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Inverse Model for Predicting Thermal Abnormality of the Transient Process Triggered by Defective Electronics Element

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Abstract

The paper presents a model for predicting thermal abnormalities in Printed Circuit Boards (PCBs) by approximating the thermal process of electric elements with heating from point sources. Circuit board heat sources are defined as point and non-point. The point heat source is a small electronic heating element that can be approximated as having originated from one point on the circuit board. A non-point source is one with heat distributed over a large area of the electronic board. The non-point heat source is approximated with a set of point sources with distances that guarantee the accuracy with such replacement. This approximation allows the tshermal process to be defined from the set of arbitrary heat elements. The mathematical expression for heat exchange from such a set of point heat sources is provided. The downside of the proposed algorithm is an increase the complexity of the algorithm by raising the number of point sources. Future research directions for algorithm optimization, such as analytical methods for defining the level of discretization of continuous heat sources, are covered. Another promising direction reveals using the theory of hypernumber for solving heat exchange partial differential equation with a small amount of non-point sources approximation for sequential hypernumber solution. The adaptive event approach is used to compute the intensity of internal heat from the electronic element in the inverse engineering of the thermal process. The adaptive approach includes the minimization of the sum of the squared differences between the theoretical direct model and the temperatures identified from the infrared image for the corresponding points. The direct model for calculating the transient thermal process from multiple heating elements uses the Green function theory. The intensities of the heat sources are computed using linear matrix equations. Before simulation, the hot spots are identified by calculating the temperature gradient over time. The gradient that would exceed the defined threshold is the condition for defining the heat source parameters with an inverse model and simulating the prediction of the temperatures. The proposed mathematical approach can be used for electronics circuit board design to minimize the size of the board by solving the optimization problem of not exceeding the defined maximum temperature.

Keywords: Point heat source, green function, inverse model, hypernumber, least square

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INTRODUCTION

The prediction of thermal abnormalities in electronics is critical for a device that requires high reliability. The study of conjugate (conduction/convection) analysis for component temperature is introduced by Eveloy and Rodgers [1].

Thermal abnormality detection using Dynamic Hypernumber and Best Criteria Fits [2,3] is greatly suited for electronic devices where heat sources related to abnormality contribute significant additions to the normal temperature field. The advantage of such an analysis is related to the low computational complexity of this approach. Such conditions are often met because of the electronics component's minimization [4]. However, when the above condition for using an integrative approach [2,3] is not met, analysis of the individual sources is necessary.

Inverse thermal re-engineering with high accuracy of a direct model allows long-term thermal process prediction. Through reverse engineering, modeling identifies unknown parameters of heat intensity generated by electronic elements.

The inverse problem solution for multiple heat sources' intensity and location identification in transient heat conduction is defined by Niliot and Lefevre [5]. The model is based on a boundary integral formulation using space and time Green functions. The numerical approach is applied for inverse engineering in two- and three-dimensional spaces.

Inverse engineering of a PCB's thermal transient processes requires a direct model that includes boundary and initial conditions for a rectangular three-dimensional (3D) plate from the bottom and top of the plate, temperature solution for multiple inner heat sources. The circuit board heat h sources include one that can be approximated with point source and heated electronics elements that such approximation is not suitable. The sources with continuous heat distribution can be replaced with a finite number of discrete sources [6]. This concept is similar to the numerical integration. The increasing the pointed sources density over electronic element leads to higher accuracy in heat source approximation.

Due to presenting circuit board heat sources with a set of point sources heat exchange for multiple sources should be defined.

The two-dimensional (2D) transient heat conduction in a rectangular domain with heat sources under different combinations of temperature and heat flux boundary conditions is formulated with the Symplectic Superposition Method (SSM) [7].

In the research paper by Gaikward et al. [8] is provided the direct 3D model of the heat exchange in PCB of the rectangular parallelepiped geometry, the third type of boundary condition in the vertical z-direction, the first type of boundary condition in horizontal directions, and initial conditions for a point heat source.

- Defining Green function

  $$G(\overline{r},\overline{r}',t,\tau)\ \overline{r},\overline{r}' - space\ variables, t,\tau - times$$

  solving the homogeneous form of the heat exchange above using the integral transform and inversion formula [9].
- The solution of the nonhomogeneous problem [9] is

  $$T(\overline{r},t) = \int_R G_{\tau=0}F(\overline{r}')\, d\overline{r}' + \frac{\alpha}{k}\int_{\tau=0}^{t} d\tau \int_R g(\overline{r}',\tau)d\overline{r}' + \alpha\int_{\tau=0}^{t} d\tau \sum_{i=1}^{s}\int \frac{G}{k_i} f_i ds_i \qquad (1)$$

where $F(\overline{r}')$ - initial condition function, f_i - boundary condition function for boundary surface *i*, *g* – heat generation function, *s* – the amount of bounding surfaces of region.

The PCB overheating failure is related to any element producing an abnormal amount of heat due to internal defects. Electronic elements and their solder joints, such as resistors, capacitors, or inductors, can be considered as a pointed source. Failure of individual electronic elements and joint failure are often the causes of overheating [10]. Because heat in joins can approximated with point source, the Green's function can be used for detecting such thermal failure.

The VGG-16 deep neural network model is applied for identifying electrical hotspots [11]. The model shows an accuracy of 99.98% in the identification of electrical hotspots. The model applicability for rapid hotspot identification should be considered per neural network computational time.

Detecting Abnormal Heat Spot

A rectangular PCB with a hot spot is shown in Figure 1.

The hot spots are defined using infrared imaging (12, 13) and the ant colony algorithm (14).

The first step in predicting failure is defined by analyzing the gradient of changing the maximum temperature at a point analyzed per unit time:

$$\frac{\Delta T}{\Delta t} = \frac{T_{t=t_2} - T_{t=t_1}}{t_2 - t_1} \quad (2)$$

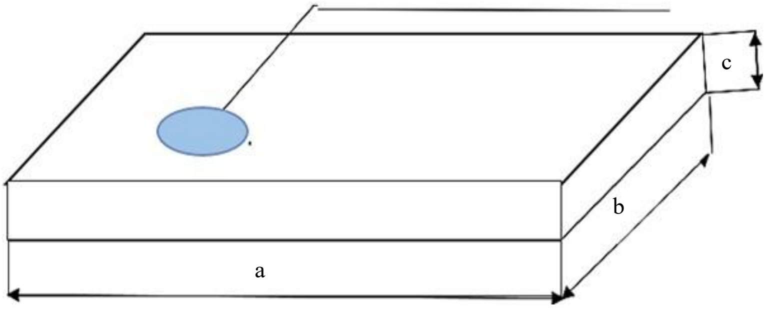

Figure 1. Schema of the rectangular circuit board with a hot spot.

Identifying Heat Source with Inverse Model

Figure 2 showed the thermal process at a rectangular plate

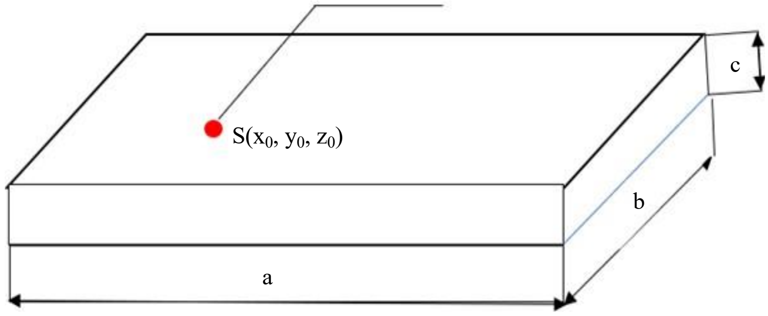

Figure 2. The thermal process at a rectangular plate with one heat source.

Per Gaikward et al.'s research paper [8], the temperature of the rectangular plate can be defined as:

$$T(t,x,y,z) = T_\infty + \int_{x'=0}^{a}\int_{y'=0}^{b}\int_{z'=0}^{c} G(x,y,z,t|x',y',z',\tau)|_{\tau=0}\,(T_0 - T_\infty)dx'dy'dz' + \frac{\alpha}{k_t}\int_{\tau=0}^{t}\int_{x'=0}^{a}\int_{y'=0}^{b}\int_{z'=0}^{c} G(x,y,z,t|x',y',z',\tau)g\,(x',y',z',\tau)\,dx'dy'dz' \quad (3)$$

Where α – is thermal diffusivity, k_t – thermal conductivity

$$\alpha = \frac{k_t}{\rho c} \quad (4)$$

where ρ is density and c is specific heat. The Green function $G(x, y, z, t|x', y', z', \tau)$ equals:

$$G(x, y, z, t|x', y', z', \tau) = \sum_{m=1}^{\infty}\sum_{n=1}^{\propto}\sum_{p=1}^{\infty}\frac{K_0(\beta_m, x)K_1(\gamma_n, y)K_2(\eta_p, z)}{S(\beta_m)S(\gamma_n)S(\eta_p)} \times$$

$$K_0(\beta_m, x')K_1(\gamma_n, y')K_2(\eta_p, z')exp(-\alpha\lambda_{mnp}^2 t), \quad (5)$$

where λ_{mnp}^2 equals:

$$\lambda_{mnp}^2 = \beta_m^2 + \gamma_n^2 + \eta_p^2 \quad (6)$$

$$K_0(\beta_m, x_0) = sin(\beta_m x_0), sin(\beta_m a) = 0 \quad (7)$$

$$K_1(\gamma_n, y) = sin(\gamma_n y), sin(\gamma_n b) = 0 \quad (8)$$

$$K_2(\eta_p, z) = \eta_p cos(\eta_p z) + h_{s1}\eta_p z, \quad (9)$$

$$\tan(\eta_p c) = \frac{\eta_p (h_{s1} + h_{s2})}{\eta_p^2 - h_{s1}h_{s2}}$$

Where h_{s1}, h_{s2} – coefficients of heat transfer equals:

$$h_{s1} = \frac{h_{c1}}{k_t} \quad (10$$

$$h_{s2} = \frac{h_{c2}}{k_t} \quad (11)$$

$$S(\beta_m) = \frac{a}{2} \quad (12)$$

$$S(\gamma_n) = \frac{b}{2} \quad (13)$$

$$S(\eta_p) = \frac{A_1}{2} \quad (14)$$

$$A_1 = \left(\left(\eta_p^2 + h_{s1}^2\right)\left(c + \frac{h_{s2}}{\eta_p^2 + h_{s2}^2}\right) + h_{s1}\right) \quad (15$$

Per expression 5 is obtained per assumption that the circuit board heat source is a step function $D(t)$ defined with expression:

$$g = D(t)\delta(x - x_0)\delta(x - x_0)\delta(y - y_0)\delta(z - z_0) \quad (16)$$

Multiple heat source parameter identification for a rectangular plate is derived from the equations (1-9) defined for a point source [8]. The analytical approach for heat exchange from multiple source is an extension of the research introduced in (Figure 3).

Let the heat sources be a set of the sources S^h

$$S^h \epsilon[S_1^h, ..S_i^h, ..., S_s^h] \quad (17)$$

The heat source function for this set of sources equals to

$$g(S^h) = \sum_{i=1}^{s} D_i(t)\delta(x - x_{i0})\delta(y - y_{i0})\delta(z - z_{i0}) \quad (18)$$

$$T(t, x, y, z) = T_0 + \sum_{i=1}^{s}\frac{D_i(t)}{k_t}\sum_{m=1}^{\infty}\sum_{n=1}^{\propto}\sum_{p=1}^{\infty}\frac{K_0(\beta_m, x)K_1(\gamma_n, y)K_2(\eta_p, z)}{S(\beta_m)S(\gamma_n)S(\eta_p)}$$

$$K_0(\beta_m, x_{i0})K_1(\gamma_n, y_{i0})K_2(\eta_p, z_{i0})\left(\frac{1-exp(-\alpha\lambda_{mnp}^2 t)}{\lambda_{mnp}^2}\right) \quad (19)$$

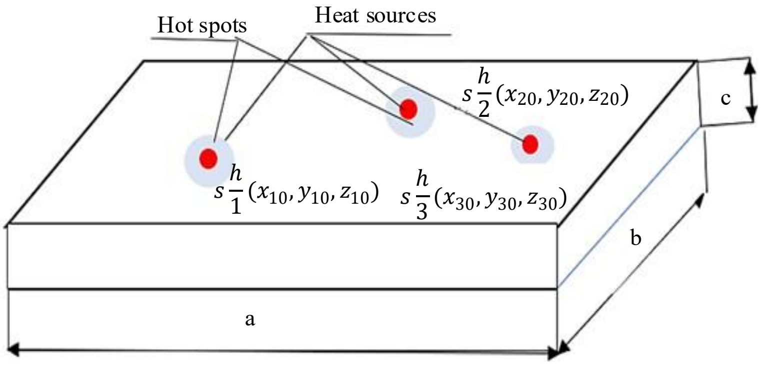

Figure 3. Circuit board with multiple hot spots.

Similarly, to [5] using the minimum least square method, the unknown intensity for each source can be found from collecting monitoring points at selected control point P by satisfying the expression.

$$L = min\sum_{l=1}^{k}\left(T_l^e(t_l, x_p, y_p, z_p) - T_l^t(t_l, x_p, y_p, z_p)\right)^2 \quad (20)$$

$$F_{i,l} = \frac{1}{k_t}\sum_{m=1}^{\infty}\sum_{n=1}^{\propto}\sum_{p=1}^{\infty}\frac{K_0(\beta_m, x)K_1(\gamma_n, y)K_2(\eta_p, z)}{S(\beta_m)S(\gamma_n)S(\eta_p)} \quad (21)$$

$$K_0(\beta_m, x_{i0})K_1(\gamma_n, y_{i0})K_2(\eta_p, z_{i0})\left(\frac{1-exp(-\alpha\lambda_{mnp}^2 t_l)}{\lambda_{mnp}^2}\right)$$

Where i – is an index of the hot spot and l – is an index of the sequential time.

$$\begin{cases} \frac{\partial L}{\partial D_1} = \sum_{l=1}^{k}\left(T_l^e(t_l, x_p, y_p, z_p) - (D_1F_{1,l} + \cdots + D_sF_{s,l})\right)\frac{\partial T_l^t(t_l, x_p, y_p, z_p)}{\partial D_1} = 0 \\ \dots \\ \frac{\partial L}{\partial D_j} = \sum_{l=1}^{k}\left(T_l^e(t_l, x_p, y_p, z_p) - (D_1F_{1,l} + \cdots + D_sF_{s,l})\right)\frac{\partial T_l^t(t_l, x_p, y_p, z_p)}{\partial D_j} = 0 \\ \dots \\ \frac{\partial L}{\partial D_s} = \sum_{l=1}^{k}\left(T_l^e(t_l, x_p, y_p, z_p) - (D_1F_{1,l} + \cdots + D_sF_{s,l})\right)\frac{\partial T_l^t(t_l, x_p, y_p, z_p)}{\partial D_s} = 0 \end{cases} \quad (22)$$

$$\begin{cases} \frac{\partial L}{\partial D_1} = \sum_{l=1}^{k}\left(T_l^e(t_l, x_p, y_p, z_p) - (D_1F_{1,l} + \cdots + D_sF_{s,l})\right)F_{1,l} = 0 \\ \dots \\ \frac{\partial L}{\partial D_j} = \sum_{l=1}^{k}\left(T_l^e(t_l, x_p, y_p, z_p) - (D_1F_{1,l} + \cdots + D_sF_{s,l})\right)F_{j,l} = 0 \\ \dots \\ \frac{\partial L}{\partial D_s} = \sum_{l=1}^{k}\left(T_l^e(t_l, x_p, y_p, z_p) - (D_1F_{1,l} + \cdots + D_sF_{s,l})\right)F_{s,l} = 0 \end{cases} \quad (23)$$

$$\begin{cases} \sum_{l=1}^{k} D_1 F_{1,l}^2 + \cdots + \sum_{l=1}^{k} D_j F_{j,l} F_{1,l} + \cdots + \sum_{l=1}^{k} D_s F_{s,l} F_{1,l} = \sum_{l=1}^{k} T_l^e F_{1,l} \\ \cdots \\ \sum_{l=1}^{k} D_j F_{1,l} F_{j,l} + \cdots + \sum_{l=1}^{k} D_j F_{j,l}^2 + \cdots + \sum_{l=1}^{k} D_s F_{s,l} F_{j,l} = \sum_{l=1}^{k} T_l^e F_{j,l} \\ \cdots \\ \sum_{l=1}^{k} D_s F_{1,l} F_{s,l} + \cdots + \sum_{l=1}^{k} D_s F_{j,l} F_{s,l} + \cdots + \sum_{l=1}^{k} D_s F_{s,l}^2 = \sum_{l=1}^{k} T_l^e F_{s,l} \end{cases} \quad (24)$$

The approach outlined above covers pointed heat sources. Solder joints, miniature capacitors, transistors, and other small components can be considered as point sources. However, microchips (microcontrollers, microprocessors, etc.) with significant dimensions can't be considered as a point source. Larger elements can be approximated by a discrete set of pointed sources [15-20].

Non-pointed Heat Sources Approximation with a Set of Pointed Sources

Figure 4 showed the schema of the circuit board with non-pointed heat sources.

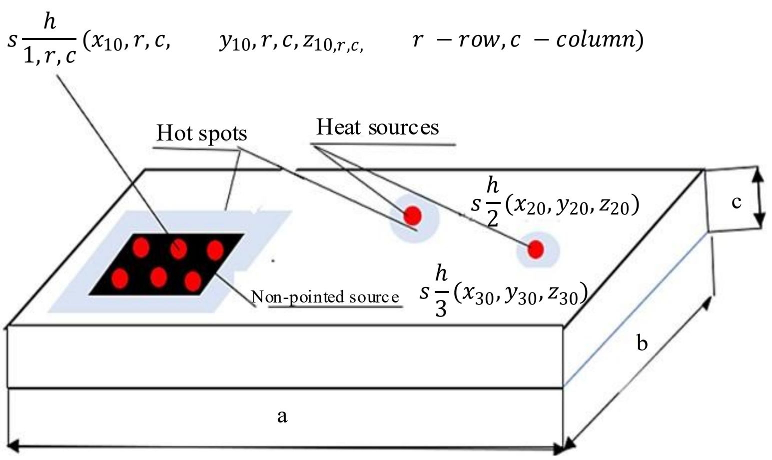

Figure 4. The schema of the circuit board with non-pointed heat sources.

$$T(t,x,y,z) = T_0 + \sum_{i=1}^{s} \frac{D_i(t)}{k_t rc} \sum_{ps=1}^{r\times c} \sum_{m=1}^{\infty} \sum_{n=1}^{\propto} \sum_{p=1}^{\infty} \frac{K_0(\beta_m, x) K_1(\gamma_n, y) K_2(\eta_p, z)}{S(\beta_m) S(\gamma_n) S(\eta_p)} \quad (25)$$

$$K_0(\beta_m, x_{i0,r,c}) K_1(\gamma_n, y_{i0,r,c}) K_2(\eta_p, z_{i0,r,c}) \left(\frac{1 - exp(-\alpha \lambda_{mnp}^2 t)}{\lambda_{mnp}^2} \right)$$

Where indexes r, c – are the row and column of the pointed source location in the non-pointed electronics element i. The amount of these elements is equal to $r \times c$

$$F_{i,l} = \frac{1}{k_t} \sum_{ps=1}^{r\times c} \sum_{m=1}^{\infty} \sum_{n=1}^{\propto} \sum_{p=1}^{\infty} \frac{K_0(\beta_m, x) K_1(\gamma_n, y) K_2(\eta_p, z)}{S(\beta_m) S(\gamma_n) S(\eta_p)} \quad (26)$$

$$K_0(\beta_m, x_{i0,r,c}) K_1(\gamma_n, y_{i0,r,c}) K_2(\eta_p, z_{i0,r,c}) \left(\frac{1 - exp(-\alpha \lambda_{mnp}^2 t_l)}{\lambda_{mnp}^2} \right)$$

CONCLUSION

The paper covers the approach of defining multiple source heat parameters for Printed Circuit Board temperature analysis.

In many cases, some neighboring heating sources can be ignored because of the significance of the sources. These exclusions depend on the intensity of the heat from the source and the distance from the validation source. The algebra of defining significant heat sources is a new theory that is subject to future research.

The proposed direct model is an extension of computing the transient heat exchange with one pointed heat source on a rectangular plate to multiple sources. The method, as shown, can be extended to a complex electronic element by approximating the element as the sum of the point heat source

The heat transfer coefficients are usually unknown and can be defined using an inverse engineering approach covered in this paper. The equations for the identification of the source heat intensity and heat transfer coefficients will be nonlinear. The solution of non-linear equations for reverse engineering of the heat source strength and source location [5] is done using numerical algorithms. The limitations of using numerical methods for real-time are related to the time complexity. The nonlinear equations can be solved using the hypernumber theory analytical method [15-18]. In addition to hypernumber methods to solve nonlinear equations, the modified Newton (Kantorovich) method can be used if the convergence conditions are satisfied [19, 20]. The conditions can be verified in real time. The advantage of Kantorovich's method is the rapid computation time. However, the hypernumber method, unlike the Kantorovich method, guarantees the convergence of the solution.

The application of the method may require a significant number of point sources leading to high computational complexity. The algorithm can be optimized by identifying the amount of point sources that satisfy to required accuracy. It can be done by finding an expression for the sum of deltas squares between temperatures calculated from continuous and approximated point heat sources. From such expression can be calculated minimum amount of points that satisfy to inequality: the expression for such sum of deltas divided by an amount of sampling temperature points to be equal or less than targeting one.

The method can also be used to minimize the size of electronic devices in automated circuit board design. The minimum distance between components should not cause any circuit board temperature to exceed the defined unsafe level.

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