development of new experiments allowing, for example, the assessment of Fermi pressure and Bohm potential corrections by X-ray irradiation in forward scattering geometry. In addition, new models have been introduced that allow the progressive incorporation of relevant effects, particularly regarding spin dynamics, relativistic phenomena, and exchange-correlation energy. Much attention has been paid to the nonlinear collective behavior of quantum plasmas through analytical and numerical work toward the identification of new nonlinear structures such as vortices and solitons in such systems. In particular, nonlinear analysis is necessary in the high-intensity electromagnetic field regime, such as in plasmas arising under the action of strong laser-matter interactions and in extreme astrophysics environments. In addition, the increasing relevance of systems on an intermediate scale, such as metal clusters and thin metal films, or possible connections with promising fields such as spintronics and nanoplasmonics, provides further motivation for the development of quantum plasma.

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