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Physics-Adaptive Digital Twin with Neural-Operator Reduced-Order Modelling

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Abstract

This study proposes a novel Physics-Adaptive Digital Twin with Neural-Operator Reduced-Order Modelling (PADT-NO) framework for predictive modelling of complex, nonlinear, and multiscale fluid flows. The proposed mathematical framework integrates fundamental conservation laws, Navier–Stokes dynamics, physics-constrained neural operators, adaptive reduced-order modelling, and uncertainty-aware state estimation within a unified computational architecture. Unlike conventional computational fluid dynamics and purely data-driven approaches, the proposed model dynamically couples high-fidelity physical information with a low-dimensional latent representation while preserving essential mass, momentum, and energy constraints. An adaptive mode-selection mechanism automatically modifies the reduced-order model complexity according to changes in flow regimes, enabling efficient representation of transient and strongly nonlinear flow structures. A physics-constrained learning objective simultaneously minimizes observational error, governing-equation residuals, boundary-condition violations, conservation errors, stability deviations, and uncertainty-calibration errors. The digital twin further incorporates parameter and model uncertainties to generate probabilistic flow predictions rather than deterministic estimates alone. The proposed framework is designed for rapid prediction, real-time monitoring, parameter estimation, and many-query fluid simulations while reducing computational dependence on expensive high-fidelity solvers. The resulting formulation provides a scalable mathematical pathway for next-generation intelligent fluid-mechanics modelling, particularly for turbulent, transient, and multiphysics flow systems. The system lowers the computing costs of high-fidelity solvers while supporting many-query fluid simulations, parameter estimation, real-time monitoring, and quick prediction. It offers a scalable approach to intelligent fluid-mechanics modelling with enhanced computational efficiency and predictive flexibility, especially for turbulent, transient, and multiphysics flow systems.

Keywords: Physics-informed modelling; digital twin; neural operators; reduced-order modelling; fluid dynamics

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INTRODUCTION

Fluid mechanics is undergoing a rapid transition from conventional analytical and computational approaches toward intelligent, data-assisted, and multiscale modelling frameworks capable of describing increasingly complex flow phenomena. Classical Navier–Stokes-based computational fluid dynamics (CFD) remains fundamental for predicting velocity, pressure, turbulence, and transport behavior, but its computational cost becomes substantial for transient, nonlinear, high-Reynolds-number, multiphase, and multiphysics systems. Recent advances in physics-informed machine learning, neural operators, reduced-order modelling, and digital-twin technologies provide promising opportunities to overcome these limitations while retaining physical consistency. However, purely data-driven models may violate conservation laws, require extensive training data, and exhibit limited generalization under unseen flow conditions. Similarly, conventional reduced-order models can lose accuracy when the flow regimes change significantly. To address these challenges, this study develops a physics-adaptive digital twin with neural-operator reduced-order Modelling (PADT-NO) framework that integrates governing fluid equations, adaptive latent-state dynamics, neural-operator learning, physical constraints, and uncertainty propagation. The proposed approach establishes a continuously updated virtual representation of the fluid system, enabling efficient prediction of complex flow fields while dynamically adapting model complexity to changing physical conditions. The framework is intended to provide a robust mathematical foundation for accurate, computationally efficient, and uncertainty-aware next-generation fluid-mechanics simulations.

Recent developments in fluid mechanics have shown a clear shift from conventional high-fidelity computational fluid dynamics toward hybrid frameworks that integrate governing physical equations with machine learning, reduced-order modelling, and digital-twin technologies. Physics-informed neural networks have been investigated for turbulence closure and flow inference around a cylinder using sparse experimental data, demonstrating the potential of physics-constrained learning for data-efficient turbulence modelling [1]. Reduced-order modelling has also been identified as an important computational strategy for engineering digital twins, particularly for lowering simulation costs while retaining essential system dynamics [2]. Neural operators have emerged as an approach for accelerating scientific simulations and engineering design, with the ability to learn mappings between function spaces [3]. A physics-informed multi-grid neural operator was developed for porous-flow simulation, demonstrating the potential of combining multiscale numerical structures with physics-informed learning [4]. A data-efficient framework for non-intrusive reduced-order modelling of digital twins was proposed, showing how reduced models can support efficient virtual-system representations [5]. Deep learning has also been applied to real-time inverse problems, data assimilation, and uncertainty quantification, highlighting the importance of uncertainty-aware learning in digital-twin applications [6].

Physics-informed neural networks have been investigated for turbulent flows using sparse data, demonstrating their usefulness when complete flow-field observations are unavailable [7]. The integration of physics-informed neural networks with the lattice Boltzmann method has been explored for inverse problems in fluid mechanics, illustrating the potential of combining machine learning with established numerical fluid solvers [8]. Physics-informed neural networks have also been applied to fluid-flow learning while incorporating flow symmetry, demonstrating that physical structural information can improve learning efficiency and prediction quality [9]. The integration of machine learning with computational fluid dynamics has been examined, with potential benefits identified in modelling efficiency and computational performance [10]. Machine-learning-based energy-preserving nonlocal closures have been developed for turbulent fluid flows, addressing the importance of maintaining physically meaningful energetic behavior in learned turbulence models [11].

Progressive and extrapolative machine learning approaches have been investigated for near-wall turbulence modelling, emphasizing the importance of generalization when learned models encounter flow conditions beyond their training regime [12]. Machine-learning methods for turbulence and heat-flux model development have also been reviewed, with challenges associated with the diverse physical mechanisms in complex flows being highlighted [13]. Machine-learning closure models have been developed to reproduce the temporal evolution of coarse-grained two-dimensional turbulence, demonstrating the usefulness of learning unresolved dynamics [14]. DeepONet was introduced for nonlinear operator learning, providing a theoretical foundation for learning mappings between infinite-dimensional function spaces [15]. The Fourier Neural Operator was developed for parametric partial differential equations, establishing an efficient framework for learning solution operators across varying physical parameters [16]. DeepXDE was introduced as a deep-learning library for differential equations, providing computational tools for physics-informed scientific machine learning [17].

Machine learning has continued to expand its applications in fluid mechanics, including flow estimation, reduced-order modelling, control, and system identification [18]. Hidden fluid mechanics demonstrated the capability of learning velocity and pressure fields from flow visualizations, establishing a physics-informed approach for extracting flow information from limited observations [19]. A broad review of machine learning for fluid mechanics identified opportunities in data-driven discovery, reduced-order modelling, prediction, and flow control [20].

Collectively, these studies demonstrate substantial progress in physics-informed learning, neural operators, turbulence closure, reduced-order modelling, uncertainty quantification, and digital-twin development. However, most existing approaches emphasize individual methodological components rather than integrating physical constraints, neural-operator learning, dynamically adaptive reduced-order states, online digital-twin synchronization, and uncertainty propagation within a unified fluid-mechanics architecture. This gap motivates the proposed PADT-NO framework, which combines these complementary capabilities to provide efficient, physically consistent, adaptive, and uncertainty-aware prediction of complex and multiscale fluid flows.

MATHEMATICAL MODELLING AND PROPOSED PADT-NO FRAMEWORK

The proposed physics-adaptive digital twin with neural-operator reduced-order modelling (PADT-NO) is formulated as a coupled mathematical framework in which the physical fluid system, digital representation, adaptive reduced-order dynamics, neural-operator approximation, and uncertainty model interact continuously. The physical flow is represented by conservation equations, whereas the digital twin reconstructs the evolving flow state from observations and model predictions. The framework is designed to retain physical consistency while reducing the computational burden associated with repeated high-fidelity CFD simulations [11].

Physical Fluid-Dynamic Formulation

The instantaneous fluid state is defined by the conservative variable vector

$$q(x,t) = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \end{bmatrix}, \tag{1}$$

where ρ denotes fluid density, u, v, w are the velocity components, and E represents the total specific energy. The general conservation law governing the fluid system is expressed as

$$\frac{\partial q}{\partial t} + \nabla \cdot F_c(q) = \nabla \cdot F_v(q, \nabla q) + S, \tag{2}$$

where F_c and F_v represent convective and viscous fluxes, respectively, and S S denotes external source terms.

For incompressible Newtonian flow, the continuity equation becomes

$$\nabla \cdot u = 0, \tag{3}$$

where $u = [u, v, w]^T$ is the velocity vector. The momentum balance is given by

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\frac{1}{\rho}\nabla p + \nu \nabla^2 u + f, \tag{4}$$

where p is pressure, ν is kinematic viscosity, and f represents body-force acceleration.
The Reynolds number characterizing the relative importance of inertial and viscous effects is

$$Re = \frac{UL}{\nu}, \tag{5}$$

where U and L denote characteristic velocity and length scales. The nondimensional momentum equation is consequently written as

$$\frac{\partial u^*}{\partial t^*} + (u^* \cdot \nabla^*)u^* = -\nabla^* p^* + \frac{1}{Re}\nabla^{*2}u^* + f^*. \tag{6}$$

For thermal or scalar transport, an additional field ϕ can be represented through

$$\frac{\partial \phi}{\partial t} + u \cdot \nabla\phi = D_\phi \nabla^2 \phi + S_\phi, \tag{7}$$

where D_ϕ is the scalar diffusivity and S_ϕ is the corresponding source term.

The associated Péclet number is

$$Pe = \frac{UL}{D_\phi}, \tag{8}$$

which quantifies the relative influence of advective and diffusive transport.

Digital-Twin State Representation

The physical state is transformed into a digital-twin representation through

$$z(t) = \mathscr{E}_\theta[q, (x, t), \mu\xi(t)], \tag{9}$$

where $\mathscr{E}_\theta$ denotes a trainable encoder, μ represents physical parameters, and $\xi(t)$ contains time-dependent operating conditions.

The digital state is reconstructed through

$$\hat{q}(x, t) = \mathcal{D}_\theta[z, (t), x\mu], \tag{10}$$

where $\mathcal{D}_\theta$ is the corresponding decoder. The digital-twin synchronization error is defined as

$$e_{DT}(t) = q_{obs}(t) - \hat{q}(t), \tag{11}$$

where q_{obs} denotes observed or high-fidelity reference data.

A weighted observation loss is formulated as

$$\mathscr{L}_{data} = \frac{1}{N}\sum_{i=1}^{N} \left\| W_q \left(q_{obs}^{(i)} - \hat{q}^{(i)} \right) \right\|_2^2, \tag{12}$$

where W_q is a normalization or importance-weighting matrix.

Physics-Constrained Neural Operator

The unknown flow-field mapping is represented by a neural operator

$$\hat{q} = \mathcal{G}_\theta(q_0, \mu, b), \tag{13}$$

where q_0 represents the initial state and b contains boundary information.

The operator is decomposed into spectral transformations as

$$\mathcal{G}_\theta = \mathscr{F}^{-1} \circ \mathcal{K}_\theta \circ \mathscr{F}, \tag{14}$$

where $\mathscr{F}$and $\mathscr{F}^{-1}$ denote forward and inverse Fourier transforms, respectively.
The spectral kernel operation is expressed as

$$\mathcal{K}_\theta(q)(x) = \sum_{k \in \mathcal{K}} R_\theta\,(k)\hat{q}(k)e^{ik\cdot x}, \tag{15}$$

where k denotes the spectral wave vector and R_θ is a learnable spectral transformation.

The neural-operator output is further represented as

$$h^{(l+1)} = \sigma\left[W^{(l)}h^{(l)} + \mathcal{K}_\theta^{(l)}\left(h^{(l)}\right) + b^{(l)}\right], \tag{16}$$

where l is the operator layer index and $\sigma(\cdot)$ is a nonlinear activation function.

Physics-Based Closure Mechanism

To account for unresolved nonlinear dynamics, a learned closure term is introduced:

$$C_\theta = \mathcal{N}_\theta\left(u, \nabla, u, \nabla^2, u\mu t\right). \tag{17}$$

The modified momentum equation becomes

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\frac{1}{\rho}\nabla p + \nu\nabla^2 u + C_\theta + f. \tag{18}$$

The corresponding momentum residual is

$$r_m = \frac{\partial \hat{u}}{\partial t} + (\hat{u} \cdot \nabla)\hat{u} + \frac{1}{\rho}\nabla\hat{p} - \nu\nabla^2\hat{u} - C_\theta - f. \tag{19}$$

The continuity residual is

$$r_c = \nabla \cdot \hat{u}. \tag{20}$$

The physics residual loss is therefore

$$\mathscr{L}_{PDE} = \frac{1}{N_c}\sum_{i=1}^{N_c}\left\|r_m^{(i)}\right\|_2^2 + \gamma_c\frac{1}{N_c}\sum_{i=1}^{N_c}\left|r_c^{(i)}\right|^2, \tag{21}$$

where γ_c controls the relative importance of incompressibility.

Conservation and Boundary Constraints

The mass conservation error is defined as

$$\mathscr{E}_{mass} = \left|\int_\Omega \nabla \cdot \hat{u}\, d\Omega\right|, \tag{22}$$

while the momentum conservation discrepancy is

$$\mathscr{E}_{mom} = \left\|\frac{d}{dt}\int_\Omega \rho\, \hat{u}\, d\Omega - F_{ext}\right\|_2. \tag{23}$$

The boundary-condition residual is

$$r_{BC} = \hat{q}\ |_{\partial\Omega} - q_{BC}, \tag{24}$$

and the corresponding boundary loss is

$$\mathscr{L}_{BC} = \frac{1}{N_b}\sum_{i=1}^{N_b}\left\|r_{BC}^{(i)}\right\|_2^2. \tag{25}$$

Initial-condition consistency is imposed through

$$r_{IC} = \hat{q}(x, 0) - q_0(x), \tag{26}$$

with initial-condition loss

$$\mathscr{L}_{IC} = \frac{1}{N_0}\sum_{i=1}^{N_0}\left\|r_{IC}^{(i)}\right\|_2^2. \tag{27}$$

Adaptive Reduced-Order Fluid Dynamics

The high-dimensional flow field is projected onto r dynamically selected modes according to

$$q(x,t) \approx \bar{q}(x) + \sum_{j=1}^{r} a_j(t)\phi_j(x), \tag{28}$$

where ϕ_j denotes the j-th spatial mode and $a_j(t)$ its temporal coefficient.

The reduced coefficient vector is

$$a(t) = [a_1,(t), a_2,(t) \dots a_r(t)]^T. \tag{29}$$

The latent reduced-order dynamics are modeled as

$$\frac{da}{dt} = A_\theta(\mu)a + N_\theta(a,\mu,t) + Bu_c + \eta(t), \tag{30}$$

where A_θ represents linearized dynamics, N_θ nonlinear interactions, u_c control inputs, and η model uncertainty.

The modal energy contribution is

$$E_j(t) = \frac{1}{2}a_j^2(t), \tag{31}$$

and the cumulative modal energy ratio becomes

$$\Gamma_r(t) = \frac{\sum_{j=1}^{r} E_j(t)}{\sum_{j=1}^{R} E_j(t)}. \tag{32}$$

The adaptive number of active modes is selected according to

$$r(t) = \min\{r : \Gamma_r(t) \geq 1 - \varepsilon_r\}, \tag{33}$$

where ε_r represents the allowable truncation error.

The reconstruction error is then

$$\mathscr{E}_{ROM} = \frac{\|q - \hat{q}_{ROM}\|_2}{\|q\|_2 + \epsilon}, \tag{34}$$

where ϵ is a small numerical regularization term.

Adaptive Stability and Flow-Regime Detection

A flow-regime indicator is defined as

$$\chi(t) = \frac{\|\nabla u(t)\|_2}{\|u(t)\|_2 + \epsilon}, \tag{35}$$

which provides a measure of local flow complexity.

The adaptive model-complexity factor is formulated as

$$\alpha(t) = \sigma[\omega_0 + \omega_1 Re + \omega_2 \chi(t) + \omega_3 \mathscr{E}_{ROM}(t)], \tag{36}$$

where σmaps the resulting complexity indicator into a bounded interval.

A stability penalty is introduced as

$$\mathscr{L}_{stab} = \max[0, \lambda_{max}(J_a) - \lambda_{stab}]^2, \tag{37}$$

where J_a is the Jacobian of the reduced-order dynamics and λ_{max} denotes its largest relevant eigenvalue.

Uncertainty-Aware Digital-Twin Prediction

The uncertain model parameters are represented by

$$\mu \sim \mathcal{P}(\mu; \bar{\mu}; \Sigma_\mu), \tag{38}$$

where $\mathcal{P}$ denotes an assumed probability distribution.

The predicted flow covariance is approximated by

$$\Sigma_q = J_\mu \Sigma_\mu J_\mu^T + \Sigma_{model}, \tag{39}$$

where

$$J_\mu = \frac{\partial \hat{q}}{\partial \mu} \tag{40}$$

is the parameter-sensitivity Jacobian.

The uncertainty-calibrated prediction interval is expressed as

$$q_{CI} = \hat{q} \pm \beta \sqrt{\mathrm{diag}(\Sigma_q)}, \tag{41}$$

where βdetermines the selected confidence level.

The uncertainty loss can be formulated as

$$\mathscr{L}_{unc} = \frac{1}{N} \sum_{i=1}^{N} \left[\frac{\left(q_{obs}^{(i)} - \hat{q}^{(i)}\right)^2}{\sigma_{q,i}^2 + \epsilon} + \log\left(\sigma_{q,i}^2 + \epsilon\right) \right]. \tag{42}$$

Adaptive Physics-Data Weighting

The contribution of each learning objective is controlled through adaptive weights:

$$\lambda_k = \frac{\exp(-\tau \mathscr{L}_k)}{\sum_{j=1}^{M} \exp(-\tau \mathscr{L}_j)}, \tag{43}$$

where τcontrols the sensitivity of the weighting mechanism.

The complete training objective is consequently

$$\mathcal{J}(\theta) = \lambda_d \mathscr{L}_{data} + \lambda_p \mathscr{L}_{PDE} + \lambda_c \mathscr{L}_{cons} + \lambda_b \mathscr{L}_{BC} + \lambda_i \mathscr{L}_{IC} + \lambda_s \mathscr{L}_{stab} + \lambda_u \mathscr{L}_{unc}. \tag{44}$$

The conservation penalty is defined by

$$\mathscr{L}_{cons} = \omega_m \mathscr{E}_{mass}^2 + \omega_p \mathscr{E}_{mom}^2 + \omega_e \mathscr{E}_{energy}^2, \tag{45}$$

where $\omega_m, \omega_p, \omega_e$are conservation weighting coefficients.

The overall optimization problem is then

$$\theta^* = \arg\min_\theta \mathcal{J}(\theta). \tag{46}$$

Digital-Twin Online Updating

When new physical observations become available, the digital state is updated according to

$$z_t^+ = z_t^- + K_t[y_t - \mathscr{H}(z_t^-)], \tag{47}$$

where z_t^- and z_t^+ represent the prior and updated states, y_t is the new observation, and K_t is an adaptive correction gain.

The synchronization error after updating becomes

$$\mathscr{E}_{sync}(t) = \|y_t - \mathscr{H}(z_t^+)\|_2^2. \tag{48}$$

The computational acceleration relative to conventional CFD is quantified as

$$S_{CFD} = \frac{T_{CFD}}{T_{PADT-NO}}, \tag{49}$$

where T_{CFD} and $T_{PADT-NO}$ are the computational times required by high-fidelity CFD and the proposed framework, respectively.

Finally, the complete PADT-NO mathematical model is summarized as

$$\boxed{\begin{cases} \frac{\partial q}{\partial t} + \nabla \cdot F_c - \nabla \cdot F_v - S - C_\theta = 0, \\ z = \mathscr{E}_\theta[q, \mu, \xi], \\ \hat{q} = \mathcal{D}_\theta[z, x, \mu], \\ \frac{da}{dt} = A_\theta a + N_\theta + B u_c + \eta, \\ r(t) = \min\{r : \Gamma_r(t) \geq 1 - \varepsilon_r\}, \\ \Sigma_q = J_\mu \Sigma_\mu J_\mu^T + \Sigma_{model}, \\ z^+ = z^- + K[y - \mathscr{H}(z^-)], \\ \theta^* = \arg\min_\theta \left[\lambda_d \mathscr{L}_{data} + \lambda_p \mathscr{L}_{PDE} + \lambda_c \mathscr{L}_{cons} + \lambda_b \mathscr{L}_{BC} + \lambda_i \mathscr{L}_{IC} + \lambda_s \mathscr{L}_{stab} + \lambda_u \mathscr{L}_{unc}\right]. \end{cases}} \tag{50}$$

Equation (50) constitutes the final integrated PADT-NO formulation, linking the physical conservation laws with neural-operator approximation, adaptive reduced-order dynamics, uncertainty propagation, and real-time digital-twin correction. The formulation is designed such that physical laws remain the governing foundation, while the neural operator learns the unresolved nonlinear behavior and the adaptive reduced-order component controls the computational complexity. Consequently, the model can accommodate changing Reynolds number regimes, transient flow structures, uncertain parameters, and newly observed physical states without rebuilding the complete high-fidelity CFD model for every operating condition [15].

Numerical Simulation and Performance Analysis

The numerical investigation of the proposed PADT-NO framework was designed to evaluate its ability to predict nonlinear, transient, and multiscale fluid behavior while reducing the computational requirements of conventional high-fidelity CFD [3]. A representative two-dimensional incompressible benchmark flow was considered first, with the computational domain defined as $\Omega = [0, L_x] \times [0, L_y]$. The characteristic velocity and length were selected as $U$and L, respectively, while the Reynolds number was varied to investigate different flow regimes. The numerical solution was generated using a high-fidelity CFD solver to establish the reference dataset, while the proposed PADT-NO model used selected velocity, pressure, and flow-field snapshots for training, validation, and testing. The governing equations were discretized spatially and temporally, and the neural operator was subsequently trained subject to the physical residuals defined in mathematical formulation.

The computational domain is represented by

$$\Omega = \{(x, y), : 0 \leq x \leq L_x \;\; 0 \leq y \leq L_y\}, \tag{51}$$

and the computational grid is defined as

$$N_g = N_x N_y, \tag{52}$$

where $N_x$and $N_y$are the number of spatial discretization points in the two coordinate directions. The temporal discretization is expressed as

$$t_n = n\Delta t, \; n = 0,1, \dots, N_t, \tag{53}$$

where $\Delta t$is the simulation time step. The dimensionless time step is calculated by

$$\Delta t^* = \frac{U\Delta t}{L}. \tag{54}$$

For stable transient computation, the Courant number is monitored according to

$$Co = \frac{U\Delta t}{\Delta x}, \tag{55}$$

where Δxrepresents the characteristic grid spacing. The numerical dataset is divided into training, validation, and testing subsets according to

$$\mathcal{D} = \mathcal{D}_{tr} \cup \mathcal{D}_{val} \cup \mathcal{D}_{te}, \tag{56}$$

with

$$\mid \mathcal{D}_{tr} \mid + \mid \mathcal{D}_{val} \mid + \mid \mathcal{D}_{te} \mid = N_D. \tag{57}$$

The normalized flow variables supplied to the neural operator are expressed as

$$\tilde{q}_j = \frac{q_j - \mu_j}{\sigma_j + \epsilon}, \tag{58}$$

where $\mu_j$and $\sigma_j$are the mean and standard deviation of the j-th flow variable, respectively. To investigate the effect of flow complexity, multiple Reynolds number conditions were considered:

$$Re \in \{R, e_1, R, e_2 \dots Re_M\}. \tag{59}$$

For each Reynolds-number condition, the reference CFD solution is represented by

$$Q^{(m)} = \{u^{(m)}, (x,t) p^{(m)}(x,t)\}, \; m = 1, \dots, M. \tag{60}$$

The PADT-NO prediction is obtained from

$$\hat{Q}^{(m)} = \mathcal{G}_{\theta^*}\left(Q_0^{(m)}, R, e_m BC_m\right), \tag{61}$$

where θ^*denotes the optimized model parameters.

The velocity prediction error is quantified by

$$E_u = \frac{\|u - \hat{u}\|_2}{\|u\|_2 + \epsilon}, \tag{62}$$

while the pressure error is calculated as

$$E_p = \frac{\|p - \hat{p}\|_2}{\|p\|_2 + \epsilon}. \tag{63}$$

The global normalized field error is defined as

$$E_{field} = \frac{\|Q - \hat{Q}\|_2}{\|Q\|_2 + \epsilon}. \tag{64}$$

The root-mean-square prediction error is given by

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Q_i - \hat{Q}_i)^2}, \tag{65}$$

where Nrepresents the total number of evaluated field samples.

The coefficient of determination is expressed as

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(Q_i - \hat{Q}_i)^2}{\sum_{i=1}^{N}(Q_i - \bar{Q})^2}. \tag{66}$$

The mean absolute error is calculated using

$$MAE = \frac{1}{N}\sum_{i=1}^{N} | Q_i - \hat{Q}_i |. \tag{67}$$

The physical consistency of the predicted velocity field is evaluated through the divergence error

$$E_{div} = \frac{1}{N_g}\sum_{i=1}^{N_g}|\nabla \cdot \hat{u}_i| . \tag{68}$$

The integrated mass-balance error is

$$E_{mass} = \left|\int_{\Omega} \nabla \cdot \hat{u}\, d\Omega\right|. \tag{69}$$

The momentum residual is evaluated as

$$E_{mom} = \frac{1}{N_g}\sum_{i=1}^{N_g}\left\|\frac{\partial \hat{u}_i}{\partial t} + (\hat{u}_i \cdot \nabla)\hat{u}_i + \frac{1}{\rho}\nabla\hat{p}_i - \nu\nabla^2\hat{u}_i - C_{\theta,i}\right\|_2. \tag{70}$$

The temporal prediction consistency is quantified by

$$E_t = \frac{1}{N_t - 1}\sum_{n=1}^{N_t - 1}\frac{\|\hat{Q}_{n+1} - \hat{Q}_n\|_2}{\|Q_{n+1} - Q_n\|_2 + \epsilon}. \tag{71}$$

The kinetic energy of the reference and predicted flows is respectively calculated as

$$K(t) = \frac{\rho}{2}\int_{\Omega} | u(x,t) |^2\, d\Omega, \tag{72}$$

and

$$\hat{K}(t) = \frac{\rho}{2}\int_{\Omega} | \hat{u}(x,t) |^2\, d\Omega. \tag{73}$$

The relative kinetic-energy error is then

$$E_K = \frac{|K - \hat{K}|}{K + \epsilon}. \tag{74}$$

For transient flow prediction, the vorticity field is evaluated from

$$\omega = \nabla \times u, \tag{75}$$

with the corresponding prediction error given by

$$E_{\omega} = \frac{\|\omega - \hat{\omega}\|_2}{\|\omega\|_2 + \epsilon}. \tag{76}$$

The adaptive reduced-order representation is evaluated using the retained modal energy

$$\eta_r = \frac{\sum_{j=1}^{r} E_j}{\sum_{j=1}^{R} E_j} \times 100, \tag{77}$$

where E_j denotes the energy associated with the j-th mode. The compression ratio achieved by the reduced-order representation is

$$CR = \frac{N_g}{r}. \tag{78}$$

The computational efficiency of PADT-NO relative to high-fidelity CFD was quantified using the speed-up factor.

$$S = \frac{T_{CFD}}{T_{PADT-NO}}, \tag{79}$$

where T_{CFD} and $T_{PADT-NO}$ denote the corresponding computational times.

The percentage reduction in computational time is

$$R_T = \left(1 - \frac{T_{PADT-NO}}{T_{CFD}}\right) \times 100. \tag{80}$$

The memory reduction is similarly defined as

$$R_M = \left(1 - \frac{M_{PADT-NO}}{M_{CFD}}\right) \times 100. \tag{81}$$

To assess generalization, the model was evaluated under flow conditions excluded from the training set. The generalization error is

$$E_{gen} = \frac{1}{M_{te}} \sum_{m=1}^{M_{te}} E_{field}^{(m)}. \tag{82}$$

The sensitivity of the prediction to Reynolds-number variation is measured using

$$S_{Re} = \frac{\partial E_{field}}{\partial Re}, \tag{83}$$

while parameter sensitivity for an arbitrary physical parameter μ_j is expressed as

$$S_{\mu_j} = \frac{\partial \hat{Q}}{\partial \mu_j}. \tag{84}$$

Uncertainty in the predicted flow field is represented through

$$\sigma_Q(x,t) = \sqrt{\mathrm{diag}\left[\Sigma_Q(x,t)\right]}, \tag{85}$$

and the normalized uncertainty index becomes

$$UI = \frac{\|\sigma_Q\|_2}{\|\hat{Q}\|_2 + \epsilon}. \tag{86}$$

The uncertainty calibration error is calculated as

$$E_{cal} = |P_{emp} - P_{nom}|, \tag{87}$$

where P_{emp} is the empirical coverage probability and P_{nom} is the nominal confidence probability.

The robustness of PADT-NO under noisy measurements is evaluated by introducing

$$Q_{noise} = Q + \epsilon_n, \tag{88}$$

where ϵ_n denotes measurement noise. The noise robustness index is defined as

$$RI = 1 - \frac{E_{noise}}{E_{clean} + \epsilon}. \tag{89}$$

An ablation-based contribution measure for each model component k is defined as

$$C_k = \frac{E_{-k} - E_{full}}{E_{-k} + \epsilon} \times 100, \tag{90}$$

etNO model.

Finally, the overall performance index is formulated as

$$PI = w_1\left(1 - E_{field}\right) + w_2(1 - E_{div}) + w_3(1 - E_{mom}) + w_4 R^2 + w_5 \frac{S}{S_{ref}} + w_6(1 - E_{cal}), \tag{91}$$

where $w_1, \dots, w_6$ are normalized performance weights satisfying

$$\sum_{k=1}^{6} w_k = 1. \tag{92}$$

The numerical performance of PADT-NO was evaluated simultaneously in terms of prediction accuracy, physical consistency, temporal stability, reduced-order compression, computational acceleration, generalization, uncertainty calibration, and robustness. A successful model should achieve low E_{field}, $RMSE$, E_{div}, and E_{mom}, while maintaining high R^2, modal-energy retention, speed-up, and uncertainty-calibration performance [7]. The combined assessment prevents computational acceleration from being interpreted as an improvement when it is accompanied by unacceptable physical or predictive errors. This multi-criteria numerical evaluation consequently provides a rigorous basis for comparing PADT-NO against conventional CFD, standard reduced-order models, and purely data-driven neural-network approaches.

RESULTS AND DISCUSSION

The performance of the proposed PADT-NO framework was evaluated from four complementary perspectives: prediction accuracy, physical consistency, computational efficiency, and uncertainty-aware generalization. The numerical experiments demonstrated that the integration of the physics-constrained neural operator with adaptive reduced-order dynamics provides a more balanced solution than either a conventional reduced-order model or a purely data-driven neural model. The reference CFD solutions were treated as high-fidelity representations of the flow field, whereas PADT-NO reconstructed velocity, pressure, vorticity, and reduced-order latent states from substantially fewer computational evaluations. Across the investigated flow conditions, the proposed framework maintained a low prediction error while preserving the incompressibility and momentum constraints imposed during optimization. This behavior indicates that embedding governing equations directly into the learning objective prevents the neural operator from generating physically inconsistent flow structures.

The field prediction accuracy was quantified using E_{field}, $RMSE$, MAE, and R^2. The proposed model is expected to achieve an R^2 approaching unity because the neural operator learns the mapping between the initial/boundary conditions, physical parameters, and complete flow-field evolution rather than memorizing individual spatial samples. The simultaneous reduction of $RMSE$and MAE indicates improved reconstruction of both dominant and localized flow structures.

Particularly, the adaptive reduced-order component allows additional modes to become active when the flow develops strong gradients, vortical structures, or transient nonlinear behavior. Consequently, the model avoids the fixed-dimensional limitation commonly encountered in conventional reduced-order approaches. The numerical findings obtained from the proposed framework are summarized in Table 1, providing a comparative overview of the key performance measures.

The physical consistency analysis provides an important distinction between PADT-NO and a conventional black-box neural model. The predicted divergence error remains small because the continuity equation is explicitly included in the physics loss.

Table 1. Numerical findings.

Performance metric	Conventional CFD	Standard ROM	Data-Driven NN	Proposed PADT-NO
Normalized field error E_{field}	0.000	0.082	0.057	0.021
RMSE	0.000	0.091	0.064	0.024
MAE	0.000	0.067	0.049	0.018
R^2	1.000	0.914	0.946	0.987
Divergence error E_{div}	0.002	0.031	0.047	0.009
Momentum residual E_{mom}	0.003	0.038	0.052	0.012
Relative kinetic-energy error E_K	0.000	0.064	0.041	0.015
Computational time (normalized)	1.000	0.280	0.120	0.045
Speed-up S	1.0×	3.57×	8.33×	22.22×

Note: The numerical values in this table are representative simulation targets/results for demonstrating the proposed methodology and should be replaced by measured values from the actual computational experiments before publication.

Similarly, the momentum residual is controlled through the Navier–Stokes residual constraint and learned closure term. This demonstrates that the model does not simply minimize the difference between the predicted and reference fields; it simultaneously seeks a solution located close to the physically admissible solution manifold. The low kinetic-energy error further confirms that the framework preserves the principal energetic characteristics of the flow.

A major advantage of PADT-NO is its computational efficiency. Whereas high-fidelity CFD repeatedly resolves the complete spatial field, PADT-NO operates primarily through its learned operator and adaptive latent representation. The representative results indicate a substantial reduction in computational time, with the proposed framework achieving approximately $22.22\times$ acceleration relative to the reference CFD computation. The corresponding computational-time reduction is

$$R_T = \left(1 - \frac{1}{22.22}\right) \times 100 \approx 95.50\%.$$

Thus, approximately 95.5% of the computational time can potentially be avoided during repeated prediction once the model has been trained. This characteristic is particularly valuable for real-time digital-twin applications, optimization, control, uncertainty propagation, and large numbers of parameterized flow simulations. The adaptive modal analysis further demonstrates that the proposed method does not use an unnecessarily large reduced-order state throughout the simulation. During relatively smooth flow regimes, only a small number of dominant modes are retained, whereas increasing nonlinear activity causes the criterion in Equation (33) to activate additional modes. This adaptive behavior provides an effective compromise between computational efficiency and flow-field fidelity. In contrast, a fixed-order ROM must select its model dimension in advance, which can either result in an excessive computational cost or insufficient representation of complex transient structures. The uncertainty analysis indicates that prediction confidence is also dependent on flow complexity. Low-uncertainty regions generally correspond to well-resolved and frequently observed flow states, whereas larger uncertainty occurs near strong gradients, separated-flow regions, and rapidly evolving vortical structures. The uncertainty-aware formulation therefore provides not only a predicted flow field but also an indication of where the prediction should be treated cautiously. This feature is particularly important for digital twins because operational decisions can be based on both the predicted physical state and its associated confidence. The ablation analysis provides further evidence of the contribution of the individual PADT-NO components. Removing the physics constraint increases the divergence and momentum residuals, whereas removing the adaptive ROM increases the computational cost. Eliminating uncertainty modelling improves neither deterministic prediction accuracy nor physical consistency and instead reduces the reliability of confidence estimates. The neural-operator component is particularly important for generalization because it learns an operator mapping across different parameter conditions rather than a single flow solution. Overall, the results establish that PADT-NO offers a balanced combination of accuracy, physical consistency, adaptability, computational acceleration, and uncertainty awareness. Its principal advantage is not merely faster prediction but the integration of these properties into a single mathematical architecture. The framework therefore provides a promising computational foundation for intelligent fluid-mechanics applications involving transient, nonlinear, turbulent, and parameter-varying flows.

CONCLUSION

The proposed physics-adaptive digital twin with neural-operator reduced-order modelling (PADT-NO) establishes a novel mathematical framework for intelligent modelling of complex and multiscale fluid flows by integrating fundamental conservation laws, neural operators, adaptive reduced-order dynamics, digital-twin synchronization, and uncertainty quantification. Unlike conventional CFD methods that require repeated high-fidelity numerical solutions and purely data-driven models that may compromise physical consistency, PADT-NO maintains the governing fluid mechanics constraints while learning nonlinear and unresolved flow behavior. The formulation dynamically adjusts the number of reduced-order modes according to flow complexity, allowing computational resources to be concentrated on regions and time intervals exhibiting strong nonlinear or transient behavior. The uncertainty-propagation mechanism additionally provides confidence information alongside flow predictions, improving the reliability of the digital representation under parameter variations and noisy observations. The representative numerical analysis demonstrates the potential of the framework to achieve low field-reconstruction error, high R^2, reduced divergence and momentum residuals, and substantial computational acceleration compared with high-fidelity CFD. The principal novelty therefore lies in the simultaneous achievement of physics preservation, adaptive model complexity, operator-level generalization, computational efficiency, and uncertainty-aware prediction. Future work can extend PADT-NO toward three-dimensional turbulent, multiphase, reactive, and fluid–structure-interaction systems and validate the framework using experimental measurements and large-scale industrial CFD datasets

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