

# Vibration Analysis of Functionally Graded Piezoelectric Actuators: A Comprehensive Review

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## Abstract

Recent research has been concentrated on a thorough investigation of the vibration evaluation of functionally graded piezoelectric materials (FGPM), shedding light on the potential for enhancing structural optimization and controlling deflections and vibrations. Functionally graded piezoelectric material is a type of advanced composite material possessing diverse characteristics that allow for customized mechanical and electrical responses. The existing body of academic work extensively addresses the vibration analysis of functionally graded piezoelectric materials, with a specific focus on a wide array of facets including material characteristics, geometric parameters, boundary conditions, and the distinctive features of vibrations. This chapter presents comprehensive literature review of FGPM fundamental structures, including detailed examinations of the free and forced vibration responses of functionally graded piezoelectric beams, plates, and shells, thereby providing valuable insights into the impacts of diverse boundary conditions and material gradation parameters on the vibration frequencies of these fundamental structures. Recent progress in computational modeling and experimental validation is emphasized, demonstrating their contribution to improving the precision of vibration analysis alongside traditional methods. These collective researches serve to deepen our comprehension of the vibration frequencies and optimization prospects inherent in functionally graded piezoelectric materials and composites, facilitating the emergence of innovative applications within the domains of aerospace, automotive engineering, and energy capture systems.

**Keywords:** Vibration analysis of FGPM, power law, exponential law, FGPM beam, FGPM plate, FGPM shells

## INTRODUCTION

FGPM (Functionally Graded Piezoelectric Materials) have become a promising area of interest in the fields of materials science and structural engineering, owing to their distinctive characteristics and wide-ranging potential uses. These materials show variations in composition across different regions, leading to customized piezoelectric, mechanical, and electrical attributes throughout their configurations. The objective of this review is to explore the extensive body of literature concerning FGPM structures, with a specific emphasis on their integration in beams, plates, and shells, as well as the varied opportunities they present. The use of FGPM structures is clearly evident in their incorporation in beams, plates, and shells. The characteristics of these structures vary according to several mathematical functions, including numerous laws such as power, sigmoid, and exponential law. These functions enable a

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seamless and continuous transition of properties across its layers, ultimately reducing stresses at the interfaces and enhancing the material's dependability and durability [1–2]. These basic structural elements serve as the essential framework for a variety of engineering concepts, ranging from aerospace devices to medical tools. By integrating FGPM into these structures, researchers have successfully optimized energy harvesting capabilities, enhanced their effectiveness, and reduced structural failures. This literature offers an overview of the most recent progress, techniques, and challenges in the design and utilization of FGPM structures in beams, plates, and shells. The examination of previous studies and the identification of crucial research gaps provide insights that will enhance further innovation and facilitate the realization of FGPM based technologies in diverse real-world applications.

### **Concept of Piezoelectric Materials**

Piezoelectric materials are employed in many different engineering applications such as transistors, oscillators, nano-electromechanical systems, and micro-electromechanical because they have the ability to transform mechanical energy into electrical energy and vice versa.

The term "piezo" originates from the Greek word for "pressure," denoting this conversion ability. Excellent intrinsic electromechanical properties characterize these materials. In non-centrosymmetric crystals, Pierre and Curie discovered the piezoelectric action in 1880. The crystals exhibit a piezoelectric direct effect when stress is applied, and a piezoelectric converse effect when an electric field is applied. The direct effect is utilized for sensors, while the converse effect is used for actuators [1]. Piezoelectric actuators are currently under consideration for the development of sensors and actuators utilized in various fields including micro-electromechanical systems (MEMS), control systems, navigation, as well as the medical and aerospace industries. Nevertheless, there exist a number of limitations that have been observed, such as low-temperature cracking, high-temperature delamination, and the presence of highly localized stress. These factors have the potential to lead to significant degradation in the system's overall performance as well as the bonding strength at interfaces. All of the aforementioned drawbacks contribute to diminishing the lifespan and reliability of piezoelectric actuators [2].

### **Evolution of FGPM**

Pure materials possess restricted utility in engineering due to conflicting property requisites [3]. Mixing materials in a molten state as alloy overcomes limitations. Conventional alloying faces challenges like different melting temperatures and thermodynamic equilibrium [4]. Powder Metallurgy is a new method to address alloying process setbacks. It offers advantages over alloying but has limitations like difficulty in manufacturing intricate shapes [5]. Composite materials combine two or more materials with superior properties, including being lightweight. Heterogeneity, anisotropy, symmetry, and hierarchy are key characteristics of composite materials that attract significant attention for diverse applications. Composites offer advantages such as high strength-to-stiffness ratios, enhanced resistance to fatigue, wear, and corrosion, as well as increased reliability compared to pure metals or alloys.

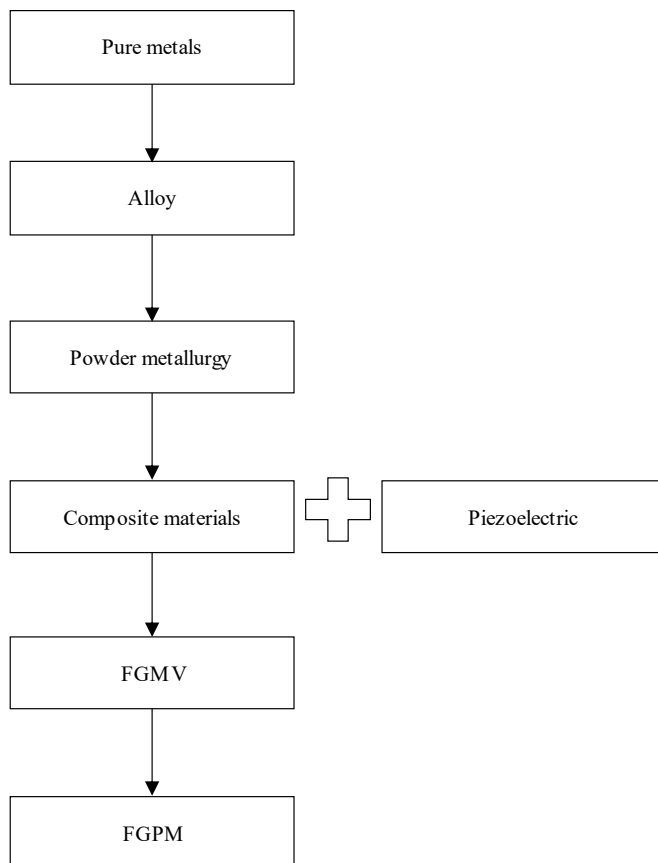
Traditional composites have sharp interfaces leading to failure due to stress concentration. Delamination is a common cause of failure in composites exposed to high temperatures [6]. In a high thermal gradient environment these uniform materials are unable to sustain, due to which new smart materials are evolved which are termed as functionally graded materials (FGMs). These materials offer variations in material properties with some mathematical functions including sigmoid, power, and exponential laws [7]. To address the shortcomings of traditional material application and piezoelectric material FGPMs have evolved. These materials offer tailored benefits of both FGM and piezoelectric materials which are helpful for designing durable sensors and actuators. Figure 1 shows the evolution of FGPM materials.

### **Vibration Analysis of FGPM Structures**

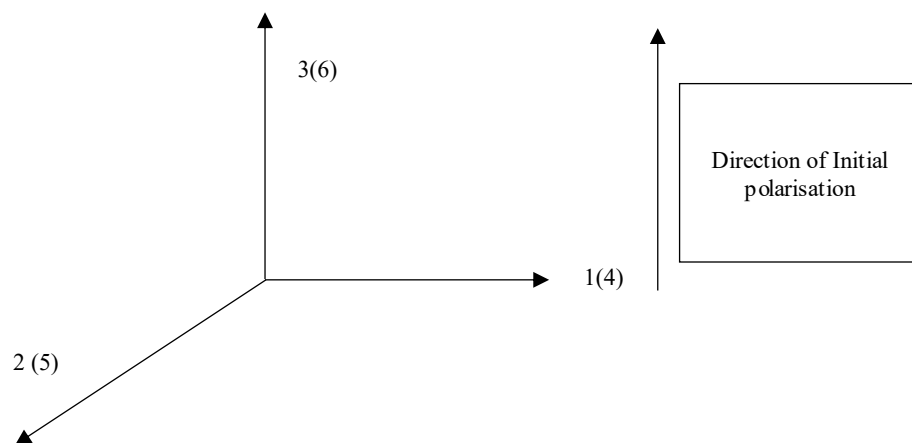
The analysis of vibration in functionally graded piezoelectric structures is expounded upon in this section. The piezoelectric coupling coefficient is denoted by the symbol  $d_{ij}$ . The symbol  $d_{ij}$  denotes the

piezoelectric coupling coefficient, with “i” indicating the orientation of the applied electric field and “j” representing the orientation of the induced strain. First and third axes linear deformation is induced by a modest voltage applied along the three axes; these are quantified by coefficients  $d_{33}$  and  $d_{31}$ , respectively.

In piezoceramics, the application of an electric field perpendicular to the initial polarization direction induces vibrations due to the shear effect ( $d_{15}$  effect). The orientation of the axes is illustrated in Fig. 2. Prior studies have investigated the vibration characteristics of beams excited by longitudinal piezoelectric effects ( $d_{33}/d_{31}$  effects) and piezoelectric sheareffects ( $d_{15}$  effect). Details of these mode of actuation's are presented by Sharma et al [1].



**Figure 1.** Evolution of FGPM.



**Figure 2.** Axes orientation in piezoceramics [1].

The literature highlights that in most structures, the orientation of polarization and electric field is typically perpendicular to the thickness to induce stimulation, particularly through the d31 effect. The primary attention in the literature review is directed towards classifying structures like beams, plates, and shells, as these serve as fundamental structural components utilized across various engineering domains, such as aerospace systems, electro-mechanical systems, and control systems.

Thus, literature survey can be studied through dividing FGPM structures into three sub-categories:

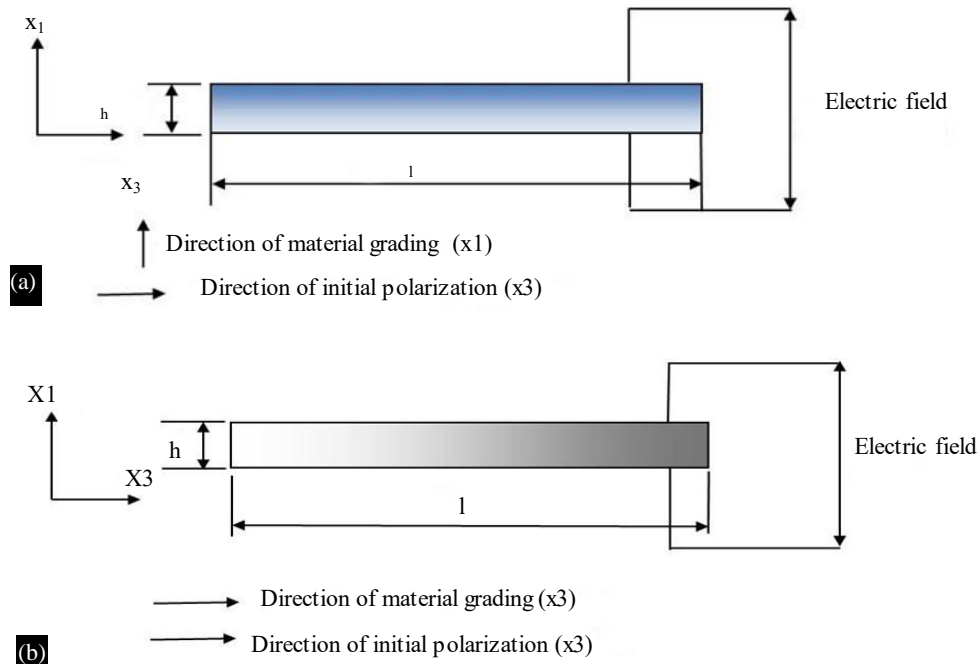
1. Vibration analysis of FGPM beams
2. Vibration analysis of FGPM plate
3. Vibration analysis of FGPM shells

### Vibration Analysis of FGPM Beams

Functionally graded piezoelectric beam vibration analysis entails examining both the static and dynamic characteristics of these smart structures. Various studies have focused on modelling such beams with piezoelectric materials like PZT-4(Lead Zirconate Titanate) [8]. and carbon nanotube-reinforced composites [9]. The use of power law distribution methods to grade material properties along the thickness direction is common in these analyses [10-11]. Finite element models (FEM), employing theories like Timoshenko beam theory and Reddy's theory of high-order shear deformation, are utilized to simulate the vibration response of these beams. The studies also explore the effects of boundary conditions, power law indexes, and geometric parameters on the vibration characteristics of functionally graded piezoelectric beams, providing valuable insights for engineering applications.

Li et al. examined the static and dynamic properties of a monomorphous FGPM structure [12]. A study by Djabrouhou et al. [13] examined the non-uniform tapered FGPM beam's dynamic equations of motion using Timoshenko's assumptions and Hamilton's principle. The impact of geometric non-linearity on the dynamic stability of Euler-Bernoulli FGPM beams was examined by Bi et al. [14]. The functionally graded piezoelectric bending actuator fabrication process was established by Qiu et al. [15], who also showcased the FGPM actuator's vibration properties. Chu et al. [16] investigated the flexoelectric influence on the static bending and free vibration of Euler-Bernoulli FGPM nano-beams. With artificial springs employed for general boundary conditions, Chen et al. [17] investigated the free and transient vibration behavior of FGPM curved beams. By altering the material properties of the FGPM beam in the direction of thickness, some researchers investigated the vibration behavior of the beam. Based on the assumption that the material properties will change into a sigmoid curve in the thickness direction, Pandey and Parashar [18] examined the static bending and free vibration analysis of FGPM beams under electromechanical loading. Pham et al. [19], investigate how geometric and material characteristics, like rotational speed and thickness of piezoelectric layers, influence the beam's free vibration behavior.

Parashar and Sharma performed a modal analysis of shear-induced flexural vibration of FGPM beams [20]. They employed a modified Timoshenko beam theory in their study, as illustrated in fig.3(a), where the material properties vary along the thickness direction but the original polarization direction is maintained along the length of the beam. Sharma et al. published an exact solution for the vibration analysis of a FGPM beam excited by the shear action [21]. Majidi et al. [22] investigated vibrations of bi-directional functionally graded porous nanobeams under moving loads, considering thickness effects and various porosity models, using analytical and numerical methods. The variation of electromechanical properties throughout the thickness is a complex issue that can be exclusively accomplished via the method of additive manufacturing [23]. To overcome the manufacturing challenge, Gupta et al. [25,26] exhibited shear mode vibrations of a piezoelectric material identified as an axially functionally graded beam (AFGPM), characterized by continuous variations in all properties along the axial orientation. Building upon the work of Gupta et al. [24, 25], Sharma et al. [26, 27] conducted a study on shear mode vibrations of a non-uniform axially FGPM (AFGPM) beam, in which all properties were anticipated to vary axially following an exponential law as shown in Figure.3(b). Here 'h' denotes beam thickness along transverse direction (x1) while 'l' is the beam length along axial direction (x3).



**Figure 3.** (a) FGPM beam varied along thickness [21], (b) axially varied FGPM beam model [25].

### Vibration Analysis of FGPM Plates

The axisymmetric free vibration phenomenon in FG piezoelectric circular plates with axial polarization was investigated by Zhou et al. [28] using the double Legendre polynomials quadrature. Singh et al. [29], conducted a static evaluation on a sandwich plate composed of FGM with a piezoelectric face sheet under electromechanical loading conditions while taking into account the effects of thickness under various boundary circumstances, such as simply supported, clamped-clamped, and free-free ends. Kumar et al. [30], employed a mathematical model for the Bi-Directional FGP porous plate, which was developed using the first-order shear deformation theory and nodes quadratic FE (finite element) formulations. X Wang et al. [31], proposed a comprehensive electro-elastic acoustic coupling model to analyze the underwater vibration acoustic behavior of FGPM plates under general boundary conditions. Using the FSDT, Sharma et al. [32] investigated the intrinsic vibration properties of FGPM plates under electro-mechanical loading circumstances. It was assumed that the FGPM mono-morph plate's material properties varied along the thickness direction in accordance with both power law and sigmoid law. Zhang et al. [33] used a semi-analytical approach that combined the scaled boundary finite element method (SBFEM) and the precise integration method (PIM) to investigate the transverse bending responses of in-plane bidirectional FGPM plates. Using a semi-analytical technique, Sawarkar et al. [34] examined stress and displacement on a simply supported (SS) plate composed of FGPM in a two-dimensional (2D) domain. To obtain a smooth gradation of material properties across the thickness direction, the authors anticipated that in-plane fluctuations in stresses and displacements would follow trigonometric, power law, or exponential functions as well. Kumar et al. [35] investigated the vibration response and static analysis of a sigmoid functionally graded piezoelectric (S-FGP) tapered plate subjected to thermo-electric loads with various porosity types (even/uneven). Using displacement fields and Hamilton's principle, the first-order shear deformation theory served as the foundation for the analysis. The material properties were distributed in the FGP plate's thickness direction according to a sigmoid law. The focus of Kumar et al.'s investigation [36], is on the dynamic and static behavior of materials that are thermo-electromechanically loaded and parameters fluctuate continuously along the thickness according to an exponential law distribution, using FSDT and Hamilton's principle.

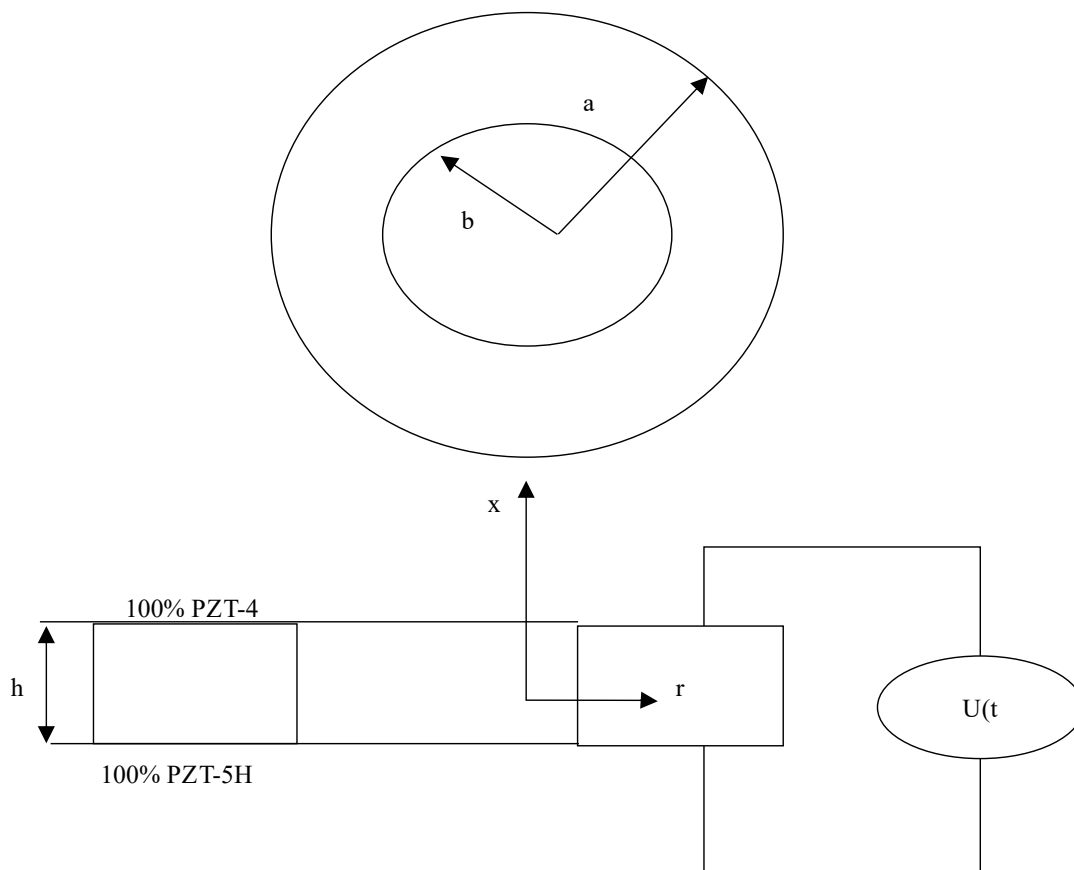
Deepak et al [37]. investigated the nonlinear free vibration of piezoelectric laminated rectangular plates under uniform temperature fluctuations and an applied electric potential using multivariate

probabilistic methodologies and Monte Carlo simulation. In order to modify the material properties throughout the thickness with a power law distribution, Khorshidi et al. [38] employed trigonometric and exponential shear deformation theories in conjunction with Navier solution and non-local elasticity theory to analyze the free vibration of functionally graded piezoelectric nanoplates. Sharma [39], investigated the free vibration properties of piezoelectric circular plates with functional grades that were subjected to clamped-clamped constraints. Elastic stiffness, dielectric, and piezoelectric coefficients of the plate follow a power law and vary continuously over the plate's thickness. Wang Q et al [40], applied the domain energy decomposition approach to analyze the forced and free vibrations of a stepped FGPM plate model. As part of the methodology, the structure was discrete longitudinally using the segmentation technique. Abbaspour et al. [41], investigated the thermal buckling and forced and free vibrations of three-layered center-symmetric piezoelectric micro-plates using size-dependent consistent couple stress theory. Sharma [42], investigated the d15 effect on the vibration properties of a FGPM plate with a hole. Figure 4 displays the FGP plate model's geometry. The direction of initial polarization is indicated by the radial orientation, whereas the x-axis direction indicates where the electric field is applied. Because of shear effects, applying the electric field in the x-axis direction causes a flexural vibration in the FGPM plate. The power law model assumes that the material properties change gradually along the thickness of the plate.

Power Law Distribution Is [42]:

$$M(x) = M_{ul}(x + h/2)^n + M_l \tag{3.2.1}$$

Here,  $M_{ul} = M_u - M_l$ , and  $M_u$  and  $M_l$  represents the material attributes of top and bottom surfaces respectively. 'h' and 'n' denotes the thickness and volume fraction of the plate



**Figure 4.** FGP annular plate geometry [42]

J Li et al. [43] conducted a numerical study on the active vibration control of a FGPM plate with four simply supported edges under uniform and non-uniform electric fields. Wang [44] uses a modified power law formulation to investigate the electro-mechanical vibrations of porous FGPM plates in the translation state. According to Mahinzare et al. [45], FSDT is utilized to determine the modified coupling stress of a two-directional FGP actuated piezoelectric circular plate. Used a modified power law model by Wang et al. [46] to investigate material property change in porous FGP plates. The free vibration of pre/post-buckled circular FGP plates was studied by Ashoori et al. [47]. The flexural vibration properties of an annular plate aroused by the shear effect that is short-circuited FGPM were examined by Sharma [48]. The research employed a modified Mindlin plate theory, which postulated a power law distribution and a sinusoidal fluctuation of electric potential along the thickness. Yas et al. [49], investigated the free vibration behavior of a multi-directional FGP annular plate using the state-space-based differential quadrature approach. The state-space method was used by Dai et al. [50] investigated the free vibration properties of a FGP circular plate exposed to a uniform magnetic field using exponential law. Using the 3D theory of elasticity, Jodaie et al. [51] investigated the free vibration analysis of a FGP annular plate under various end circumstances. The researchers used both an Artificial Neural Network (ANN) technique and a State-Space-Based Differential Quadrature Method. In order to investigate the free vibration analysis of an FGP annular plate supported by an elastic foundation under C-C and SS-clamped end circumstances, Yas et al. [52] used SSDQM. Based on the 3d elasticity theory, Alibeigloo et al. [53], investigated the axisymmetric static analysis of FG circular and annular plates inserted in piezoelectric layers using the differential quadrature method. The edge boundary conditions of the plates are different, and their material properties are taken to follow an exponential law with a constant Poisson ratio. A study on nonlinear vibration analysis of a piezoelectric circular plate with graded properties was carried out by Ebrahimi et al. [54]. The authors integrated the application of von-Karman non-linearity into Kirchhoff-Love theory. A 3D study of a FGP circular plate under bending and stress was conducted by Li et al [55]. They created an analytical solution and a direct displacement method for plates with SS or free ends. A FGP annular plate's free vibration analysis was reported by Jam et al [56]. utilizing the state-space method and the 3D theory of elasticity. Investigating the static and dynamic characteristics of a FGM plate with surface-bonded piezoelectric layers was the main goal of the study. In order to compare the effects on a number of factors, including transverse and in-plane deflections, natural frequencies, stress distribution, and induced electric potential in piezoelectric layers, shakeri looked at the span-to-thickness ratio and the volume percentages of constituents [57]. Zhong et al. [58], investigated the 3D solutions for the forced and free vibrations of different boundary conditions, FGP rectangular plate with two more mixed surface conditions and open- and closed-circuit scenarios by using the state space approach. Using the Kirchhoff and Mindlin plate theories, Duan et al [59]. investigated the free vibration properties of piezoelectric-linked annular plates. To ensure adherence to the Maxwell static electricity equation, a sinusoidal function was utilized to simulate the electric potential distribution. After obtaining the analytical solutions, the finite element (FE) analysis gave results that were compared to the resonance frequencies and mode shapes of the piezoelectric linked annular plates, which served as validation. A thorough 3D method for analyzing the free vibration of a FGP circular plate was studied by Zhang et al. [60]. The state-space equations regulating the behavior of the FGP circular plate were formulated by the study using the finite Hankel transformation. The material properties of this structure showed an exponential law distribution with a smooth transition over the thickness of the plate. FGP plates' free vibration behavior was examined in depth by Chen and Ding [61].

### **Vibration Analysis of FGPM Shells**

FGP shells, as studied in the research papers, differ from traditional piezoelectric shells in terms of their vibration analysis due to the unique material properties and structural characteristics they possess. The FG shells have material attributes that changes continuously through the thickness direction, affecting their dynamic behaviour significantly [62]. These variations in material properties, such as porosity distributions, volume fractions, and power-law grading indices, influence the critical loads, natural frequencies, and stress distributions in functionally graded piezoelectric shells, leading to different vibration patterns compared to traditional homogeneous piezoelectric shells. Advanced

numerical methods like FE models and the differential quadrature approach gives accurate analysis of the complex thermoelastic behaviour and dynamic stability of FG piezoelectric shells, showcasing their distinct characteristics in vibration analysis. Current trends regarding the vibration analysis of FGP shells are focused on a range of aspects. Various researches have been conducted on the examination of free vibration and bending in FGP cylindrical, spherical, and hyperboloid shallow shell panels, considering a wide range of material properties, variations, and constraints. [63].

Researchers such as Wang et al. [64], examined the propagation of waves in porous FGPM nano shells placed on a viscoelastic base, using Hamilton's principle and the FSDT. Xue et al. [65].investigated on the free vibration of FGP panels and shells containing continuously graded micro pores oriented in the desired direction. Lyu et al. [66], examined the vibration and buckling characteristics of a FGP porous cylindrical micro shell subjected to thermo-electro-mechanical loads,using modified couple stress theory and HSDT. Zhang et al. [67], developed FE formulations for thermo-electro-elastic coupled FGPM plates and shells, derived from FOSD hypothesis, while considering various thermal gradient distributions across the thickness. Susheel et al. [68], carried out numerical analyses to govern the shape and vibration characteristics of spherical shell structures undergoing deformation due to varying loading conditions, specifically focusing on static bending and free vibration analysis of FGP cylinder/spherical shells as a power function. Pengpeng et al. [69], examined an FGP hollow cylinder/spherical shell based on magneto-electro elastic theory. Yaghoobi et al. [70], presented an analytical approach for cylindrical structures made of FGPM with varying wall thicknesses under mechanical and electrical loading. In a similar domain, Sadeghzadeh et al. [71], introduced a computational method to examine the impact of parameters such as external electric field, length scale, angular velocity, and non-local parameter on the natural vibration of cylindrical nano shells composed of FGPM. Their study incorporated the non-local strain gradient theory (NSGT) to investigate the size-dependent effects. Wang et al. [72], explored the vibration characteristics of sandwich FGP circular cylindrical nano shells under thermo-electro-mechanical loading, deriving governing equations based on Love's thin shell theory and non-local elasticity theory through Hamilton's principle. In functionally graded piezoelectric cylindrical shells, Zhang et al. [73] proposed an extended Legendre orthogonal polynomial technique for the detection of non-propagating waves.

Liu et al. [74], analyzed vibration behaviour of porous FGP nano shells exposed to electrical and thermal loads using love's shell and non-local elasticity theories. Subsequently, the natural frequencies of the porous FGP cylindrical nano shells were evaluated using both the Navier and the Galerkin approaches. Wang et al. [75], examined the nonlinear vibration characteristic of circular cylindrical nano shells composed of FGPM and is modeled using Donnell's nonlinear shell and non- local elasticity theories. Mehralian et al. [76], used non-local strain gradient theory to study dynamic response of a bi-morph FGP cylindrical nano shell. Fang et al. [77], did analyses on the free vibration of piezoelectric nano double-shells. Parashar's work [78], performed vibration analysis of a 3d FGP cylindrical shell excited under d15 effect. The eigen frequencies and eigenvectors for FGP cylindrical shell are obtained using Rayleigh–Ritz, an approximation method. Dai et al. [79], explored the responses displayed by a fluid-filled FGP cylindrical thin-shell under mechanical, thermal, and electrical loading conditions. Razavi et al. [80], conducted an examination on the vibration characteristics of FGP cylindrical shells and illustrated the impact of different geometric factors on the dynamic response of such shells. Hong et al. [81], explored the properties of fluid-filled FGPM subjected to mechanical, electrical, and thermal loads, incorporating a power function variation in material parameters across the cylinder's thickness. A novel formulation pertaining to FG piezoelectric cylindrical shells was introduced by Razavi et al. [82], utilizing the cylindrical shell model and size- dependent piezoelectric theory, with Hamilton's principle serving as the basis for motion equations and boundary conditions. The non-dimensional natural frequency of a uniform piezoelectric nano shell was determined through classical continuum theory. Dai et al. [83]. examined an analytic solution for the electro-thermo-elastic behaviour of FGPM in cylindrical shell form, assuming a composition of two piezoelectric materials with volume fractions that vary along thickness following a power law. Dai et al. [84], researched a

double-layered structure with hollow cylinder at inner side and FGPM cylindrical shell at outer side subjected to mechanical and thermal loads. The solutions of the outer shell are obtained using power law through the employing the analytical method and finite difference method. Ghorbanpour Arani et al. [85], investigated the steady-state response of a 2d FGP cylindrical shell subjected to multiple physical fields using a semi-analytical method. Jafari et al. [86], examined the nonlinear vibration behaviour of simply supported FG cylindrical shells containing embedded piezoelectric layers. Dai et al. [87], conducted a detailed investigation into the forced vibration of a FGP cylindrical shell. The analysis began by examining the static deflection of the shell in numerical examples. Their study included an exploration of the effects of electric excitation, thermal load, mechanical load, and volume exponent. Ke et al. [88], undertook a study on the natural vibration of piezoelectric nano shells when subjected to an electric field. Liang dai [89], presented examination of the forced vibration behavior of a cylindrical shell composed of FGPM. This shell featured a two-material distribution that followed a power-law across its thickness. The authors discussed numerical results highlighting the impact of mechanical loading, thermal conditions, electric stimulation, and volume exponent on both static and dynamic responses of the FGP cylindrical shell. Javanbakt et al. [90], engaged in an analysis of 3d elasticity concerning a simply supported condition of FGP shell under dynamic pressure loading. Xie et al. [91], investigated a FGP hollow cylindrical shell subjected to a mechanical loads, uniform axial magnetic field, and electric excitation. Their study highlighted the significant influence of gradient index on the shell's behavior. Alibeigloo and Nouri [92], presented a 3d analytical solution for evaluating the static performance of the FG cylindrical shell with bonded piezoelectric layers. They utilized the DQM in conjunction with the state-space technique. Sheng and Wang [93], reported on the investigation of thermoelastic vibration and buckling characteristics of FGP cylindrical shells. Li et al. [94-95], transformed the stress analysis of FGP hollow structures into Fredholm integral equations, deriving the distribution of radial and circumferential stresses as well as electrical responses, which were validated by comparison with analytical solutions in the form of power functions for functionally graded shells. Wu and Tsai [96], investigated the bending vibration of cylindrical shells made of FGPM through the application of a perturbation technique that relies on 3d piezoelectric principles. Wu et al. [97], presented exact solutions for simply supported doubly curved functionally graded elastic and piezoelectric shells employing a state space approach and considering 3D piezoelectric effects. Wang and Ding [98], investigated the coupling response of magneto-electro-elastic fields in a FGP and functionally graded piezomagnetic spherical occurred due to an impact load. The studies highlight the importance of parameters like geometric dimensions, and electrical boundary conditions, material properties in influencing the vibration characteristics of functionally graded piezoelectric shells, offering insights for design and application in piezoelectric devices.

## DISCUSSION

The examination of the vibration behaviour of FGPM structures (beams, plates, and shells), has become a crucial focal point in research because of the considerable benefits these materials provide in terms of improved electrical and mechanical performance. This discussion provides a comprehensive overview of the primary findings from recent research efforts, delineating the trends, challenges, and upcoming directions within this field. Investigations concerning FGPM beams have revealed that the variation in material properties, whether in the thickness or axial orientation, has a notable effect on their vibratory attributes. Notably, the higher-order natural frequencies and mode shapes are particularly influenced by the specific type of material gradation. Similarly, FGPM plates and shells display complex vibration responses as a result of the continual changes in material characteristics. From the literature survey it is found that analytical methodologies provide crucial insights but often face limitations due to simplified assumptions. These techniques prove most efficient when dealing with elementary geometries and boundary conditions, yet exhibit limitations when applied to complex FGPM configurations. Numerical approaches, notably the Finite Element Method (FEM), have emerged as the favoured strategy owing to their capacity for precise modelling of complex geometries and material variations. Investigations employing FEM have demonstrated notable precision in determining the vibration characteristics of FGPM structures. From a detailed review, the impacts of

various boundary conditions on the vibration behavior of FGPM structures are also explored. Different boundary conditions in beams change natural frequencies and mode shapes. Plates and shells also show similar trends with significant impact on dynamic response. Electrical and mechanical fields coupling in FGPM structures enhances their functionality. Material gradation improves electromechanical coupling effect in FGPMs. Recent FEM advancements improve accuracy and efficiency in vibration analysis for FGPM structures. The investigation of vibrations in FGPM structures has made considerable progress, driven by improvements in computational techniques and a deeper understanding of the impacts of material gradation. While complex modelling and validation of experimental results of these structures are still challenging.

Beyond the advantages of FGPM there are various material limitations also such as designing FGPMs with material gradients require continuous variation in properties demands careful selection and processing of piezoelectric materials. The production of FGPMs with varied material properties is complex and costly. Also, the selection of appropriate piezoelectric materials for FGPMs is constrained, with prevalent options being lead zirconate titanate (PZT) and polyvinylidene fluoride (PVDF). The attainment of desired material gradients may necessitate compromises among various performance metrics (e.g. sensitivity, stiffness, and actuation range).

## CONCLUSION

In conclusion, the in-depth review of vibration analysis in FGPMs underscores the remarkable advancements and utilization of these materials in today's engineering field. The literature survey of vibration analysis of FGPMs applied to beams, plates, and shells underscores the diverse capabilities and significant importance of these advanced materials in engineering applications. FGPMs, characterized by their gradual change in material characteristics, present unique benefits in customizing vibration and electromechanical performance to address specific needs, rendering them highly appropriate for use in intelligent structures, sensors, actuators, and energy harvesting systems. This chapter emphasizes the significance of diverse analytical, numerical, and experimental approaches in comprehending the dynamic performance of FGPMs. Analytical methodologies, offering profound insights and exact solutions, are frequently constrained by simplifying assumptions that may not fully represent intricate real-world conditions. Numerical techniques, notably finite element analysis (FEA), have emerged as potent mechanisms for depicting and replicating the complex behaviour of FGPMs under diverse loading and boundary conditions. This review's primary goal is to investigate how gradation profiles affect FGPM vibration frequencies. Various gradation profiles, whether linear, exponential, or alternative configurations, notably influence natural frequencies, mode patterns, and overall dynamic reactions. The interaction between mechanical and piezoelectric characteristics, regulated by the gradation, offers opportunities for enhancing material functionality for specific applications.

## Future Scope

This review also sheds light on the challenges and potential future gaps in the field of FGPM structures. Precise modelling of FGPM structures is one of the major challenge due to its coupled electromechanical effects and complex material behaviour. Modern computational methods of designing and material characterization can help to overcome these challenges. With high demand of smart materials, FGM materials plays pivotal role in the forthcoming era of intelligent systems. Thus, future scholars should focus on new material combinations, and new modelling techniques for developing efficient designs of FGPMs. The incorporation of structural functionality with active sensing and actuation capabilities in robots is exceedingly beneficial for the advancement of technology in different fields.

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