

Optimal PMU Placement in Power Systems Using Graph Theory and PSAT

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Abstract

Phasor measurement units (PMUs) play a vital role in modern power systems by delivering synchronized, real-time measurements. These devices enhance system reliability by supporting functions such as monitoring, protection, and control. By accurately capturing voltage and current phasors across different locations, PMUs enable better situational awareness and more effective decision-making in grid operations and management. Determining the optimal location of PMUs is essential to ensure system observability, reduce installation costs, and improve network reliability and efficiency. It was discovered that there is no one methodology that is always better; rather, the best approach is determined by the needs and limitations of the system. Instruction-Level Parallelism (ILP) and heuristic methods are effective for small to medium-sized networks, whereas metaheuristic and AI-based approaches are more suitable for large and complex networks. This paper determines the optimal placement of PMUs using graph theory methods implemented within the power system analysis toolbox (PSAT). By applying these analytical techniques, the study enhances the monitoring and observability of power systems, ensuring more efficient data acquisition and system analysis for improved reliability and performance.

Keywords: PMU, optimal placement problem, integer linear programming, heuristic algorithms, genetic algorithm, particle swarm optimization, power system observability

INTRODUCTION

The integration of PMUs in power systems has significantly enhanced monitoring and control mechanisms. The PMUs provide high-resolution, time-synchronized measurements that facilitate better situational awareness and stability in power systems. However, because of the cost and complexity of PMU deployment, determining the optimal placement is critical. Owing to technological advancements and improvements in global living standards, electricity demand is increasing. To satisfy this requirement, the power output must be increased. However, problems such as the scarcity of fossil fuels, environmental concerns, and the challenge of constructing new power plants make it necessary to integrate renewable energy sources into the grid and effectively manage the current transmission and distribution infrastructure. To monitor the state of the system, accurate measuring tools must be deployed. Historically, supervisory control and data acquisition (SCADA) systems have been used to

accomplish this. The most popular use of SCADA is for asynchronous and delayed-measurement-based power grid status estimation. The slow duty cycle and communication bias of SCADA-based measurements can cause delays and other problems. A solution to the problems and constraints of SCADA is wide-area monitoring, protection, and control (WAMPAC). The phasor measurement unit (PMU), the central component of the WAMPAC, enables synchronized phasor measurements of voltage and current in the power grid, as well as real-time computation. Global Positioning System (GPS) offers accurate time-reference signals that

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allow PMUs to attain authenticity and precision. The measurements of the voltage and current phasors were synchronized during this time. PMUs have been increasingly used to enhance power grid monitoring. However, PMUs are expensive devices; therefore, optimizing their number and placement is essential. Instead of installing these devices on every bus, PMUs can be strategically placed on selected buses to achieve optimal coverage and observability at minimal cost [1–3]. In this study, the power system analysis toolbox (PSAT) was used for the placement of the PMU.

PHASOR MEASUREMENT UNITS IN THE POWER GRID

Phasor measurement units (PMUs) are critical components in the modern management of power grids because of their ability to provide synchronized, real-time measurements of key electrical parameters, such as voltage and current phasors. These devices play an essential role in enhancing the reliability, stability, and efficiency of the electrical power systems. What distinguishes PMUs from traditional SCADA systems is their use of GPS signals for precise time synchronization. This GPS-based synchronization enables the PMUs to collect data at very high sampling rates with exceptional accuracy, allowing operators to observe dynamic changes in the grid with minimal delay. As a result, PMUs support better situational awareness, facilitate faster and more informed decision-making, and contribute to improved fault detection and system protection. While SCADA systems typically offer slower and less coordinated monitoring capabilities, PMUs provide time-stamped data that can be synchronized across multiple locations. This makes them particularly valuable in wide-area monitoring systems (WAMS), where real-time grid visibility is crucial for maintaining stability in increasingly complex and distributed power networks [4, 5].

Advantages of PMUs over SCADA Systems

1. *Real-time data*: PMUs provide real-time data, offering a more immediate understanding of the grid's status than SCADA systems' slower, delayed measurements.
2. *High authenticity*: GPS-based synchronization in PMUs ensures that measurements from different locations are time-stamped, enabling a more accurate analysis of the grid status.
3. *Improved observability*: PMUs enable better visibility of the power grid, allowing quicker and more effective responses to disruptions and operational challenges.

Challenges and Optimization

Although PMUs provide substantial advantages for power system monitoring and control, their high implementation costs require a carefully optimized deployment strategy. The primary goal is to ensure full or near-complete observability of the electrical grid using the fewest possible number of PMUs. This is achieved by strategically placing units at key locations within the network to maximize coverage and data accuracy. This approach balances performance with cost efficiency, making large-scale deployment more feasible. The optimization process involves various factors, including system topology, measurement redundancy, and critical node identification, as detailed in the following considerations [6, 7].

1. *Optimal locations*: Determine the most effective locations for the PMUs to maximize grid observability and minimize redundancy.
2. *Cost-benefit analysis*: Balancing the benefits of improved grid monitoring and control with the financial cost of the PMUs.
3. *Integration with existing systems*: Ensuring PMUs complement SCADA systems and other infrastructure to provide a comprehensive monitoring solution.

The deployment of PMUs across India is growing rapidly as part of efforts to strengthen power grid monitoring and management. PMUs play a crucial role in enhancing the reliability, stability, and real-time observability of electrical grids. By providing synchronized measurements of the electrical parameters, PMUs allow operators to detect disturbances and respond to dynamic changes in the grid with greater precision and speed. However, given the high cost associated with each PMU, their

placement across the network is being carried out in a strategic and optimized manner. The goal is to achieve the highest possible level of system observability while minimizing the number of units required. This involves detailed studies to determine the optimal locations for installation, considering the topology of the grid and operational scenarios. Ongoing research and pilot initiatives address the complex challenges associated with PMU deployment, such as the presence of zero-injection buses, which do not inject current into the system, and single-line contingencies that may affect grid stability and observability under fault conditions.

FORMULATION OF THE OPP PROBLEM

At the bus where it is mounted, a PMU measures the voltage and current phasor values. Figure 1 shows all the components of the WAMPAC system, including the PMUs connected to the GPS. The measured and time-stamped phasor signals are sent via the PMUs to a unit called the Phasor Data Concentrator (PDC). The purpose of PDC is to collect raw phasor data and synchronize it over time to generate consistent real-time information. The system signal processor converts phasor data from the PMUs into useful information. A human-machine interface can display this information to the operator, enabling them to easily monitor and record critical grid conditions and take corrective actions in the future. The operator can fully observe the network operation by placing PMUs on the calculated bus locations. These PMUs can measure the current phasors in each branch that is connected to the bus, as well as the voltage phasor at each bus. Therefore, increasing or achieving network observability, rather than merely lowering the number of PMUs, is the goal of Optimal PMU Placement (OPP), which aims to maximize the number of deployed PMUs [8–10].

$$F = \text{Min } \sum w_i x_i \quad (1)$$

Thus, the problem is to determine a solution vector with a minimum number of x_i that satisfies Equation (1). A binary connection matrix A (matrix A) is used to create the constraint optimization function. The grid bus connectivity information is provided by this matrix. The placement problem of the current flow and injection PMUs can be further formulated with constraints on the constraints and optimization function. Some of these constraints are as follows.

1. Effect of conventional measurements,
2. Single or multiple PMU loss contingencies,
3. Single branch outages,
4. Single-line disconnection contingencies,
5. Effect of PMU channel limits, and
6. Single PMU loss.

RESULTS AND DISCUSSIONS

Figure 1 shows the voltage magnitude profile, and Table 1 presents the power flow analysis. Figure 2 shows a graphical representation of the voltage phase profile, and Table 2 lists the measurements from the PMU.

Additionally, Figure 3 depicts a graph representation of the “d-024-md1” power network. Here, each bus is modelled as a node, and each transmission line is an edge connecting the nodes. This graph-based model streamlines the application of graph theory for phasor measurement unit (PMU) placement. The network exhibited a meshed structure. It provides multiple paths between buses, augmenting the system’s robustness and intensifying the complexity of achieving full observability with a minimal number of PMUs. High-degree buses, which are connected to multiple neighboring buses, are recognized as key candidates for PMU installation. By deliberately selecting these buses, it is possible to observe neighboring nodes indirectly through graph connectivity rules. The graphical approach not only offers visual clarity but also helps to systematically apply the placement algorithm to achieve optimal system observability with minimized resource usage.

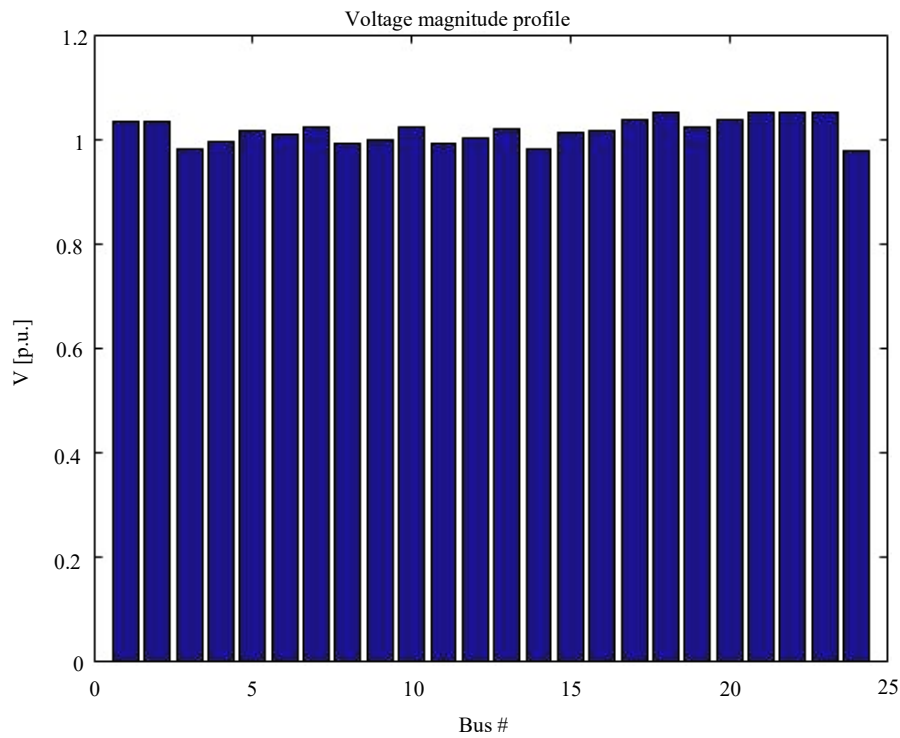


Figure 1. Voltage profile for 24 bus system.

Table 1. Power flow analysis result.

Bus no.	Voltage (p.u.)	Phase angle (rad.)	P (p.u.)	Q (p.u.)
1	1.035	-0.12691	0.64	0.05431
2	1.035	-0.12876	0.75	-0.01861
3	0.9824	-0.08705	-1.8	-0.37
4	0.9968	-0.16973	-0.74	-0.15
5	1.0167	-0.17503	-0.71	-0.14
6	1.0095	-0.21883	-1.36	-1.2991
7	1.025	-0.13084	1.15	0.28897
8	0.99141	-0.19571	-1.71	-0.35
9	0.9993	-0.13061	-1.75	-0.36
10	1.0248	-0.16815	-1.95	-0.4
11	0.9906	-0.04718	0	0
12	1.0016	-0.0248	0	0
13	1.02	0	-0.77677	0.83863
14	0.98	0.0134	-1.94	-0.47432
15	1.014	0.2274	-1.02	-0.69127
16	1.017	0.20943	0.55	0.10542
17	1.0383	0.28704	0	0
18	1.05	0.31038	0.67	0.72151
19	1.0232	0.17648	-1.81	-0.37
20	1.0385	0.18194	-1.28	-0.26
21	1.05	0.32498	4	1.0776
22	1.05	0.42365	3	-0.2927
23	1.05	0.19736	6.6	1.4146
24	0.97921	0.11172	0	0

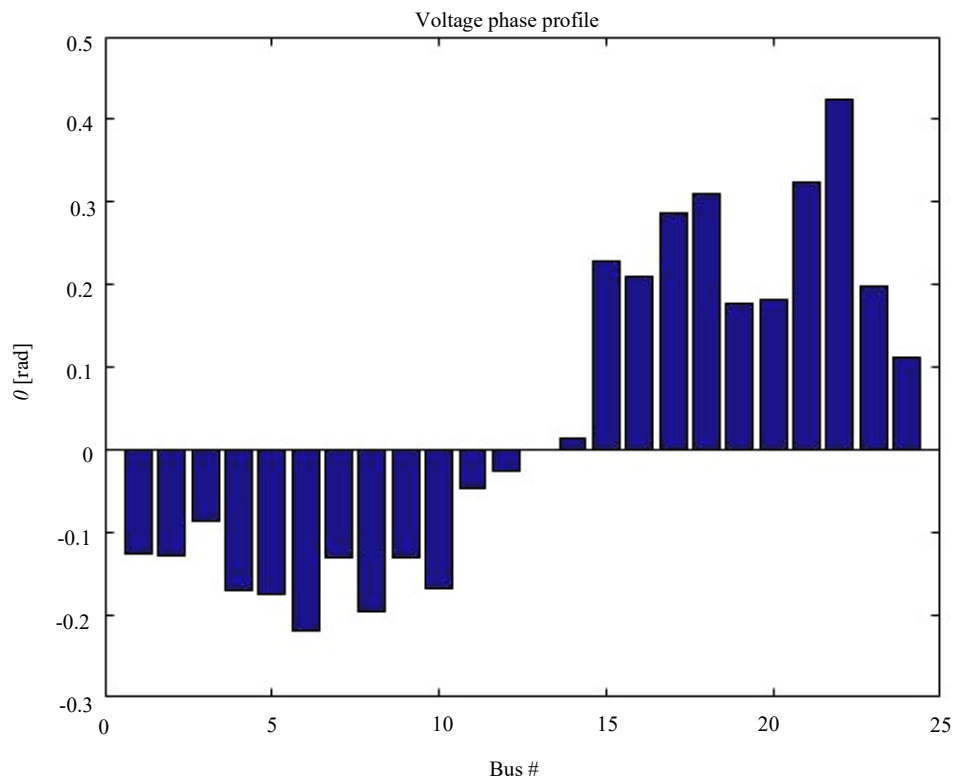


Figure 2. Angle profile of 24 bus system.

Table 2. Measurements from PMU.

Estimated V	Estimated theta	PMU bus location
1.004	0.00481	Bus 1
PMU 6 #1.0047	PMU 6# 0.00859	PMU 6 Bus 2
1.0024	-0.05492	Bus 3
0.99028	-0.06108	Bus 4
0.99166	-0.03502	Bus 5
0.97922	-0.10189	Bus 6
PMU 8 1.0006	PMU 8 #-0.00575	PMU 8 Bus 7
0.98508	-0.0775	Bus 8
PMU 3 0.99727	PMU 3 #-0.03276	PMU 3 #Bus 9
PMU 2 0.9994	PMU 2 #-0.01263	PMU 2 Bus 10
0.99912	-0.13735	Bus 11
0.99907	-0.13697	Bus 12
1.0008		Bus 13
		Bus 14
		Bus 15
PMU 5 1.0096	PMU 5 #0.06167	PMU 5 Bus 16
1.0017	0.00988	Bus 17
1.0076	0.04772	Bus 18
1.0056	0.03525	Bus 19
1.0041	0.02273	Bus 20
	PMU 1 #0.08202	
	0.03107	
PMU 1 # 1.0141	PMU 4 # 0.04488	PMU 1 #Bus 21
	PMU 7 # 0.14215	Bus 22
		PMU 4# Bus 23
		# Bus 24

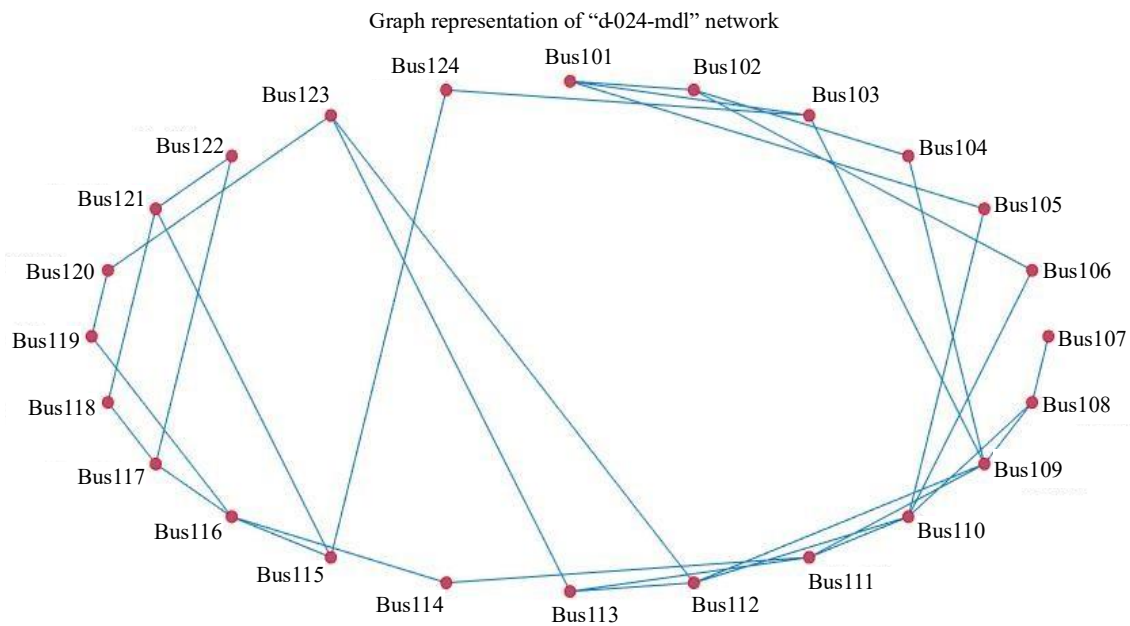


Figure 3. Graph representation.

Actual PMU 8

Non-observable buses -0

Applications

These voltage and angle measurements are used for the following purposes:

1. *Power flow analysis:* To study the flow of electricity across different parts of the system.
2. *Voltage stability assessment:* To assess whether the system is secure from a stability perspective.
3. *Fault detection:* To identify issues in case of any electrical faults.
4. *Real-time monitoring:* To keep an eye on how the electrical power system is operating right now.

Through these measurements, the performance and stability of the power system were monitored to ensure reliability and efficiency of the electricity supply.

CONCLUSION

In this study, a graph-theoretical approach was successfully applied to the optimal deployment of PMUs using PSAT. A graph of buses and transmission lines was used to represent the network. The strategy decreases the number of PMUs while guaranteeing full system observability. Thereby, the cost-effectiveness is increased. The graphical illustration offers a clear system connectivity. This aids in making smart deployment choices. This method validates that the graph theory, collectively with the PSAT tool, proposes an influential structure for boosting system observability in modern grids. Future research should consider factors such as installation cost.

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