Trends in Electrical Engineering
ISSN: 2249-4774

Volume – 14
Issue-03
Review Article

First Order Sliding Mode Control of Coupled Tank System

B. Amarendra Reddy[1], Urimiti Prathyusha[2*], Manapragada Umakameswari[2], Ch. Nayak Bhukya, A. Venkatesh

[1]Associate Professor, Department of Electrical Engineering, Andhra University College of Engineering, Vishakhapatnam, Andhra Pradesh, India
[2]Student, Department of Electrical Engineering, Andhra University College of Engineering, Vishakhapatnam, Andhra Pradesh, India
[3]PhD Scholar, Department of Electrical Engineering, Andhra University College of Engineering, Vishakhapatnam, Andhra Pradesh, India

Mail for corresponding author: prathyulatha5814@gmail.com

Receiving Date: 11th November 2024
Accepted Date: 27th November 2024
Published Date: 3rd December 2024

ABSTRACT: -
This paper investigates the challenging problem of liquid level control in Coupled Tank Systems (CTS). CTS, characterized by its nonlinear behaviour and sensitivity to disturbances, presents significant control challenges. This report suggests and assesses several control measures to solve these problems. Mathematical models are developed to capture the system's nonlinear dynamics, and controllers such as Proportional-Integral (PI) and Sliding Mode Controllers (SMC) are designed. The PI controller effectively regulates the amount of fluid in the second tank by modifying the water pump in the initial tank. It is well known that the sliding mode controller (SMC) is robust to uncertainties and disturbances. The simulation results are carried out for FOSMC and PI controllers for various tracking signals with MATLAB. Because of its intrinsic nonlinearities and tank-to-tank interactions, the coupled tank system is a dynamic system that is extensively researched in control engineering. It is a useful field of study because of its use in industrial operations where liquid levels need to be kept at predetermined levels in spite of disruptions. Because the system is frequently modelled as a multi-input, multi-output (MIMO) system, control solutions face particular difficulties.

Keywords: - Coupled-tank, Sliding Mode Control, Proportional-Integral

I. INTRODUCTION

Managing liquid level systems in an industrial process control system can present various challenges, here, excessive liquid levels can lead to spills, which pose safety and environmental risks. These problems often require careful design, effective control strategies, and continuous monitoring to ensure optimal operation. Implementing robust controllers, regular maintenance, can help mitigate many of these issues. Sliding mode control can be applied in various scenarios, such as managing the inflow and outflow in water treatment plants, chemical processing, and other industrial applications where precise flow control is critical.

The robust control method known as Sliding Mode Control (SMC) is particularly suitable for systems that are subject to external disturbances, nonlinearities, and uncertainties [8]. A closed-loop feedback control method called a Proportional-Integral controller (PI) seeks to modify the process variable, by adjusting it in response to the difference between the process variable and the setpoint.

This paper is arranged as follows. (i). Section II explains the system description of a connected tank. (ii). Section III explains the Mathematical modelling of the coupled tank system. (iii) Section IV explains the Proportional-Integral Controller and how it is derived. [6-10] (iv) Proportional-Integral Controller for coupled tank system is explained in V.(v) Section VI explains the Sliding Mode Control and how it is derived for the generalized systems. [11-12] (v)Section VII explains SMC design for a coupled tank system. [13-18] (vi). The MATLAB- based simulation is described in section VIII. (vii) Section IX presents the conclusions.

II. SYSTEM DESCRIPTION

The objective of a coupled tank system is to regulate the flow and liquid levels between two interconnected tanks. Such systems are commonly used in various industrial applications, including water treatment, chemical processing, and hydraulic systems. Tanks are connected via pipes that facilitate the flow of liquid between them. Control valves may be included in these pipes to regulate the flow rate. To ensure effective liquid transport, these pipelines design should take elevation, length, and diameter into consideration. Through the connecting pipes, which are managed by valves, the liquid enters and exists the tanks. The flow rates are adjusted to maintain desired levels in each tank. Fig .1 demonstrates the representation of a coupled tank system. The next section provides an explanation of coupled tank system modelling and analysis.

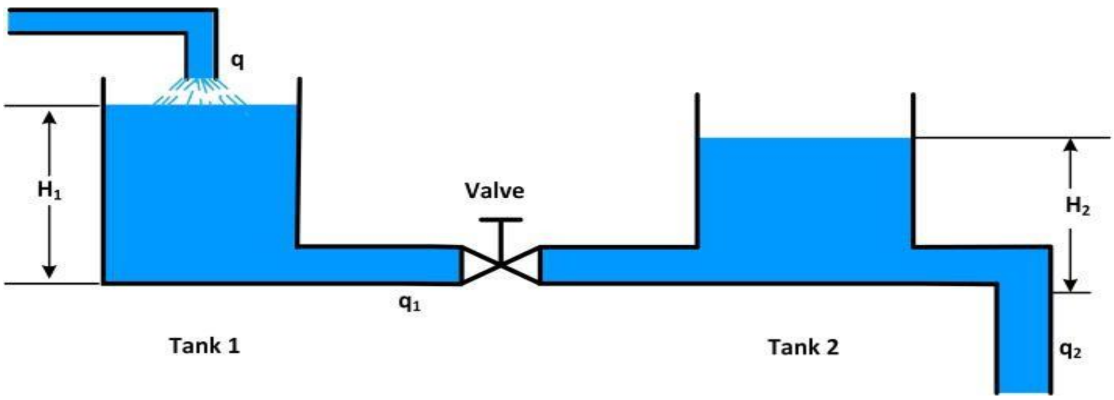

Fig 1. A Coupled Tank System

III. MODELLING OF COUPLED TANK SYSTEM

Taking into consideration that the first tank (q) is pumped with water. A connecting pipe (q1) fills the second tank from the first tank. The second tank has a channel at the bottom to change the output flow (q2). The coupled-tank system is represented mathematically as a nonlinear system [1]. Applying the equation for flow balance to tank 1 and tank 2, so the following equations are produced. H_1 and H_2 represent the liquid levels in tank 1 and tank 2, are measured in relation to the respective outflow. Thus, for tanks 1 and 2, the dynamic equation is developed and provided in equation (1).

$$\frac{dH_1}{dt} = \frac{1}{A_1}(q - q_1)$$

$$\frac{dH_2}{dt} = \frac{1}{A_2}(q - q_2) \quad (1)$$

Where H_1, H_2 represent the liquid levels in tank 1 and tank 2. $A_1$and A_2 represent the cross-sectional areas of two tanks respectively. Where q_1, q_2 in equation (1) are expressed as

$$q_1 = a_{12}\sqrt{2g(H_1 - H_2)}\ for\ H_1 < H_2$$

$$q_2 = a_2\sqrt{2gH_2}\ for\ H_2 < 0 \tag{2}$$

Where the proportionality constants, a_{12} and a_2 are determined by the cross-sectional area of each tank's orifice and the coefficients of discharge, respectively [2].

The following differential equations can be used to represent the coupled tank system:

$$A_1 \frac{dH_1}{dt} = q - B_{12}\sqrt{|H_1 - H_2|}\ sign\ (H_1 - H_2) \tag{3}$$

$$A_2 \frac{dH_2}{dt} = -B_2\sqrt{H_2} + B_{12}\sqrt{|H_1 - H_2|}\ sign\ (H_1 - H_2) \tag{4}$$

Since there is no leakage in tank 1, hence, where $B_1 = 0$. Equation (5) provides the output discharge proportionality constant, B_2, while in equation (6) provides the discharge proportionality constant between tanks 1 and 2.

$$B_2 = a_2\sqrt{2g} \tag{5}$$

$$B_{12} = a_{12}\sqrt{2g} \tag{6}$$

The dynamic model is represented as

$$\frac{dH_1}{dt} = \frac{U}{A_1} - \frac{B_{12}}{A_1}\sqrt{|H_1 - H_2|}\, sign\,(H_1 - H_2) \tag{7}$$

$$\frac{dH_2}{dt} = -\frac{B_2}{A_2}\sqrt{H_2} + \frac{B_{12}}{A_2}\sqrt{|H_1 - H_2|}\, sign\,(H_1 - H_2) \tag{8}$$

The nonlinear model is first developed using Simulink using the dynamic equations (7) and (8). The modelling of the valve pump actuator is given in the following differential equation (dynamics).

$$T_c\frac{dq_i(t)}{dt} + q_i(t) = Q_c(t) \tag{9}$$

The time constant of the valve/pump actuator is denoted by T_c. The time-varying input flow rate is denoted by $q_i(t)$.$Q_c(t)$ represents the computed or commanded flowrate.

IV. PROPORTIONAL – INTEGRAL (PI) CONTROLLER DESIGN

To controlling different operations in control system, a Proportional-Integral (PI) controller is used for regulating processes. By combining proportional and integral control actions, a PI controller aims to reduce the discrepancy between a system's actual output and a desired setpoint.

The following are the steps involved in designing a Proportional-Integral controller for a coupled tank system:

i) Define the nonlinear system and its mathematical model, including differential equations as discussed in section II and section III.

ii) Determine the system parameters, such as tank 1 and tank 2 cross-sectional areas, the height-to-volume relationship, and flow dynamics as described in section III.

iii) The gains K_p and K_i are used for coupled tank systems. To tune the PI controller parameters, trial and error methods are used for nonlinear models.

iv) Start with K_p to achieve a desired transient response, then adjust K_i to eliminate steady-state error.

v) To show the controller performance when the system is subjected to disturbance rejection

V. PI CONTROLLER FOR COUPLED-TANK SYSTEM

The manipulated variable is affected by a control signal applied to actuator, which is one of the feedback controller's outputs and results in change in the controlled variable [4].

$$G(s) = \frac{U_{(s)}}{E_{(s)}} = K_p + \frac{K_i}{s}$$

Table.1 The effect of increasing the controller parameters

Parameter	Rise time	Overshoot	Settling time	Steady state error
K_p	Decrease	Increase	Small change	Decrease
K_i	Decrease	Increase	Increase	Eliminate

The Proportional-Integral controller parameters for the nonlinear model are adjusted by trial-and-error method. Table 1 represents the effect of increasing the controller parameters. The difference between the desired setpoint and the actual process variable once the system has stabilized and functioning under steady conditions is known as steady-state error. To achieve the required control performance in a proportional-integral (PI) controller, steady-state error must be understood and managed. Additionally, a motor pump or valve is used to connect the commanded input to the actual input flow entering the first tank.

VI. SLIDING MODE CONTROLLER DESIGN

A system is forced to slide along a predetermined surface in state space using the nonlinear control method known as sliding mode control (SMC). It's a variable structure control method that uses a discontinuous control signal to alter the system's dynamics [5]. The control signal switches between different structures based on the system's position in state space. The way the system slides around the boundaries of these structures is called sliding mode. The fundamental concept of sliding mode control is to push the system states to a predetermined "sliding surface" (or manifold) in the state space and then make sure that the system stays there in spite of disruptions or inaccurate model representations.

The following are the steps involved in designing a Sliding mode controller (SMC) for a coupled tank system:

i) Define the nonlinear system and its mathematical model, including differential equations as discussed in section II and section III.

ii) Determine the system parameters, such as tank 1 and tank 2 cross-sectional areas, the height-to-volume relationship, and flow dynamics as described in section III.

iii) In the system's nonlinear model, FOSMC is used.

iv) Define the system error and the sliding function of the system.

v) Define the discontinuous control law and SMC control law. The chattering phenomenon is FOSMC's primary drawback in this case. In this case, the unwanted chattering is adjusted using the chattering suppression factor, as illustrated in section VII.

VII. FIRST ORDER SMC AND STABILITY CONDITION FOR COUPLED TANK SYSTEM:

The theoretical definition of the system error is the difference between the desired and the actual heights as the aim is to have both the system error and its derivative equal to zero. It can be stated Mathematically as

$$e(t) = H_d(t) - H_2(t) \tag{10}$$

Where $H_d(t)$ is the desired height, $H_2(t)$ is the actual height in tank 2.

The n^{th} order sliding function is expressed as

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e(t) \tag{11}$$

A 2^{nd} order sliding function (n-2) can be expressed as

$$S = \dot{e}(t) + \lambda e(t) \tag{12}$$

Where the slope of sliding surface $\lambda > 0$

Then,

$$S(t) = \dot{H}_2(t) + \lambda\,(H_d(t) - H_2(t)) \tag{13}$$

differentiate 's' with respect to time

$$\dot{S}(t) = \ddot{e}(t) + \lambda\,\dot{e}\,(t) \tag{14}$$

Here,

$$\dot{S}(t) = \ddot{H}_2(t) + \lambda \dot{H}_2(t) \tag{15}$$

From the state equation of the CTS, we can express $\ddot{H}_2$ as

$$\ddot{H}_2(t) = f(t) + b(t)u \tag{16}$$

Where,

$$f(t) = \frac{bc}{2}\left(\frac{\sqrt{H_2}}{\sqrt{H_1 - H_2}} - \frac{\sqrt{H_1 - H_2}}{\sqrt{H_2}}\right) + \frac{b^2}{2} - c^2$$

$$b(t) = \frac{c}{2A}\left(\frac{1}{\sqrt{H_1 - H_2}}\right) \tag{17}$$

By substituting eq.14 in eq.15

$$\dot{S} = f(t) + b(t)u + \lambda \dot{H_2}(t) \quad (18)$$

If the system stays on a sliding surface, then $e(t)$ is always zero. Therefore, it is intended to make

$$\dot{S}(t) = \frac{dS(t)}{dt} = 0 \quad (19)$$

The control law's continuous part will be (from equation 16)

$$u(t) = \frac{1}{b(t)} [-f(t) - \lambda \dot{H}_2(t)] \quad (20)$$

The expression for SMC's basic discontinuous control law is

$$U_D = Ksgn(S) \tag{21}$$

Where the reaching mode is caused by K, which is the constant manual tuning parameter. The chattering phenomena is the major drawback of FOSMC.

$$U_{disc} = K\frac{s}{|s|+\delta} \tag{22}$$

When the undesired chattering is eliminated by manually adjusting the chattering suppression factor, or parameter δ [3]. The following is the representation of the SMC law:

$$U = \frac{1}{b(t)}\left[-f(t) - \lambda\dot{H}_2(t)\right] + K\frac{s}{|s|+\delta} \tag{23}$$

VIII. SIMULATION RESULTS

The objective of this paper is to regulate the liquid level of the second tank H_2 for the nonlinear model. The liquid level in the second tank is 10cm, which is a desired setpoint. Zero steady state errors should be achieved, along with a good transient response. The PI controller simulation result for various K_p and K_i values is given in figure 2(a). Through trial and error that has been adjusted. The best performance was provided by the controller at gains K_p=15.6 and K_i=1.3. To demonstrate the controller's ability to reject disturbances affecting the system, the simulation is run with the disturbance loaded. The second motor pump in the second tank, activates and produces a flow rate of magnitude 17cm³/sec. After 60 seconds of running, the disturbance is applied to the system.

Parameters of the Coupled Tank system

	Symbol	**Parametric Values**		
Cross sectional area of each tank	$A_1 and A_2$ cm^2	32		
Proportionality constant which depends upon the cross - sectional area of tank and gravitational constant	B_i $cm^{\frac{5}{2}}/sec$	B_1	B_2	B_{12}
		14.3	14.3	20
Pump motor time constant	T_c sec	1		
Maximum allowable volumetric flowrate pumped by motor	Qi_{max} cm^3/sec	300		
Gravitational constant	g cm/sec^2	981		
Constant	λ	0.35		
Tuning parameter	k	350		
Tuning parameter	δ	0.20		

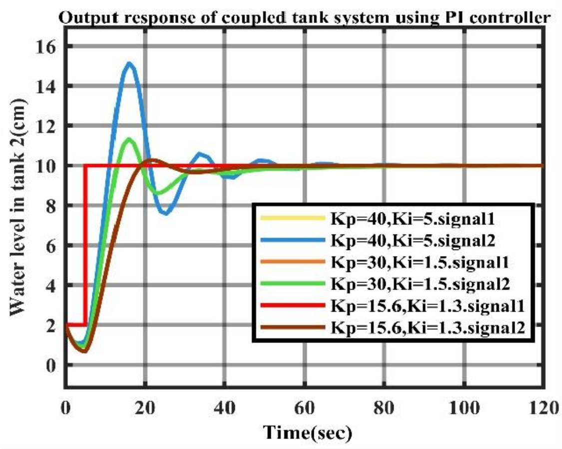

Fig.2(b) Output response of coupled tank system using PI controller.

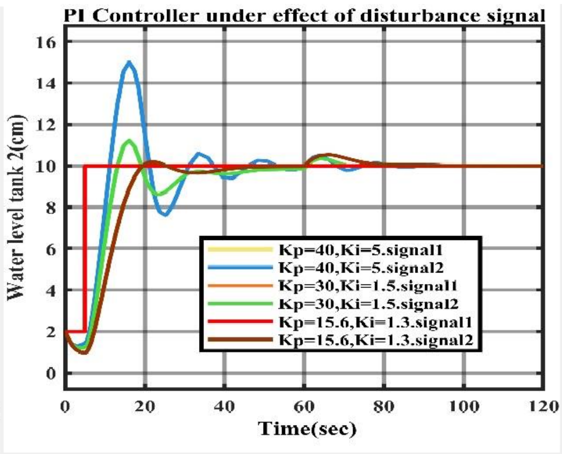

Fig.2(a) Output response of coupled tank system using PI controller under effect of disturbance

The step response of the system under effect of the disturbance and the Simulink diagram of the controlled nonlinear model are displayed in Fig 2(b). The controller has clearly rejected the disturbance.

The first order sliding mode controller (FOSMC) is a simplified version of the sliding mode control strategy, aimed at controlling systems in a way that achieves robustness and stability while minimizing complexity. This controller focuses on achieving a basic level of sliding mode control with a simpler approach, which is useful for systems where computational resources or system complexity is a concern. The liquid level in second tank is controlled by the sliding mode controller, which maintains a constant level of 10 cm. In tank 2, this liquid level and the required level must be reached, the sliding mode controller is made to control the flow rate “q” at tank 1. To obtain the control signal, the simulation’s saturation function always stays inside its limits. The tuning parameters $'K = 350'$ and $'\delta = 0.20'$ are taken into consideration. Here, the simulation is done using sliding mode control. The following represents the simulation response of step signal, sine signal, square signal, sawtooth signal, disturbance signal and without disturbance signal.

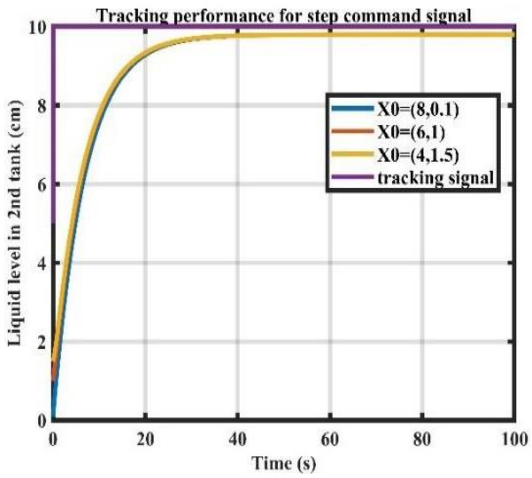

Fig.3(a) Tracking performance for step command signal

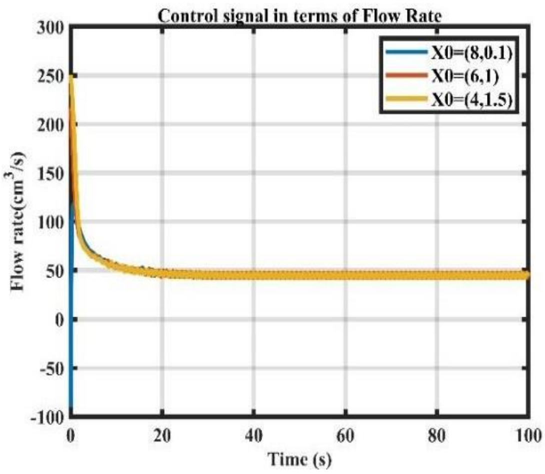

Fig.3(b) Control signal in terms of flowrate

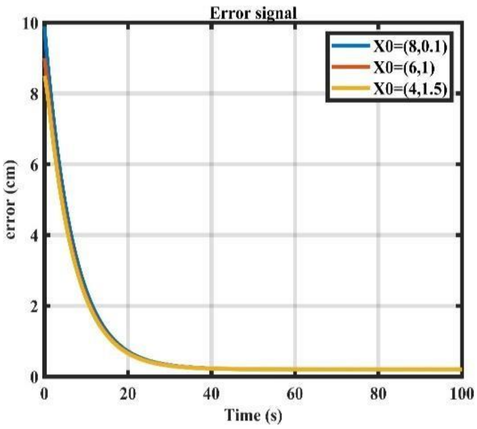

Fig.3(c) Error signal

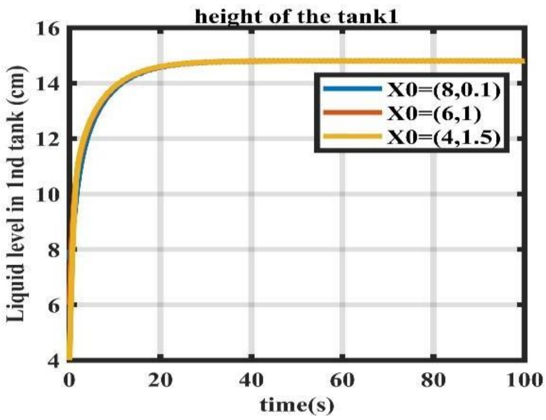

Fig.3(d) Liquid level of the tank 1 versus time

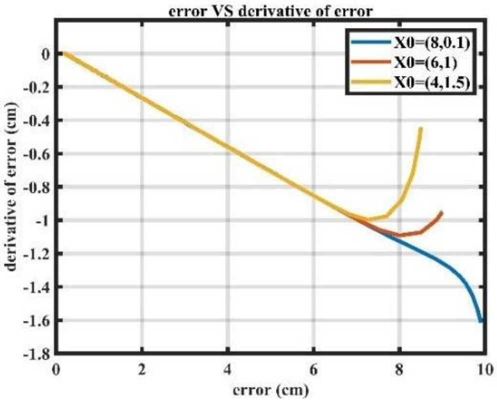

Fig.3(e) Phase trajectory for step signal

Here, Fig.3(a) illustrates the tracking performance of step signal with initial conditions. The control signal, error signal, liquid level of the tank 1 and the phase trajectory for step signal are plotted as shown in Fig 3(b) - 3(e).

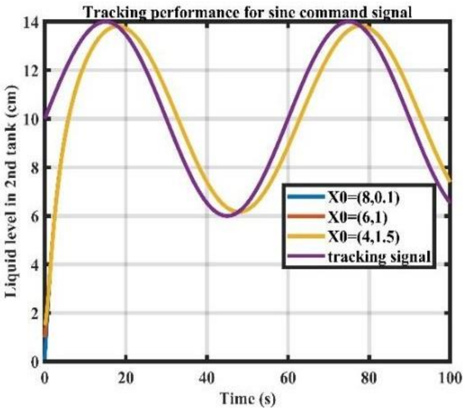

Fig.4(a) Tracking performance for sine command

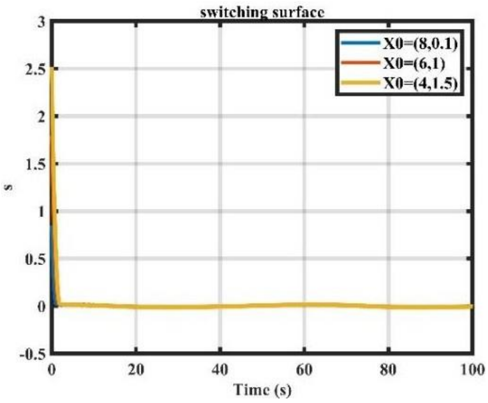

Fig.4(b) Switching surface versus time

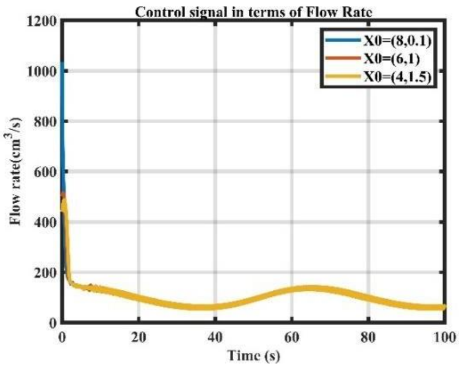

Fig.4(c) Control signal in terms of flowrate Signal

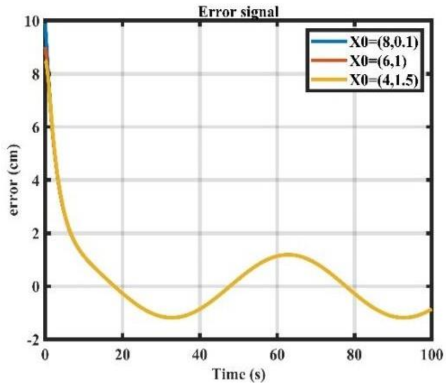

Fig.4(d) Error signal

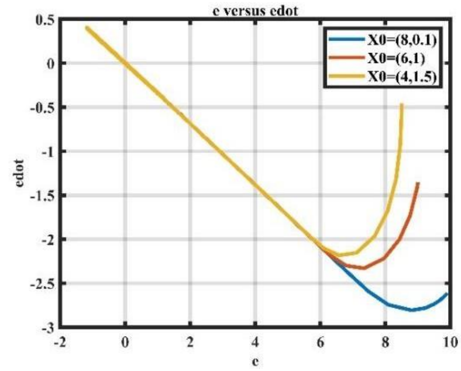

Fig.4(e)Phase trajectory for sine signal

Here, Fig.4(a) illustrates the tracking performance of sine command signal with initial conditions. Fig.4(b) - 4(e) illustrates the switching surface, control signal, error signal, and the phase trajectory for sine signal.

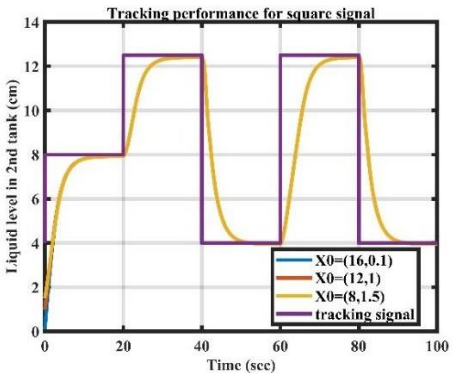

Fig.5(a)Tracking performance for square signal

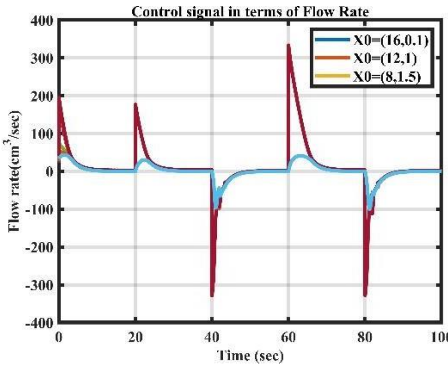

Fig.5(b) Control signal in terms of flowrate signal

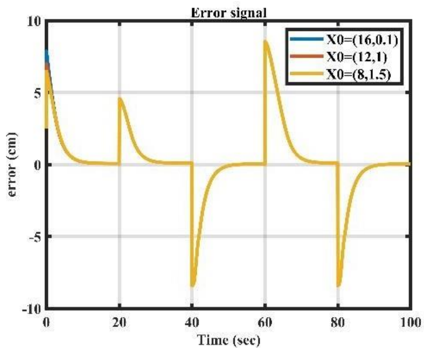

Fig. 5(c)Error signal

Here, Fig.5(a) illustrates the tracking performance of square signal with initial conditions. Fig.5(b) and 5(c) illustrates the control signal and error signal.

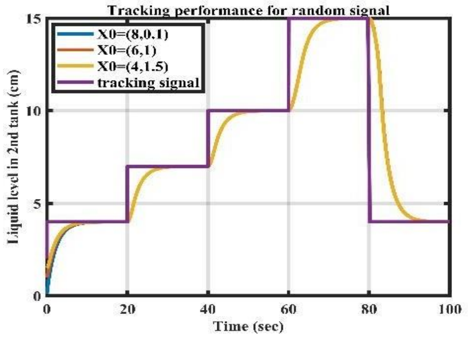

Fig.6(a)Tracking performance for random signal

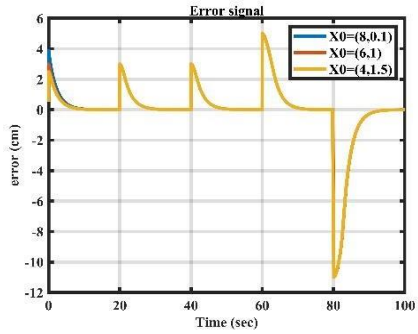

Fig.6(b) Error signal

Here, Fig.6(a) illustrates the tracking performance of random signals with initial conditions. Fig 6(b) illustrates the control signal.

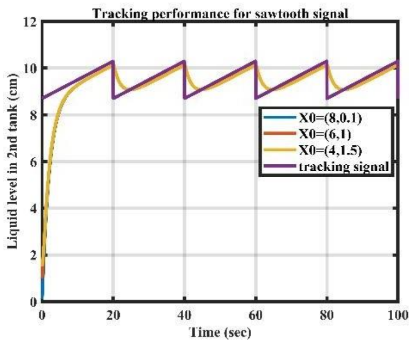

Fig.7(a)Tracking performance for sawtooth signal

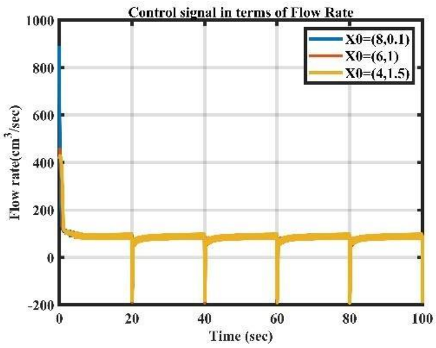

Fig.7(b) Control signal in terms of flowrate signal

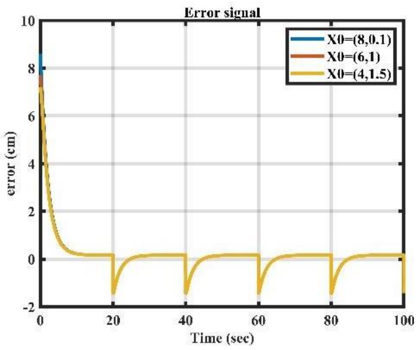

Fig. 7(c)Error signal

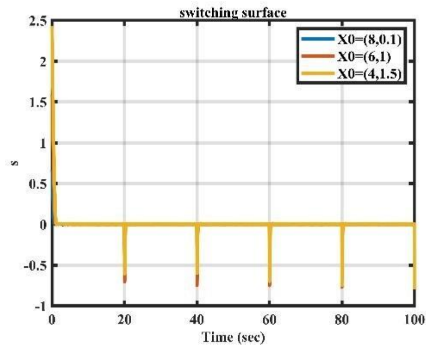

Fig.7(d)Switching surface versus time

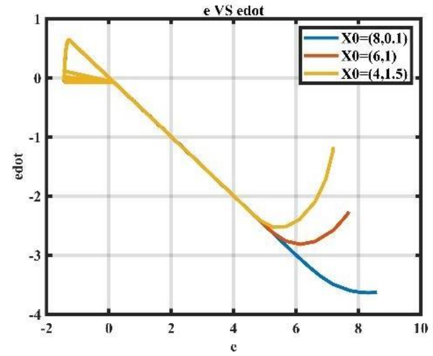

Fig.7(e)Phase trajectory for sawtooth signal

Here, Fig 7(a) illustrates the tracking performance for sawtooth signal with initial conditions. Fig 7(b) - 7(e) illustrates the switching surface, control signal, error signal, and the phase trajectory for sawtooth signal.

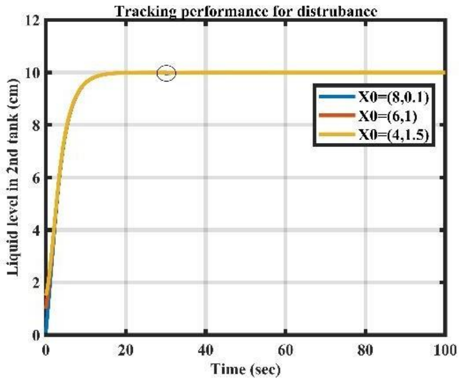

Fig.8(a)Tracking performance for disturbance signal

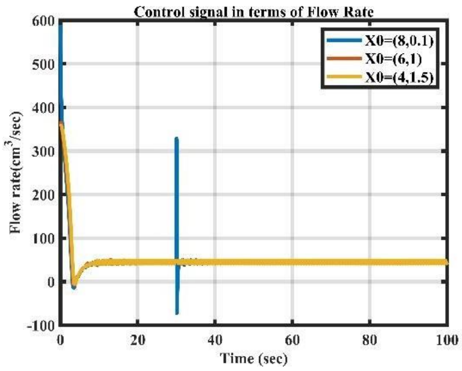

Fig.8(b) Control signal in terms of flowrate signal

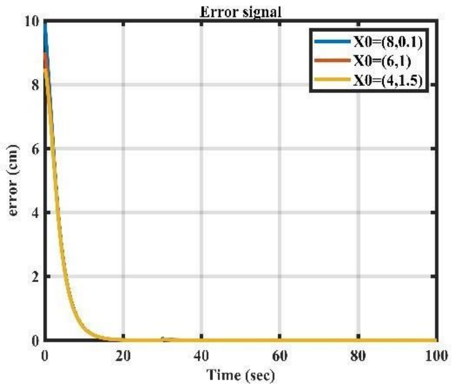

Fig.8(c) Error signal

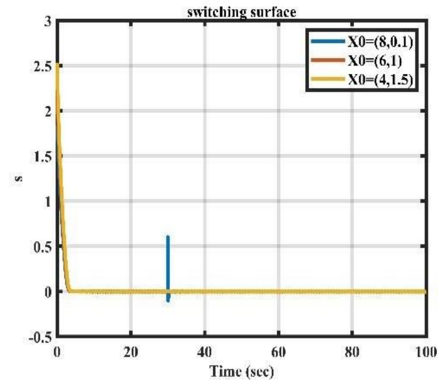

Fig8(d)Switching surface versus time

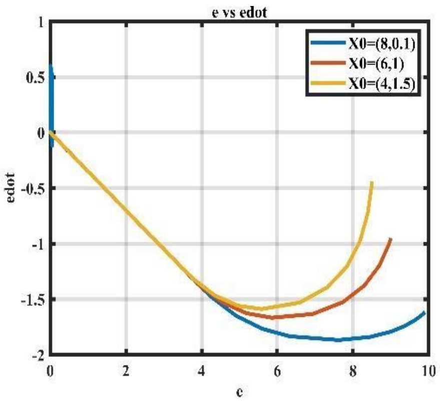

Fig.8(e) Phase trajectory for disturbance signal

Here, Fig 8(a) illustrates the tracking performance for a disturbance signal with initial conditions. Fig 8(b) - 8(e) illustrate the switching surface, control signal, error signal, and the phase trajectory for the sawtooth signal.

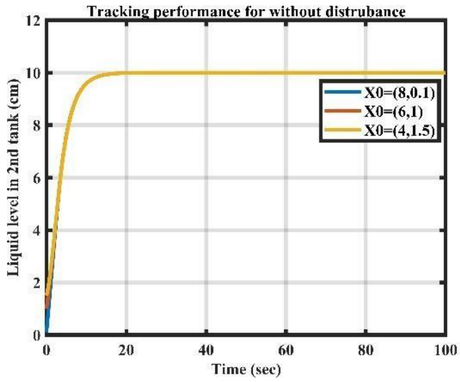

Fig.9(a)Tracking performance for without disturbance signal

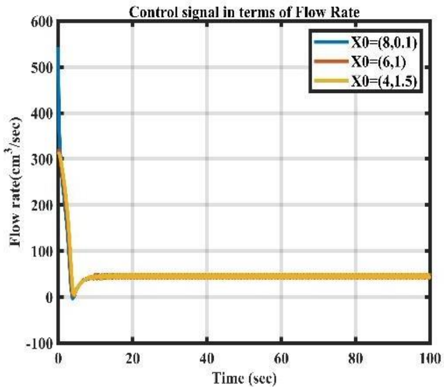

Fig.9(b)Control signal in terms of flowrate signal

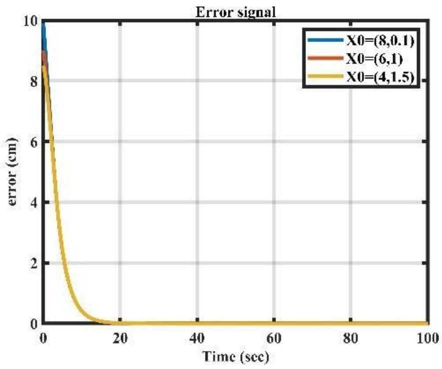

Fig.9(c) Error signal

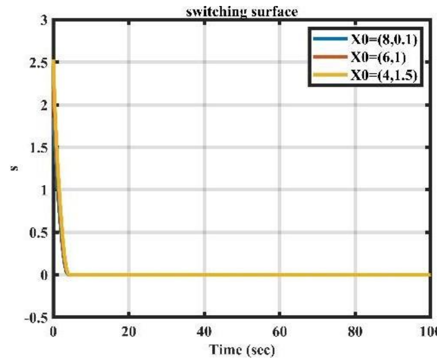

Fig.9(d)Switching surface versus time

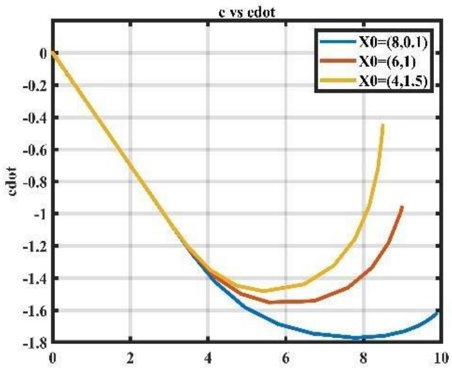

Fig.9(e) Phase trajectory for without disturbance signal

Here, Fig 9(a) illustrates the tracking performance for the without disturbance signal with initial conditions. Fig 9(b) - 9(e) illustrate the switching surface, control signal, error signal, and the phase trajectory without the disturbance signal.

IX. CONCLUSIONS

Based on the effective design and development of the nonlinear model, the liquid level in second tank can be adjusted to any desired level. A variety of input conditions have been successfully used and designed for the coupled tank system with PI controller and FOSMC controllers. The PI controller adjusts the PI parameters using a trial-and-error method. There is a minor improvement in the integral term of the controller and the transient response. As a result, the controller is more robust to reject the disturbance. In SMC, several input signals including step, square wave, sawtooth, sinusoidal, and random signals as well as in disturbance cases, were taken into consideration when designing the switching surface and sliding mode control law.

REFERENCES

1. B. Amarendra Reddy, G. Ananda Rao, K. Anusha, B. Pavan Kumar "Control of Coupled Tank Liquid Level System Using Sliding Mode Control", Helix. February 2018; 8(2): 3119-3125.
2. Reddy, B. Amarendra, and P. Vamsi Krishna. "Comparison of second order sliding mode control strategies for coupled tank system." International Journal of Innovative Technology and Exploring Engineering 8.6S3 (2019): 344 -349.
3. A. R. Gopisetti, K. Anusha, B. A. Reddy and B. P. Kumar, "Performance Evaluation of Second Order Sliding Mode Control Strategies for a Coupled Tank System," 2018 International Conference on Current Trends towards Converging Technologies (ICCTCT), Coimbatore, India, 2018, pp. 1-7, doi: 10.1109/ICCTCT.2018.8551080.
4. M. Saad, "Performance Analysis of a Nonlinear Coupled-Tank System using PI Controller," Universal Journal of Control and Automation. 2017; 5(4): 55-62.
5. H. Abbas, S. Asghar and S. Qamar, "Sliding Mode Control for Coupled-Tank Liquid Level Control System," *2012 10th International Conference on Frontiers of Information Technology*, Islamabad, Pakistan, 2012, pp. 325-330, doi: 10.1109/FIT.2012.65.
6. M. Saad, A. Albagul, and Y. Abueejela, "Performance Comparison between PI and MRAC for Coupled-Tank System," Journal of Automation and Control Engineering, 2 (3): 316-321, September 2014.
7. J. Malmborg and J. Eker, "Hybrid control of a double tank system," *Proceedings of the 1997 IEEE International Conference on Control Applications*, Hartford, CT, USA, 1997, pp. 133-138, doi: 10.1109/CCA.1997.627515.
8. B. Stenlund, and A. Medvedev, "Level Control of Cascade Coupled Flotation Tanks IFAC Proceedings Volumes. August 2000; 33(22): 185-190.
9. M. S. Ramli, R. M. T. R. Ismail, M. A. Ahmad, S. M. Nawi, and M. A. M. Hussin, "Improved coupled tank liquid levels system based on swarm adaptive tuning of hybrid proportional-integral neural network controller," American J. of Engineering and Applied Sciences, 669-675, 2009.
10. N. B. Almutairi, and M. Zribi, "Sliding Mode Control of Coupled Tanks," Elsevier J. Mechatronics, 427-441, 2006.
11. T. L. Seng, M. Khalid, and R. Yusof, "Tuning of a Neuro-Fuzzy Controller by Genetic Algorithms with an Application to a Coupled-Tank Liquid-Level Control System. Engineering Applications of Artificial Intelligence," In press, 517-529, 1998.
12. S. N. Engin, J. Kuvulmaz, and V. E. Ömürlü, "Modeling of a Coupled Industrial Tank System with ANFIS," in MICAI Advances in Artificial Intelligence, 804-812, 2004.
13. M. N. b Mahyuddin, Direct Model Reference Adaptive Control of Coupled Tank Liquid Level Control System, Ms. Eng. Thesis, Dept. Mechatronics and Automatic Control, Universiti Teknologi Malaysia, October 2005.
14. Mircea Dulau, Stelian-Emilian Oltean and Adrian- Vasile Duka, "Robust Control of a Multivariable System," 10th International Conference Interdisciplinarity in Engineering, INTER-ENG 2016; Procedia Engineering. 181 (2017): 626-633.
15. R. Dorf and R. Bishop, Modern Control Systems, 11th ed., Prentice Hall, 2008.
16. Hayriye Tugba, Kaan, Abdullah, "Real-Time Application of Sliding Mode Controller for Coupled Tank Liquid Level System", International Journal of Applied Mathematics, Electronics and Computers. 2016; 4(special issue): 301-306.
17. Christopher Edwards and Sarah K. Spurgeon "Sliding mode control: theory and applications". 1ST ed. 1998. CRC Press; London.
18. Ligang Wu, Peng shi and Xiaojie Su "sliding mode control of uncertain parameter-switching hybrid systems". John Wiley & Sons, Ltd. 2014.