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Review TEE

Animated Transient Response of High-voltage Power Line Towers Equipped with Nonlinear Footing Resistances

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Abstract

This study focuses, among other objectives, on the use of animation techniques for visualizing the electromagnetic transients in high-voltage power line towers. The model includes parameters such as the tower's footing resistance and its location-dependent surge impedance. It describes the governing algebraic/partial differential equations expressed in terms of the current and voltage distributions as functions of time and location. The transients are assumed initiated by a lightning discharge of a given time waveform, which is mostly of a double-exponential shape that exhibits a distinct peak value as well as a wavefront and a wave-tail of durations in the microsecond range. The waveform can be described in terms of two-time constants. The computer code is written in the Mathematica programming language (Versions 11 and 12). It solves the above equations numerically using one of the known computer codes for the numerical inversion of the Laplace transform. A listing of this code is given in the Appendix of this study. Moreover, this study introduces the use of an animation technique as an additional vehicle for investigating the tower transients. It augments previously published single- and three-dimensional as well as the pertinent contour plots available in the literature. The presented model and computer code can handle both linear and nonlinear tower footing resistances. The obtained results indicate that by using animations alongside other visualization tools, a deeper understanding of complex systems like high-voltage power line towers can be gained. It is believed that the investigation can be of great help to the researchers and practicing electrical power engineers and trainees specialized in power system protection and high-voltage technology.

Keywords: High-voltage line towers, overhead, lightning, surges, grounding, earthing, electrode, transients, electromagnetic, numerical solutions, Mathematica, inverse Laplace transform, animation, voltage and current distributions, footing resistance, nonlinear, Temporal and spatial distributions

INTRODUCTION

The analysis of electromagnetic transients along the towers of high-voltage and extra-high-voltage overhead power transmission lines has attracted the attention of many researchers [1–12]. In particular, the determination of the transient current and voltage stresses resulting from lightning discharges has been the topic of numerous studies, as manifested in sample investigations [7–12]. In terms of circuit analysis, power line towers can be considered lossless non-uniform transmission lines with location-dependent inductance and capacitance distributions. In most cases, they are modeled in either the time or frequency domains. The latter includes the Laplace or *s*-domain approach, which is based on solving the governing simultaneous equations describing the considered equivalent circuit or system. This procedure can handle cases involving nonlinear phenomena, such as the

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presence of iron-cored coils, power electronics devices, and corona discharges [12]. Frequency- or Laplace *s*-domain solutions are generally possible only in linear circuits and systems. This imposes a serious limitation on the practical applicability. The solutions were based on numerically inverting the *s*-domain equations and expressions simulating the tower. Many efficient algorithms, such as the well-known Hosono algorithm, the Graver-Stehfest, Fourier-Euler, and Talbot methods, are available for this purpose [7]. The available technical literature includes only the transient analysis of line towers with electrodes that have linear earthing resistances. To the best of the author's knowledge, no information is available on the analysis of power line towers with possible nonlinear earthing electrodes. This study intends to serve as an introductory contribution to this topic. Moreover, the advantages of using animation techniques for a detailed display of the results are explored.

THE METHOD OF ANALYSIS

The study also includes a comparison between the transient current and voltage distributions along the tower in cases comprising both usual linear or nonlinear earthing electrodes.

As shown in Figure 1, the top of the high-voltage tower of height *h* and the footing resistance *Rg* is hit by a direct lightning strike, which is described by the indicated time-dependent ideal current source i_{source} *(t)* and the lightning shunt internal resistance R_{source} in the form of Norton's equivalent circuit. The source current of the peak value of 30 kA is assumed to be a double-exponential time waveform:

$$isource\ (t) = 30.3097[e^{-t/T1} - e^{-t/T1}] \quad (1)$$

Where *e* denotes the base of the natural logarithms and the two-time constants T_1 and T_2 are assumed 17.63 and 0.316 µs, respectively. If multiplied by the equivalent surge impedance of the lightning strike (assumed to have a typical value of 250 Ω), the plot depicted in Figure 2 is obtained for the source voltage $e(t)_{source}$ over the time range $0 \le t \le 0.5$µs, respectively. Its peak value was approximately 7.5 MV.

In terms of its equivalent circuit parameters, the series inductance *l (x)* and shunt capacitance *c(x)* per meter can be easily derived from the assumed dependence of the surge impedance *zo(x)* on the coordinate *x*. This follows from the following two relations:

$$Zo(x) = \sqrt{[l(x)/c(x)]} \quad (2)$$

and

$$\text{the speed of light}^2 = 1/[l(x).c(x)] \quad (3)$$

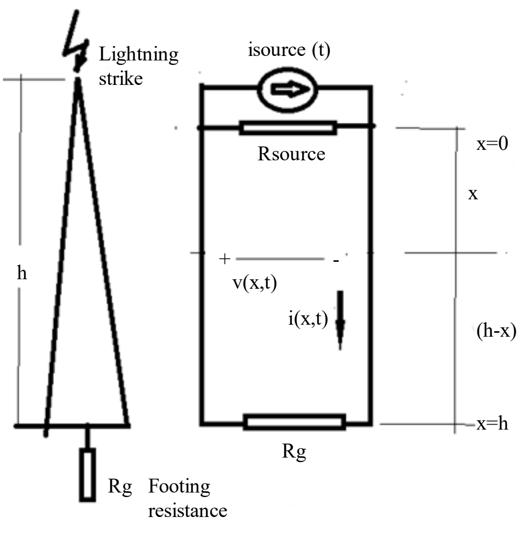


Figure 1. Schematic diagram of the considered power line tower and its equivalent circuit [7–10].

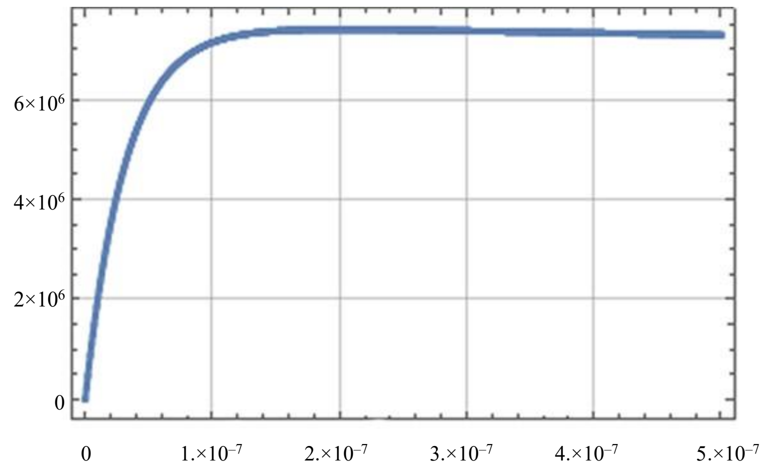


Figure 2. The time waveform of the lighting strike's equivalent source voltage $e(t)_{source}$ versus the time *t* over the time range between 0.5 and 5μs.

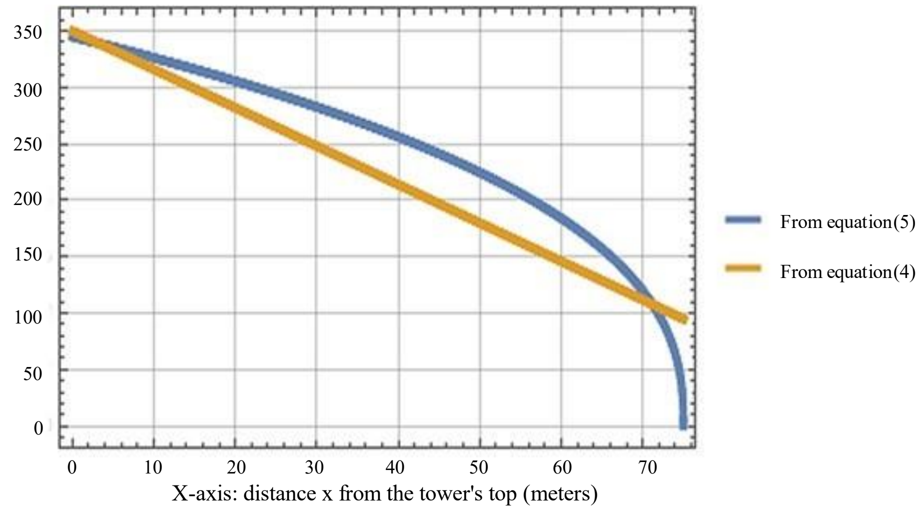


Figure 3. Assumed dependencies of the tower's surge impedance *zo*, in Ohms, on the coordinate *x i*n meters, measured from the tower's top.

The plot in Figure 3 depicts the results corresponding to the sample tower discussed in references [7–10].

The following equation for zo*(x)* in terms of coordinate *x* will be assumed:

$$zo(x) = zomax - \left[\frac{(zomax - zomin)}{height}\right] x \tag{4}$$

where the values *height*=75m, *zomax*= 350 Ω, and *zomin*=95 Ω are assumed. This approximates the original equation

$$zo(x)=50+35\sqrt{(height-x)} \tag{5}$$

Similarly, zo$(x) = 64.04\times(height-x)^{0.39}$, which are the results of simple regression analyses.

The current and voltage are functions of both location *x* and time *t and* are denoted *i (x,t)* and *v(x,t)*, respectively. These are governed by two simultaneous partial differential equations.

$$\partial v[x,t] / \partial x = -l(x).\partial i[x,t] / \partial t \tag{6}$$

$$\partial i[x,t] / \partial x = -c(x).\partial v[x,t] / \partial t \tag{7}$$

Moreover, the two following equations express the zero initial conditions in the case of an initialization transient following a lightning strike:

$i[x,0]=0$ (8a)

and

$v[x,0]=0$ (8b)

and based on Kirchhoff's circuit laws, the boundary conditions are expressed as

$v[h,t]=Rg.i[h,t]$ at the tower footing (9a)

and

$v[0,t]=2.esource(t)-rsource\times i[0,t]$ at the tower top. (9b)

Factor 2 in the last equation is derived from the definition of Thevenin's equivalent circuit at the tower top *(x=0)*. It considers the full reflection at the top under open-circuit conditions.

Reference [7] indicates the availability of an exact solution in terms of the Bessel functions for this problem only after performing a regression analysis on the expression of Equation (4). The objective is to obtain a simpler expression for the surge impedance *zo(x)* in the exponential form:

$zo(x)=zo(0).Exp(k.x)$ (10)

where, *zo(0)* is the surge impedance at the tower top (i.e., *x*=0) and *k* is a constant. Analytical Laplace *s*-domain expressions for current and voltage distributions can then be derived. This is followed by a numerical inversion procedure that yields the required solution in the time domain.

For more details on the derivation of the governing differential and algebraic equations [7, 10]. The achieved equations can be solved numerically through the use of the computer software *Mathematica* (Versions 11 and 12). Reference [13] is recommended for further detail. It also includes commands that animate the current and voltage distributions.

The resulting plots illustrate both the current *i* and voltage *v* at different locations along the tower height (i.e., between *x*=0 at the top and *x=height* at the ground level) as functions of time during the first 0.5µs.

SAMPLE RESULTS

The plots of the following three items are included in this section:

1. Animation plots for both the current $i(t)$ and voltage $v(t)$
2. Animation plots for the current $i(t)$.
3. Animation plots for the voltage $v(t)$.

As an example, the animation plot in Figure 4 depicts the conditions pertinent to case (a). It deals with an infinitely high footing resistance and cubic nonlinear voltage/current characteristic (i.e., *m*=3). It shows the current $i(t)$ and voltage $v(t)$ for an internal lightning resistance of 250 Ω. It is further assumed that the equivalent line is "hypothetically" open-circuited at its middle point (*x*=37.5m).

The horizontal axis depicts the coordinates along the tower in meters measured from its top point. During program execution, both curves propagated to the left at the speed of light. Before the lightning discharge, all tower points with $x \geq 37.5$ m remain unchanged. The four switches at the top of each animation plot are used for stopping, pausing, accelerating, and decelerating the traces of each plot.

Figures 5 and 6 show screenshots of the animated output for three linear cases (m=1) and one nonlinear case (m=3), respectively. These Figures 4 and 5 help to visualize how the current and voltage change under different circuit conditions, providing valuable insights into the transient response of line towers during lightning discharges.

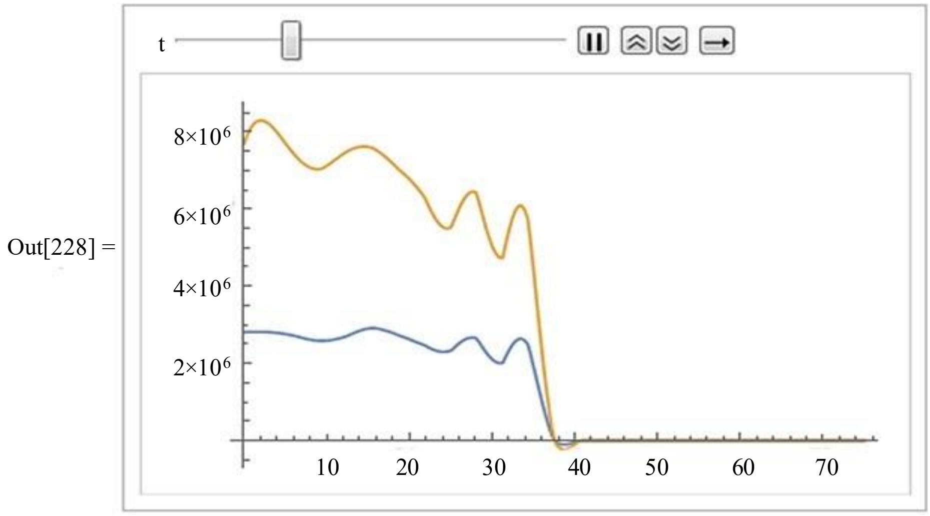
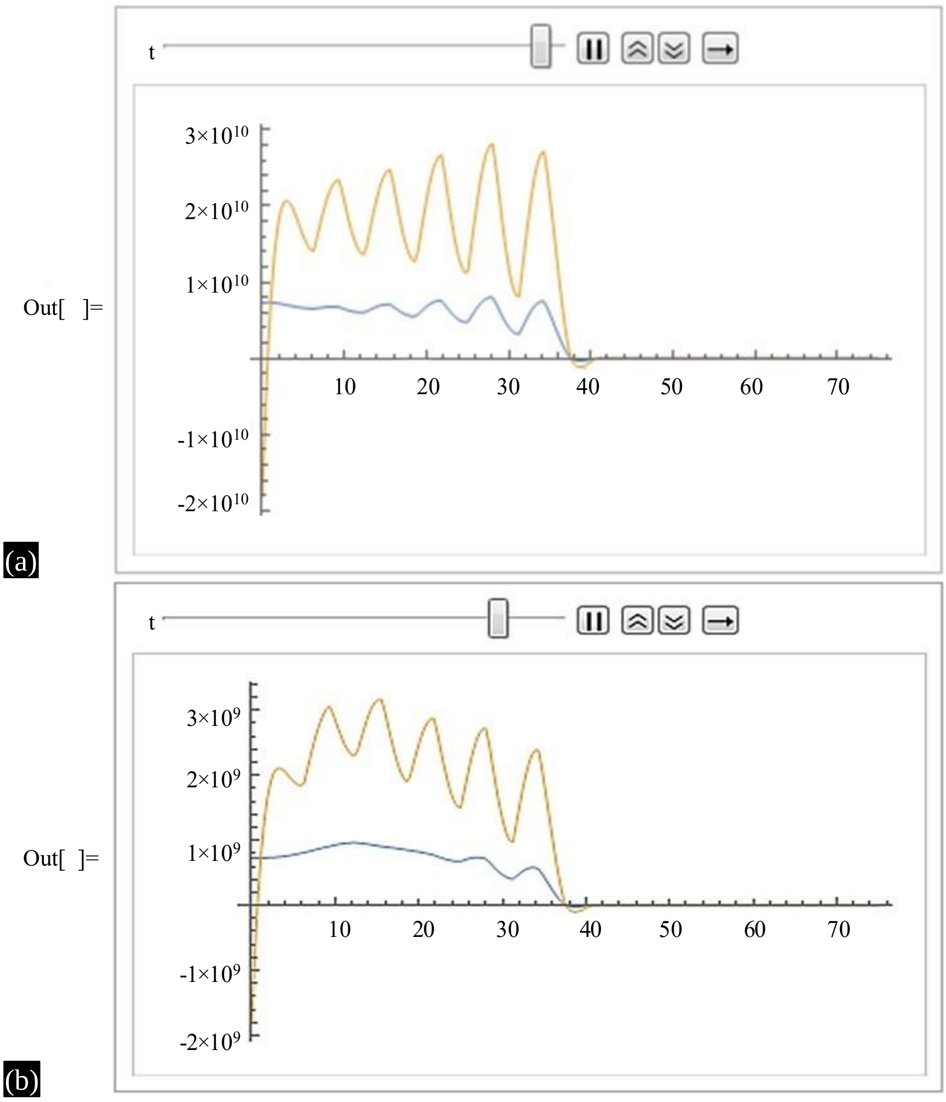
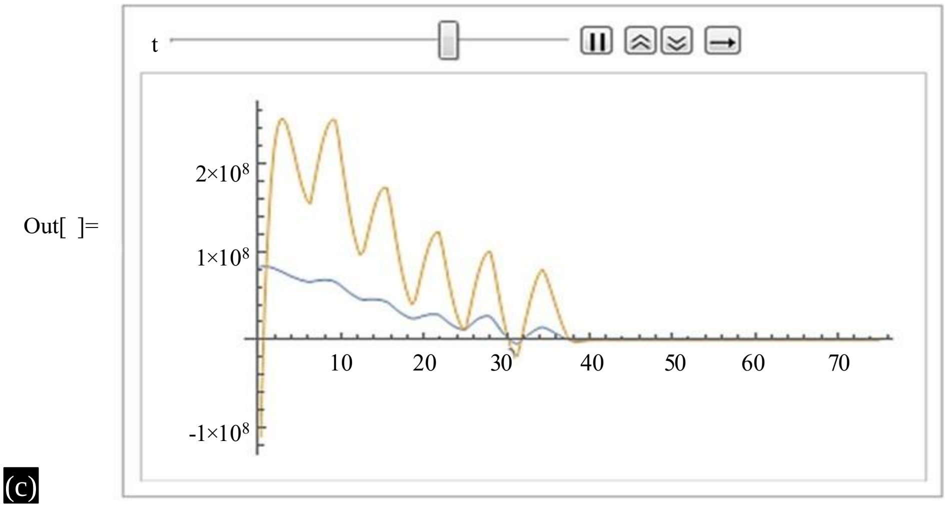


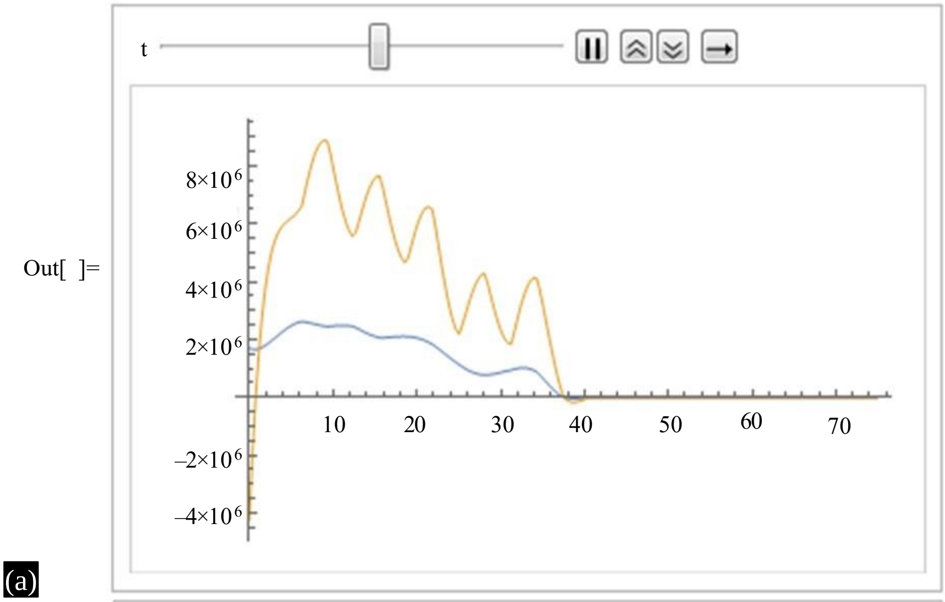
Figure 4. A screenshot of the animation plot depicting the current $i(t)$ and voltage $v(t)$ for the case involving infinitely large footing resistance but still with a cubic nonlinear voltage/current characteristic of m=3.
Note: Lower Curve: Current; Upper Curve: Voltage



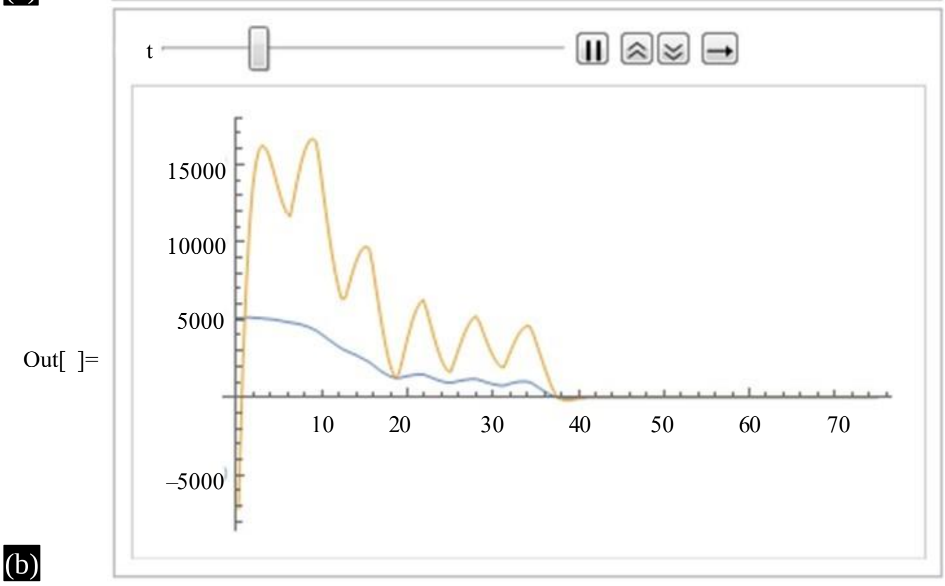


(c)

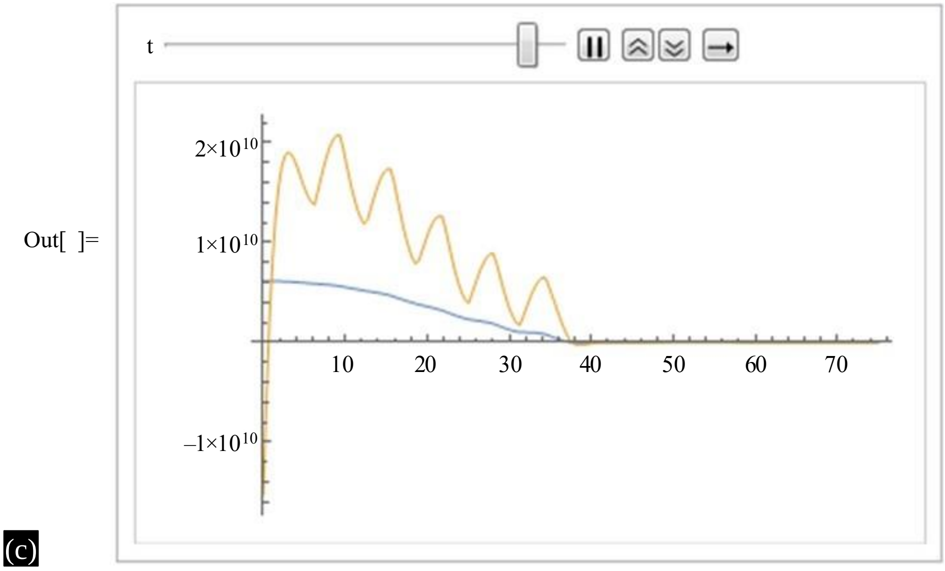
Figure 5. Screenshots of the animation output for 3 linear cases; (a) *Rg*=0, *m*=1, (b) *Rg*=0, *m*=1, *Rg*=10, *m*=1, (c) *Rg*=0, *m*=1, *Rg*=Infinity, *m*=1.



(a)



(b)



(c)

Figure 6. Screenshots of the animation output for nonlinear cases (*m*=3); (a) *Rg*=0, *m*=3, (b) *Rg*=10, *m*=3, (c) *Rg*=Infinity, *m*=3.

These animation plots, along with the given mathematical model and equations, as well as the program listed below, provide comprehensive tools for analyzing and understanding electromagnetic transients in high-voltage line towers. They can be of great help to researchers, practicing electrical power engineers, and trainees specializing in electric power system protection and high-voltage technologies.

CONCLUSION

The *Mathematica* computer program proved to be potentially helpful for detailed visualization of electromagnetic transients in the towers of high-voltage transmission lines. They considered the tower data and the parameters of the surge voltage initiating the transients. The results of applying the presented mathematical model and computer code can be displayed in both two-dimensional and three-dimensional, as well as contour plots. They illustrated time- and location-dependent current and voltage stresses. The model can handle the effect of the tower footing resistance and the eventual non-linearity of the electrode's current-voltage characteristic. The waveforms of the current and voltage transients were sensitive to and controllable by this nonlinearity. The study concludes by presenting several animating plots of the results. For convenience, the printout of the computer code is included in the Appendix.

REFERENCES

1. Bewley LV. Travelling Waves on Transmission Systems. New York: Dover; 1963.
2. Sargent MA, Darveniza M. Tower surge impedance. IEEE Trans Power App Syst. 1969 Jun;PAS–88(6):680–7. DOI: 10.1109/TPAS.1969.292357.
3. Menemenlis C, Chun ZT. Wave propagation on nonuniform lines. IEEE Trans Power App Syst. 1982;PAS–101:833–9. DOI: 10.1109/TPAS.1982.317148.
4. Chisholm WA, Chow YL, Srivastava KD. Lightning surge response of transmission towers. IEEE Trans Power App Syst. 1983 Sep;9:3232–42. DOI: 10.1109/TPAS.1983.318134.
5. Hermann S, Dommel W. Electromagnetic transients program reference manual. Portland, USA: Bonneville Power Administration; 1986. p. 2–14 to 2-20.
6. Ametani A. Electromagnetic transients program: History and future. IEEJ Trans Electr Electron Eng. 2021;16(6):1150–8. DOI: 10.1002/tee.23192.
7. Saied MM, Alfuhaid AS, El-Shandwily ME. s-Domain analysis of electromagnetic transients on nonuniform lines. IEEE Trans Power Deliv. 1990 Nov;5(4):2072–83. DOI: 10.1109/61.103703.
8. Saied MM. Transient response of power networks to pulse inputs of non-standard wave forms: Analysis and case studies. Int J Analog Integr Circuits. 2022;8(1):25–32.
9. Saied M, Safar Y, Salama M. Line transients with corona. J Univ Kuwait (Sci). 1987 Jan;14(1):77–96.

10. Saied M. A new Mathematica-based approach to the time-domain transient analysis of power line towers. J Elect Power Qual Util. 2020.
11. Stracqualursi E, Pelliccione G, Celozzi S, Araneo R. Tower models for power systems transients: A review. Energies. 2022;15:4893. DOI: 10.3390/en15134893.
12. Saied M. Electromagnetic transients on power lines due to multi-pulse lightning surges. Paper 33-101, CIGRE Meeting, Session 2002, Paris.
13. Wolfram Mathematica. Available from: https://www.wolfram.com/mathematica/new-in-12/. Accessed March 2020.

APPENDIX

The *Mathematica* Program

The two versions 11 and 12.1 have been used.

```
tend=2×10^-6; (×The Time Range×)
u[x_]:=UnitStep[x]//N;
 velocity=3×10^8;
height=75;
rarc=250; (×Internal Resistance of the Lighting Strike×)
m=3; (×Indix of Nonlinearity×)
zomax=350;
zomin=95;
K=10; (× The Earting Resistance×)
rtower=0;
(********The Governing Equations*)
esource[t_]:= 1×rarc× (Exp[-t×10^(6/17.68)]-Exp[-t×10^(6/0.0316)])//N;
(×zo1[x_]:=0.00001+64.04× (height-x)^(0.39//N; ×)
zo1[x_]:=0.00001+64.04×(height-x)^(0.39//N);
zo2[x_]:=zomax-((zomax-zomin)/height)×x//N;
zoav[x_]:=(zo1[x]+zo2[x])/2//N;
ind[x_]:=(zoav[x]/velocity)+10^5 ×u[x-height/2]//N;
cap[x_]:=(zoav[x]/velocity)/(zo2[x]^2)+10^5 ×u[x-height/2]//N;
op={D[va[x,t],x]==-ind[x] ×D[ia[x,t],t]-rtower×ia[x,t],
D[ia[x,t],x]==-cap[x] ×D[va[x,t],t] };
ics={va[x,0]==0,ia[x,0]==0};
bc={va[0,t]== 2×esource[t]-rarc×ia[0,t],
va[height,t]==K× (ia[height,t])^m};
(×The Solution×)
sol1=NDSolve[{op,ics,bc},{va,ia},{x,0, height},{t,0, tend},Method-> "Equation
Simplification"->"Residual", Accuracy Goal->1,PrecisionGoal->1, Max Steps->Infinity]
(×The Animation×)
Animate[Plot[{100×ia[x,t]/.sol1,va[x,t]/.sol1},{x,0,75}],{t,0,tend},AnimationRunning->True]
Animate[Plot[{0,100×ia[x,t]/.sol1},{x,0,75}],{t,0,tend},AnimationRunning->True]
Animate[Plot[{0,va[x,t]/.sol1},{x,0,75}],{t,0,tend},AnimationRunning->True]
```

