

History and Applications of Kalman Filter: A Review

Aiswarya Nair P.¹, Resmi R.^{2,*}

Abstract

The Kalman filter is a powerful algorithm that is used to estimate the dynamic system states with noisy measurements and uncertain behaviors. It is an optimal estimator that minimizes the average squared error between the estimated states and the true states, given the noisy data and a model of the system. The recursive algorithm is highly effective in tracking and predicting the state of complex systems over time. Kalman filters have been a very crucial part of estimation theory since their development in 1960. Originally used in missile tracking and aerospace engineering for specific purposes such as tracking and optometric estimations, they have come a long way and proved to be useful in the modern scenario in a plethora of engineering applications. Various modifications have already been brought to this recursive algorithm for state estimation, so that they can be used for a wide variety of applications in engineering such as object tracking, state of charge estimation, and localization to name a few. The main types of Kalman filters discussed in this paper are the traditional Kalman filter, the extended Kalman filter, the unscented Kalman filter, the cubature Kalman filter, and the ensemble Kalman filter. Each of these variations of the algorithm adopt unique mathematical approaches which are different from each other to solve the problem at hand. In presented paper a review of the history of Kalman filter and some of its applications is presented.

Keywords: Estimation theory, Kalman filter, extended Kalman filter, unscented Kalman filter, cubature, ensemble Kalman filter

INTRODUCTION

Estimation theory is a branch of statistics and mathematics that deals with estimation of unknown parameters on the basis of previous observations. It has changed the face of engineering over the years and has been widely applied in robotics, control systems, image processing, electric vehicular technology, and so on. Estimation theory consists of several techniques such as state estimation, curve fitting [1], robust estimation [2], etc. State estimation refers to the inferring of the internal dynamics of a system. Curve fitting or regression analysis is a technique used in statistics to formulate a relationship between two variables by fitting a function to the observed data points. Robust estimation is a statistical technique that is used for obtaining estimate values from observations accurately in the presence of disturbances. Recent updates in the area include integration of estimation techniques with deep

learning algorithms and online estimation that can handle real-time data from sensors and internet of things (IoT). The focus of this paper is the review of an estimation theory algorithm known as the Kalman filter.

Presented paper is mainly divided into three sections. In the first section, the development and modifications of the traditional Kalman filter algorithm are discussed. The second section gives some insight about various applications of the algorithms. The third and final section is the conclusion.

*Author for Correspondence

Resmi R.
E-mail: 23501@tkmce.ac.in

¹M.Tech Scholar, Department of Electrical and electronics Engineering, TKMCE, Kerala, India

²Assistant Professor, Department of Electrical and electronics Engineering, TKMCE, Kerala, India,

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THE BIRTH OF KALMAN FILTER

A New Approach to Linear Filtering

Background

The paper published in 1960 by R.E. Kalman [3] is considered a groundbreaking article since the Kalman filter (KF) algorithm was first introduced in this paper. KF is a recursive algorithm, which uses information about the previous state and noises present in the measurement of output to estimate unknown quantities. The algorithm, previously known as the optimal linear estimator, reduces the mean square error between the estimated state and the true state of the system. In the paper, Kalman formulates the problem based on linear dynamical system model with Gaussian noise.

Modification of the Conventional Approach

The Wiener filter, proposed by Wiener [4], predicts, separates, and detects random signal but not without certain shortcomings that include the practical difficulties in numerically determining the optimal impulse response of the filter. The most noticeable flaw is the obscurity of fundamental assumptions made for derivations. The new approach proposed by Kalman approaches the problem through conditional distributions and expectations which solves the lack of clarity of mathematical derivations. Another major difference between the new approach and the earlier methods is how the linear dynamic system is described. Kalman proposes to specify the systems by first order difference equations. A major assumption made in this formulation of this algorithm is that all probability density functions are Gaussian. As to what a probability density function is, it could be defined as a function representing the probability distribution of a random variable.

Inference

The major outcome of the study is the formulation as well as the solution of the Weiner problem by rectifying its drawbacks. The linear dynamic system is now represented using difference equations and the state prediction is carried out using conditional distribution from the previous estimate of state and process noise. The measurement prediction similarly takes into account measurement noise and observations.

New Algorithm for Nonlinear Systems

Background

After the introduction of the KF, which was originally intended for applications in linear systems, the KF was widely subject to studies by institutions such as the Dynamic Analysis Branch of NASA. Under the guidance of Dr. Stanley F. Schmidt, attempts were made at modifying the KF into a new form that is able to handle nonlinear systems, which the researchers at NASA were dealing with [5, 6]. In 1985, Leonard A. McGee and Stanley F. Schmidt started working on an algorithm that could solve the problem of nonlinearity effectively. The paper paved the way to a modified KF being used in nonlinear applications as well, which we know today as the extended Kalman filter (EKF), which is nothing but an extension of the traditional KF and hence the name.

Modifications of the Kalman Filter and Its First Application in Aerospace

The traditional filters, including the Wiener filter, were extensively used in navigation and guidance applications but with low precision. The KF was tested in simulated on-board optical measurements and course corrections for the manned lunar mission of Apollo, but this was challenged due to practical difficulties such as the requirement of continuous and equally spaced sequences of optical measurements made by the crew throughout the entire span of the mission. To solve this problem, the original algorithm formulation was decomposed into a discrete time update portion and a discrete time optical measurement update portion. This was based on the linearization of the reference trajectory. This proved to be successful, and the estimated state was closer to the real state values.

In the modern scenario, there have been many applications where the EKF algorithm is put to use. Methods like the Taylor series expansion, a special type of power series- the Maclaurin's series, is used

for linearization of nonlinear equations describing the system. In some cases, Jacobian matrix is used which is a simplified version of the Taylor series can also be used.

The Introduction of Unscented Transform for Nonlinear Systems

Background

The EKF algorithm was introduced in order to make the KF applicable to nonlinear systems that make up most of the systems encountered in engineering. This is done by linearizing the system using Jacobians. Jacobians are hard to calculate analytically and has higher computational complexity. The major drawback here is EKF is still heavily reliant on linear approximations. This results in inaccurate estimation of the covariance, especially since high linearity is not considered optimal for estimation, making it unreliable in some cases. In 2002, S.J. Julier and J.K. Uhlmann presented a new linear estimator, which has a better performance than the EKF [7].

The Unscented Transform

The unscented transform is introduced as a novel method used for calculations of a random variable that undergoes nonlinear transformation. A set of points, also known as sigma points are chosen and their mean and covariance are denoted using $\bar{x}$ and P_{xx} , respectively. Then the nonlinear function is applied on each sigma point so that the resultant transformed points have $\bar{y}$ and P_{yy} as their sample mean and covariance, respectively [7].

$$y_i = f[x_i] \quad (1)$$

$\bar{y}$ is given by the weighted average of the transformed points

$$\bar{y} = \sum_{i=0}^{2n} W_i y_i \quad (2)$$

P_{xx} is given by

$$\bar{y} = \sum_{i=0}^{2n} W_i \{y_i - \bar{y}\} \{y_i - \bar{y}\}^T \quad (3)$$

Which is the weighted outer product of the transformed points.

Here, n is the dimension of the random variable and W_i is the weight associated with the i th point. The filter algorithm has prediction and update steps which is similar to the traditional approach.

Results

The algorithm performs like the second order Gaussian filter but without the need to calculate the Jacobian. The results were found to be more accurate and reliable compared to linear approximation models. Unscented Kalman filter (UKF) is widely used in areas such as communication systems for channel estimation, equalization, etc. and battery management systems for estimation of parameters such as state of charge and state of health.

A Kalman Filter Based on Ensemble Method

Background

The ensemble Kalman filter (EnKF) was formulated in 1994 by Geir Evensen [8] and gained popularity instantly due to its lower computational burden and simple conceptual formulation. It is also easier to implement compared to its earlier counterparts. The error statistics are represented by an ensemble model of states. An ensemble in the context of the EnKF algorithm is a collection of state vectors. Each vector in the ensemble is a state estimate of the system at a particular instance of time.

Significance

The EnKF algorithm is basically a Monte Carlo method implemented on the Bayesian problem. Monte Carlo method is a widely used computation technique, or rather a class of techniques or

algorithms based on repeated random samplings for computational purpose. It makes use of the randomness to solve deterministic computational problems. The EnKF was developed to address two main challenges associated with using the EKF for nonlinear systems with large state spaces. The first challenge was the EKF's reliance on an approximate closure approach. The second challenge was the immense computational requirements of storing and propagating the error covariance matrix. The EnKF aims to provide a more efficient and accurate solution to these problems.

Cubature Kalman Filters

The previously discussed nonlinear filters, namely EKF and UKF, suffer from either dimensionality or divergence or both. Divergence is the decrease in estimation error covariance matrix while the actual error is larger than its theoretical value. Divergence has many negative effects on the estimation process including decrease in accuracy. Dimensionality can also have an adverse effect on the process in high dimensional state-space models. Divergence mostly occurs due to reasons such as high degree of nonlinearities in the state-space equations. In 2009, cubature integration method was introduced to propagate probability distribution through nonlinear system dynamics. It involves approximating the moments of the distribution using a set of cubature points. Cubature points are specific points in the state space of a dynamic system. To calculate the Gaussian weighted integral whose integrand is of the form *nonlinear function* $\times$ *Gaussian density* numerically, it is transformed into a spherical-radial integration form of third degree. The third-degree cubature rule gives efficiency and robustness to the computation [9]. On comparing this approach to EKF and UKF, the cubature Kalman filter (CKF) algorithm is more stable in state estimation with divergence and dimensionality eliminated.

Comparison of Mathematical Models

The general equations representing a KF are

$$x(k+1) = f[x(k), u(k), v(k), k] \quad (4)$$

$$z(k) = h[x(k), u(k), k] + w(k) \quad (5)$$

Error covariance is denoted by P_k and the Kalman gain by K .

Here $x(k)$ is the state of the system at time step k the input vector is given as $u(k)$ the state noise as $v(k)$, measurement noise as $w(k)$ and the observation or measurement vector as $z(k)$. It is to be noted that in nonlinear systems, these relationships could have nonlinear terms. For example, in the state-of-charge (SOC) estimation using EKF algorithm by Liu et al. [10], the RC (resistance capacitance) ladder voltage and SOC are the state vectors while the terminal voltage is taken as the measurement quantity, which is a function of the open circuit voltage. For this, the Jacobian is calculated and the h -matrix of the said system is thus linearized. Table 1 gives more insight about the actual mathematical steps of estimation for EKF, UKF, CKF, and EnKF [10].

From Table 1, W_m and W_c are weight vectors of mean and covariance respectively. P_{zz} and P_{xz} are innovation covariance matrix and cost covariance matrix, respectively. The former denotes the residual between the predicted observation and the actual observation while the latter represents the covariance of the difference between the true state value and the estimated state obtained. n_{en} is the number of samples. Q and R are the system noise variance and measurement noise variant, respectively.

APPLICATIONS

Kalman filter algorithms have been refined and revised according to the changing needs of each application. Few areas of applications are robotics, communication, power systems, electric vehicular technology and so on. Table 2 shows various applications.

In the next section, we discuss some of the applications in the above-mentioned fields of engineering.

Table 1. Mathematical equations of various Kalman filter (KF) operations..

Type of KF	Filter-Specific Methods/Equations	Prediction Step	Filtering Step
Extended KF	$H_k = \frac{dh}{dx} _{x_k^-}$ $F_k = \frac{df}{dx} _{x_{k-1}}$	$x_k^- = f(x_{k-1}, u_{k-1}), k \geq 1$ $P_k^- = F_{k-1} P_{k-1} F_{k-1}^T + Q$	$P_{zz} = H_k^T P_k^- H_k$ $P_{xz} = P_k^- H_k^T$ $K_k = P_{xz} (P_{zz} + R)^{-1}$ $\hat{x}_k = x_k^- K_k (z_k - h(x_k^-))$ $P_k = (I - K_k H_k) P_k^-$
Unscented KF	$n \cdot (2n + 1) \text{ -dimension Sigma points are obtained by}$ $S_{k-1} = x_{k-1} \cdot e^T + \eta$ $[0_{n,1} \cdot \sqrt{(P_{k-1})} - \sqrt{(P_{k-1})}]$	$S_k^- = f(S_{k-1}, u_{k-1})$ $x_k^- = \sum_{i=0}^{2n} W_m^i \cdot S_{i,k}^-$ $P_k^- = \sum_{i=0}^{2n} W_c^i (S_{i,k}^- - x_k^-)(S_{i,k}^- - x_k^-)^T$ $S_{xk}^- = x_k^- \cdot e^T + \eta$ $[0_{n,1} \cdot \sqrt{P_{k-1}} - \sqrt{P_{k-1}}]$ $S_{yk}^- = h(S_{xk}^-)$ $y_k^- = \sum_{i=0}^{2n} W_m^i \cdot S_{i,yk}^-$	$P_{zz} = \sum_{i=1}^{2n} W_c^i (S_{yk}^- - y_k^-) \cdot (S_{yk}^- - y_k^-)^T$ $P_{xz} = \sum_{i=0}^{2n} W_c^i (S_{i,k}^- - x_k^-) (S_{yk}^- - y_k^-)^T$ $K_k = P_{xz} (P_{zz} + R)^{-1}$ $\hat{x}_k = x_k^- + K_k (z_k - y_k^-)$ $P_k^+ = P_k^- + K_k (P_{zz} - R) K_k^T$
Cubature KF	$n \cdot 2n \text{ -order equal weight Cubature Points are obtained by}$ $C_{k-1} = x_{k-1} \cdot e^T + \sqrt{n} [\sqrt{(P_{k-1})} - \sqrt{(P_{k-1})}]$	$C_k^- = f(C_{k-1}, u_{k-1})$ $x_k^- = \frac{1}{2n} \sum_{i=1}^{2n} C_{i,k}^-$ $P_k^- = \sum_{i=1}^{2n} (C_{i,k}^- - x_k^-)(C_{i,k}^- - x_k^-)^T$	$C_{xk}^- = x_k^- \cdot e^T + \sqrt{n} [\sqrt{P_k^-} - \sqrt{P_k^-}]$ $C_{yk}^- = h(C_{xk}^-)$ $y_k^- = \frac{1}{2n} \sum_{i=1}^{2n} C_{i,k}^-$ $P_{zz} = \frac{1}{2n} \sum_{i=1}^{2n} (C_{yk}^- - y_k^-) (C_{yk}^- - y_k^-)^T$ $P_{xz} = \frac{1}{2n} \sum_{i=1}^{2n} (C_{i,k}^- - x_k^-) (C_{yk}^- - y_k^-)^T$
Ensemble KF	When $k = 1$, the sample set is given by $E_s = x_0 \cdot e^T + P_0 \cdot randn(2n, n_{en})$	$E_s^- = f(E_s, u_{k-1}) + Q \cdot randn(2n, n_{en})$ $z_k^- = h(E_s^-, u_{k-1})$	$P_{zz} = \frac{1}{n_{en}} \sum_{i=1}^{n_{en}} (z_k^{i-} - y_k^-) (z_k^{i-} - y_k^-)^T$

		$\bar{x}_k = \frac{1}{n_{en}} \sum_{i=1}^{n_{en}} E_s^{i-}$ $\bar{y}_k = \frac{1}{n_{en}} \sum_{i=1}^{n_{en}} z_k^{i-}$	$P_{xz} = \frac{1}{n_{en}} \sum_{i=1}^{n_{en}} (E_s^{i-} - \bar{x}_k) (z_k^{i-} - \bar{y}_k)^T$ $K_k = P_{xz} (P_{zz} + R)^{-1}$ $E_z = z_k + R \cdot randn(2n, n_{en})$
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Table 2. Applications of Kalman filters.

Field	Applications	Kalman Filter Algorithm
Robotics	Localization and mapping, sensor fusion, trajectory tracking	EKF, UKF
Electric vehicles	Battery management, estimation of state of charge, state of health, state of energy, etc.	EKF, UKF
Communication	Channel equalization, signal tracking, synchronization	KF, EKF, UKF
Power systems	Load forecasting, state estimation in power grids	EKF, UKF, CKF, EnKF

CKF, cubature Kalman filter; EKF, extended Kalman filter; EnKF, ensemble Kalman filter; UKF, unscented Kalman filter.

Dynamic Communication

Kalman-Consensus Filter

Howard and Qu [11] propose a novel filtering algorithm known as the Kalman-consensus filter (KCF) designed for distributed implementation over dynamic communication networks. The paper examines the issue of distributed state estimation in a network of connected sensors or agents. A consensus algorithm is a computational technique where a consensus or agreement among several nodes or agents on a common state value is achieved. The communication network has a set of nodes or agents, that are basically devices or entities that take part in computation and perform specific functions. Each agent has local measurements and communicates with neighboring agents over a changing communication network.

The goal is to create a filtering algorithm that allows each agent to estimate the overall state- of the system using only their own information and communication with nearby agents. In distributed state estimation, the KCF combines the- ideas of consensus algorithms and the KF. Through consensus iterations, each agent updates its local estimate of the system state by merging information from nearby agents using its own measurements. Despite network change-s and communication delays, the consensus step ensures that all agents converge to a common approximation of the global state. In the KCF, each agent operates autonomously based on local measurements and communication with neighbors, resulting in high computational efficiency and accuracy.

Robotics

Bioinspired Backstepping Control

In the study by Xu et al. [12], the scope of using the UKF algorithm in bioinspired backstepping of a robot is investigated. The study presents a control approach inspired by biological systems, known as bioinspired backstepping control. Backstepping control is a nonlinear control method that generates control laws recursively to stabilize dynamic systems. The bioinspired aspect enhances control performance by incorporating concepts from biological systems. The UKF for state estimation is incorporated into the bioinspired backstepping control architecture discussed in this paper. By combining sensor measurements and propagating sigma points through the nonlinear dynamics, the UKF nonlinear filtering technique provide-s accurate estimations of the robot's state. The posture of the robot in the inertial frame is described with respect to three parameters – the spatial position of the robot (x and y coordinates) and the angular orientation with respect to a fixed point. There is a desired posture which is to be obtained through the estimation and control of the parameters. The relation between the

velocities of the robot in the inertial and body-fixed frames is given by a Jacobian matrix. The paper implements KF algorithm in the inner loop consisting of the torque controller and robot dynamics as well as the UKF in the outer loop for bioinspired backstepping. Controllers for torque and backstepping are designed separately. The integration of KF and UKF provide accurate state estimates while testing the tracking through a straight line and a curved line.

Recursive Least Squares with Kalman Filter

The significance of precise system identification in robotic manipulators for control, trajectory planning, and dynamic modeling is discussed by De Souza et al. [13]. It draws attention to the shortcomings of conventional identification techniques and suggests a novel strategy based on recursive least squares (RLS) with KF. RLS is an algorithm used for parameter estimation in dynamic systems. It updates and refines parameter estimates based on observations recursively. It also maintains a memory of past data. In this paper, the main goal is to identify a manipulator joint activated by a three-phase induction motor using a hybrid approach of RLS and KF. The manipulator used in this case is a cylindrical robotic arm with 3 degrees of freedom. The first one is based on rotational movement, the second is linear and the third is horizontal. The RLS is usually used to compute a system model in real time when the system is operational. The integration of KF with RLS significantly improves the identification of the RLS. The hybrid model is then compared to RLS and extended recursive least squares (ERLS). The testing shows that the RLS-KF hybrid model is more accurate compared to ERLS and RLS models for the identification.

Electric Vehicles State-of-Charge Estimation Extended Kalman Filter

Lithium-ion batteries are well known for their positive traits like high power density, long life cycle and lesser environmental impact compared to other battery systems. This is the main reason why they are the preferred energy sources in electric vehicles. State of charge (SOC) is a very crucial parameter in determining the charging-discharging behavior and overall health of the battery. At present, there are no available technologies to directly measure SOC and therefore estimation is used for obtaining SOC with the help of previous observations.

Since Li-ion battery is a nonlinear system due to its nonlinear current-voltage characteristics, a normal KF cannot be implemented. Therefore, EKF is a better option [13]. The output voltage is typically nonlinear in nature and is a function of SOC. This relation is different from battery to battery but is assumed to stay the same throughout the battery's life cycle. The SOC estimate and the RC ladder voltage are taken as the state variables while terminal voltage is the observation vector. A first order Thevenin equivalent circuit is considered, and the nonlinearity is linearized using Jacobian. In the study, the change of battery parameters with respect to the change in SOC is considered unlike some previous models. These adaptive parameters were then given to the EKF algorithm, and from the simulation results, it can be observed that the adaptive battery parameter method increases SOC estimation accuracy of the EKF.

Adaptive Extended Kalman Filter

When using KF, the major drawback is divergence. Divergence can be defined as the decrease in process error covariance matrix while the actual error is larger than its theoretical value. Divergence has many negative effects on the estimation process including decrease in accuracy. This is where the adaptive EKF proves to be helpful [14]. It judges whether the target dynamic has altered during the recursion based on the measurement vector. If true, the adaptive filter decides if it should consider this change as a random noise before classifying it under model noise and adapt the target dynamic accordingly. It also helps in improving accuracy while process noise covariance and measurement noise covariance are unknown.

The battery model used here is an improved version of the Thevenin equivalent circuit model, with two RC ladders instead of one. The voltage across these ladders and the SOC are the state variables

while the terminal voltage of the battery is the measurement. The improved Thevenin model gives better accuracy of estimation through a longer period of time and the accuracy and robustness increased with the adaptive EKF algorithm.

Adaptive Cubature Kalman Filter

SOC estimation can be carried out using a second order Thevenin equivalent model [15] with two RC ladders as well as internal resistance, which delivers one of the best estimation accuracies over a long time period. The estimator used here, an adaptive cubature Kalman filter (ACKF), is different from other conventional estimators. A set of cubature points is used by the CKF, a KF variation, to approximate the continuous Gaussian probability distribution. Comparing these cubature points to more conventional linearization techniques like the EKF, the distribution's moments are captured with more accuracy. To estimate the system state and covariance, the CKF propagates these cubature points through the nonlinear system dynamics.

In a standard CKF, process noise covariance and measurement noise covariance are assumed to be constant. However, this is not the case with a real-life battery system due to various types of random noises. Therefore, the rules for process and measurement covariance values need to be adaptively updated for increased estimation efficiency. The parameter identification is carried out using forgetting factor least-square algorithm, where recent values or observations are given more weight. The nonlinear relationship between SOC and open circuit voltage is defined through curve fitting. After this, the ACKF algorithm is implemented by updating the process and measurement covariance matrices iteratively. On comparison with the widely used EKF algorithm and the standard CKF, more accurate estimates were obtained.

Power Systems

Comparisons on Kalman-Filter-Based Power System Estimation Methods

KF algorithms are widely used for power system applications, specifically for power system state estimation, since it is a key step in energy management. The dynamic state estimation of power systems has been a topic of discussion in recent times. Majorly used in power system applications, nonlinear KF algorithms, namely the EKF, UKF, CKF, and EnKF are compared to find the best candidate for applications demanding high accuracy and stability [10]. This particular study aims at finding the best algorithm in the context of power systems.

This study is conducted to estimate the state of the power system with power angle and speed of generates as the measurements and mechanical power along with the input voltage are given as the constant control input. As discussed, the EKF is commonly used and is based on the Taylor series expansion to linearize the nonlinearity. The algorithm, previously tested with data from synchronized phasor measurement unit has already been studied [16, 17]. A distributed EKF was also implemented based on wide area measurement system [18]. A major challenge faced during these implementations is the EKF discarding higher order terms of the Taylor series expansion which leads to large truncation errors and lower efficiency. This makes EKF slightly less preferred for this application, since other algorithms mentioned earlier approximate the posterior probability distribution of random variables instead of approximating nonlinear function. Posterior probability can be defined as the revised probability of an event on the basis of prior knowledge and observations made. This updation is calculated using Bayes theorem.

UKF generates weighted sigma points and eliminates the need for calculation of Jacobian. As already discussed in this paper, this owes to its higher approximation accuracy. CKF uses the spherical radial rule to generate set of equal weight cubature points. The major difference between UKF and CKF is that in UKF, the weight of the sample center point is greater than that of other points while in CKF, since there is no difference in weights, the sampling strategy can be assumed rather conservative.

On the other hand, the EnKF algorithm, unlike the deterministic sampling approach followed by UKF and CKF, generates a large number of sample set based on prior knowledge. This leads to a greater approximation accuracy. However, this increases the calculation complexity significantly.

In the final part of the paper, the four algorithms are compared using parameters such as root-mean-square error, precision and computation efficiency. The results of simulations show that the EKF algorithm shows a higher computation efficiency but fails to deliver decent performance when applied in large-scale power systems due to the complicated process of Jacobian matrix calculation. A similar behavior is observed in UKF although it is comparatively insensitive to measurement errors. The EnKF can be used in applications of low estimation accuracy. The CKF algorithm fairs better in all accounts with its high estimation accuracy and robustness.

CONCLUSION

The review on the history of KFs and its associated range of applications showcases the ever-evolving nature of this recursive algorithm and its usefulness in engineering. Its application ranges from state estimation and trajectory correction in aerospace and missile to accurately estimation of useful parameters of a battery. Research conducted over the years have been helpful in improving the efficiency and robustness of KF algorithm. Each of the algorithms starting from the traditional approach has its own advantages and drawbacks and each successive addition of a newer and better algorithm improves the reliability and accuracy as well as expands the range of application. Future developments such as integration of deep learning algorithms can further improve its scope and effectiveness.

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