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Harmonically Varying Moving Load on Time-Dependent Uniform Beam Resting on Pasternak Foundation

Jimoh A.[1], A. Joge E.O.[2,]*, Ajiola D.I.[3], Oni D.I.[4], Abiola Kolawole.[5], Adesanmi O.A.[6]

Abstract

In this paper, we investigated elastic beams whose properties do not vary with spatial coordinate but are constant along the span L of the beam. The moving load considered in this work is a harmonically varying moving load with non-classical boundary conditions time-dependent boundary conditions. Also, considered in this work is two parameters foundation, which are Winkler and Pasternak foundations. Closed-form solutions in plotted form are obtained using Mindlin–Goodman method, Fourier series transformation, integral transformation, and the theorem of convolution. Mindlin–Goodman method is used to transform the non-homogeneous boundary conditions to homogeneous boundary conditions. The Fourier series transformation is used to reduce the fourth-order non-homogeneous partial differential equation with singular coefficient describing the dynamical system to second order ordinary differential equation. The resulting equation is solved using the integral transformation alongside the theorem of convolution. The results are presented graphically, and it was revealed that, as the structural parameters such as Winkler foundation parameter (K), Pasternak foundation parameter (G), damping coefficient (E), axial force (N), and circular frequency (w), the response amplitude of the time-dependent uniform elastic beam reduces. Amongst all the structural parameters, shear modulus (G) and axial force (N) give more noticeable effects compared to other structural parameters. Higher values of G and N are required to reduce the resonance effect in the dynamical system, and this could guarantee the safety of lives.

***Author for Correspondence**
A. Joge E.O.
E-mail: eajoge@oauife.edu.ng

[1]Research Professor, Department of Mathematics and Statistics, Conference University of Science and Technology, Osara, Kogi State, Nigeria
[2]Research Scholar, Centre for Energy Research and Development, Obafemi Awolowo University, Ile-Ife, Osun State, Nigeria.
[3]Research Scholar, Department of Biochemistry, Chemistry, and Physics. College of Science and Mathematics, Georgia Southern University, 1332 Southern Drive Statesboro, GA 30458. USA.
[4]Research Scholar, Department of Informatics and Computer Engineering, Vietnam National University Internation School, Vietnam.
[5]Research Scholar, Department of Chemical Engineering, University of Hull, United Kingdom.
[6]Research Scholar, Department of Mathematics, Obafemi Awolowo University, Ile-Ife, Nigeria.

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INTRODUCTION

The dynamic behavior of elastic structures such as beams, plates, and shells on an elastic foundation subjected to moving loads, such as railroad rails to moving trains, response of bridges and elevated roadways to moving vehicles, rudimentary surfaces for aircraft, and guided missiles, has been investigated by many researchers in the fields of mathematical physics, physics, and applied mathematics. As a result of modern trends toward higher speeds in the field of railway engineering, accurately predicting the behavior of railway tracks has further intensified research in these areas. In most studies, only one-parameter foundation model [1–11] is considered. The one-parameter foundation consists of closely spaced linear strings subjected to a moving load. It has a short coning because it is independent of its vertical springs, which allows unloaded parts in the springs. It does not consider the bending deformation of the foundation itself.

To overcome the shortcomings of the one-parameter foundation model, we introduce some kind of interaction between independent springs with a constant parameter that characterizes the interaction between the springs. This is called the two-parameter foundation (Pasternak foundation). A two-parameter foundation model was proposed by Pasternak [12]. In view of this, Baran [13] used the transfer-matrix method to obtain a solution to the dynamic response of a simply supported Timoshenko beam resting on two-parameter foundations subjected to a moving load. He concluded from his findings that the response amplitude of the one-parameter foundation was slightly higher than that of the two-parameter foundation. His findings also revealed that the one-parameter foundation model has a lower natural frequency than the two-parameter foundation model. Free vibration analysis of clamped-clamped and simply supported beams resting on two-parameter foundations with axial material graduation was investigated by Saurabh [14]. The governing equation describing the dynamical system was derived using Timoshenko beam theory and the energy method, along with Hamilton's principle. The effects of the material model, foundation parameter, and boundary conditions on the dynamic behavior of the beam were considered in their work. His findings revealed that the effects of other parameters significantly reduce as the values of the Winkler foundation parameter increase. The dynamic response of a thick beam resting on a bi-parametric foundation subjected to a moving load was investigated by Jimoh [15]. Modal asymptotic analysis and the Struble asymptotic technique were used to obtain a closed-form solution to the dynamic problem. The effects of the prestress, foundation parameters, and moving velocity on the dynamic response of the beam were considered. The results revealed that the foundation stiffness and excitation frequency affected the critical speed of the dynamic system, and that the deformation shape of the beam strongly depended on the speed of the moving load.

In a very recent development, Giogio et al. [16] used finite element modelling of a beam resting on an elastic foundation. They based their method on a variable-reduction approach. They considered the Winkler, two-parameter, and Hetenyi foundation models. Jimoh et al. [17] investigated the effect of a two-parameter foundation on an elastic beam subjected to a concentrated load with clamped-clamped end conditions. They employed a generalized integral transform approach alongside a series representation of the Dirac-Delta function, asymptotic method, integral transformation, and convolution integral to obtain the results in plotted graphs. They concluded from their findings that the effect of shear modulus on the response amplitude of the beam is more noticeable compared to the foundation stiffness, and that the resonance effect is reached earlier in the moving mass problem than in the moving force problem. Some of the other researchers who considered two-parameter foundations in their works are Lucas et al. [18], Jimoh and Ajoge [19], Ajijola [20], Hammed et al. [21], and Awodola et al. [22].

It is interesting to note that all the researchers considered only classical boundary conditions, such as simply supported, clamped-clamped, and clamped-free boundary conditions that are homogeneous in nature, whereas in real-life situations, the boundary conditions are not homogeneous. Frequently, the problem of vibration deals with continuous systems whose one or more boundaries are not stationary but undergo a sort of displacement that varies with time. Such boundary conditions are called non-classical boundary conditions, which are time-dependent boundary conditions and elastically supported boundary conditions. Research work involves non-classical boundary conditions, time-dependent, where the moving load is harmonically maintained in the literature. This is because of the difficulties involved in finding solutions to the governing equations describing such a dynamic system.

In the present analysis, a harmonically varying moving load on a uniform elastic beam resting on a two-parameter foundation with time-dependent boundary conditions was investigated.

GOVERNING EQUATION

The dynamic response $V(x,t)$ of a uniform elastic beam resting on a two-parameter foundation subjected to a harmonically varying moving load with time-dependent boundary conditions was governed by Frybal [23].

$$EI\frac{\partial^4 V(x,t)}{\partial x^4}+\mu\frac{\partial^2 V(x,t)}{\partial t^2}-N\frac{\partial^2 V(x,t)}{\partial x^2}+\alpha\frac{\partial V(x,t)}{\partial t}+KV(x,t)-G\frac{\partial^2 V(x,t)}{\partial x^2}$$
$$=Pcos\gamma t\;\delta(x-ct) \tag{1}$$

Where
EI= flexural rigidity of the structures; μ = mass per unit length of beam.
K = foundation modulus, G = Shear modulus, N= axial force,
$Alpha\ (\alpha)$ = damping coefficient, *E*= Young's modulus, I = moment of inertia
V(x,t)= transverse displacement, *x* = spatial coordinate and *t* = time coordinate
$Omega\ (\gamma)$ = circular frequency of moving load. c = velocity of moving load.

The boundary conditions corresponding to Equation (1) were time-dependent.
Thus, at each boundary point, there are two boundary conditions.

$$D_i[V(0,t)]=f_i(t), i=1,2 \tag{2}$$
$$D_i[V(L,t)]=f_i(t), i=1,2 \tag{3}$$

Where, D_i are differential operators of the order less than or equal to three.
For example, if the beam in question has a simple support at both ends, then

$$D_1=1, D_2=\frac{\partial^2}{\partial x^2}, D_3=1\ D_4=\frac{\partial^2}{\partial x^2} \tag{4}$$

The initial conditions of the motion are specified by the functions

$$V(x,0)=0=\frac{\partial V(x,0)}{\partial t} \tag{5}$$

METHODS OF SOLUTION

To solve the above initial boundary-valued problem, we introduce the auxiliary variable $U(x,0)$ in the form

$$V(x,0)=\ U(x,0)+\textstyle\sum_{i=0}^{4} f_i(t)\,g_i(x) \tag{6}$$

The substitution of Equation (6) into the boundary-valued problems Equations (1), (2), and (3) transforms the latter into boundary-valued problem terms of $U(x,0)$. The functions $g_i(x)$ are chosen to render the boundary conditions for the boundary-valued problem in $U(x,0)$ homogeneous [24].

Substituting Equations (6) into (1) after simplification yields

$$EI\frac{\partial^4 V(x,t)}{\partial x^4}+\mu\frac{\partial^2 V(x,t)}{\partial t^2}-N\frac{\partial^2 V(x,t)}{\partial x^2}+\alpha\frac{\partial V(x,t)}{\partial t}+KV(x,t)-G\frac{\partial^2 V(x,t)}{\partial x^2}=\ Mgcos\gamma t\;\delta(x-ct)-$$
$$\sum_{i=1}^{4}\left\{\begin{matrix} f_i(t)EI\frac{\partial^4 g_i(x)}{\partial x^4}+\mu g_i(x)\ddot{f}_i(\mathrm{t})+\ \sum g_i(x)\dot{f}_i(t) \\ f_i(t)N\frac{\partial^2 g_i(x)}{\partial x^2}\ \ +f_i(t)Kg_i(x)-\ f_i(t)g\frac{\partial^2 g_i(x)}{\partial x^2}\end{matrix}\right\} \tag{7}$$

Where

$$\mathrm{P}=\mathrm{Mg} \tag{8}$$

Now, the assumed expression Equation (6) must satisfy boundary conditions Equations (2) and (3).

Thus,

$$D_i[U(0,t)] + \sum_{j=0}^{4} f_i(t)\, D_i[g_i(0)] = f_i(t),\ i = 1,2 \tag{9}$$

$$D_i[U(L,t)] + \sum_{j=0}^{4} f_i(t)\, D_i[g_i(L)] = f_i(t),\ i = 3,4 \tag{10}$$

Substituting Equation (6) into the initial conditions, Equation (5), to obtain

$$U(x,0) = -\sum_{i=0}^{4} f_i(0)\, g_i(x) \tag{11}$$

$$\frac{\partial U(x,0)}{\partial t} = -\sum_{i=0}^{4} f_i(0)\, g_i(x) \tag{12}$$

Using the Mindlin–Goodman method, the boundary conditions Equations (9) and (10) in terms of $U(x,t)$ can be made homogeneous if the function $g_i(x)$ is chosen such that the conditions given by:

$$D_i\left[g_j(0)\right] = \delta_{ij}(i = 1,2\ j = 1,2,3,4) \tag{13}$$

$$D_i\left[g_j(L)\right] = \delta_{ij}(i = 1,2\ j = 1,2,3,4) \tag{14}$$

are satisfied.

Where

$$\delta_{ij} = \begin{cases} 0, i \neq j \\ 1, i = j \end{cases} \tag{15}$$

Equations (13) and (14) are used in the non-homogeneous boundary conditions Equations (9) and (10) to obtain the homogeneous boundary conditions as follows:

$$D_i[U(0,t)] = 0, i = 1,2 \tag{16}$$

$$D_i[U(L,t)] = 0, i = 1,2 \tag{17}$$

The original problem is reduced to that of solving the non-homogeneous partial differential Equation (7) subject to homogeneous boundary conditions, Equations (16) and (17), with initial conditions Equations (11) and (12).

To solve the initial boundary-valued problems Equations (7), (16), and (17) above, one employed the method of generalized finite integral transform defined by

$$U(m,t) = \int_0^L U(x,t) \sin\frac{m\pi x}{L}\, dx, m = 1,2,3 \tag{18a}$$

With the inverse

$$U(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} U(m,t) \sin\frac{m\pi x}{L} \tag{18b}$$

By substituting Equations (18a) into (7) after simplification and rearrangement, we obtain

$$\mu U_{tt}(m,t) + \alpha U_t(m,t) + \left(\alpha_a^2 + \alpha_b^2 + \alpha_c^2 + K\right) U(x,t) = Mg cos\gamma t\, \delta(x-ct)$$

$$\left[Q_a(t) + Q_b(t) + Q_c(t) + Q_d(t) + Q_e(t) + Q_f(t)\right] \tag{19}$$

Where

$$\alpha_a^2 = EI\frac{m^4\pi^4}{L^2}, \alpha_b^2 = N\frac{m^2\pi^2}{L^2}, \alpha_c^2 = G\frac{m^2\pi^2}{L^2} \tag{20}$$

$$Q_a(t) = \sum_{i=1}^{4} EI f_i(t) \int_0^L \frac{\partial^4 g_i(x)}{\partial x^4} Sin\frac{m\pi x}{L}\, dx \tag{21a}$$

$$Q_b(t) = \sum_{i=0}^{4} \mu \ddot{f_i}(t) \int_0^L g_j(x) Sin\frac{m\pi x}{L}\, dx \tag{21b}$$

$$Q_c(t) = \sum_{i=1}^{4} E\,\dot{f}_i(t) \int_0^L g_j(x) Sin\frac{m\pi x}{L}\,dx \qquad (21c)$$

$$Q_d(t) = \sum_{i=1}^{4} N f_i(t) \int_0^L \frac{\partial^2 g_i(x)}{\partial 2} Sin\frac{m\pi x}{L}\,dx \qquad (21d)$$

$$Q_e(t) = \sum_{i=1}^{4} K\, f_i(t) \int_0^L g_j(x) Sin\frac{m\pi x}{L}\,dx \qquad (21e)$$

$$Q_f(t) = \sum_{i=1}^{4} G f_i(t) \int_0^L \frac{\partial^2 g_i(x)}{\partial 2} Sin\frac{m\pi x}{L}\,dx \qquad (21f)$$

Evaluating Equations (21a)–(21f) and substituting the results into Equation (19), after simplification and rearrangement, we obtain

$$U_{tt}(m,t) + A_1 U_t(m,t) + A_2 U(m,t) = \frac{Mg}{\mu} cos\gamma t\; Sin\frac{m\pi ct}{L} - (N_1 + N_2 + N_3) \qquad (22)$$

Where

$$A_1 = \frac{E}{\mu},\, A_2 = \frac{\alpha_a^2+\alpha_b^2+\alpha_c^2+K}{\mu},\, A_3 = \frac{K}{\mu} \qquad (23a)$$

$$N_1 = \ddot{f}_i(\mathrm{t})\,(I_1 - \frac{I_2}{L}) + \dot{f}_3(t)\frac{I_2}{L} \qquad (23b)$$

$$N_2 = A_1\left(\dot{f}_1(t)(I_1 - \frac{I_2}{L}) \;+\; \dot{f}_3\frac{I_2}{L}\right) \qquad (23c)$$

$$N_3 = A_3\left(f_i(t)(I_1 - \frac{I_2}{L}) \;+\; f_3(t)\frac{I_2}{L}\right) \qquad (23d)$$

$$I_1 = \frac{L}{m\pi}(1 - cosm\pi) \qquad (23e)$$

$$I_2 = \frac{L^2}{m\pi}((-1)^{m+1}) \qquad (23f)$$

Equation (22) is the transformed equation of the time-dependent uniform elastic beam on two parameters: the Pasternak foundation is subjected to a harmonically varying moving load.

Analysis of the Transformed Equation

Recall that we can write.

$$f_1(t) = Sin\Omega\mathrm{t} \text{ and } f_3(t) = e^{-\beta t} Sin\Omega\mathrm{t} \qquad (24)$$

Where Ω and β are frequency and parameter, respectively.
Differentiating Equation (24) with respect to t to obtain

$$\left.\begin{array}{r} \dot{f}_1(t) = \Omega Cos\Omega\mathrm{t},\, \ddot{f}_1(\mathrm{t}) = -\Omega^2 Sin\Omega\mathrm{t} \\ \dot{f}_3(t) = -\beta e^{-\beta t} Sin\Omega\mathrm{t} + \Omega e^{-\beta t} Cos\Omega\mathrm{t} \\ \ddot{f}_3(\mathrm{t}) = \beta^2 e^{-\beta t} Sin\Omega\mathrm{t} - 2\beta\Omega e^{-\beta t} Cos\Omega\mathrm{t} - \Omega^2 e^{-\beta t} Sin\Omega\mathrm{t} \end{array}\right\} \qquad (25)$$

Using Equation (25) in Equation (22) yields

$$U_{tt}(m,t) + A_1 U_t(m,t) + A_2 U(m,t) = \frac{Mg}{\mu} cos\gamma t\; Sin\frac{m\pi ct}{L}$$

$$-\left\{\begin{array}{c} -\mathrm{C_{f1}}\Omega^2 Sin\Omega\mathrm{t} + \mathrm{C_{f2}}\left(\beta^2 e^{-\beta t} Sin\Omega\mathrm{t} - 2\beta\Omega e^{-\beta t} Cos\Omega\mathrm{t} - \Omega^2 e^{-\beta t} Sin\Omega\mathrm{t}\right) \\ +A_1\left[\mathrm{C_{f1}}\Omega Cos\Omega\mathrm{t} + \mathrm{C_{f2}}\left(\Omega e^{-\beta t} Cos\Omega\mathrm{t} - \beta e^{-\beta t} Sin\Omega\mathrm{t}\right)\right] \\ A_3(\mathrm{C_{f1}} Sin\Omega\mathrm{t} + \mathrm{C_{f2}} e^{-\beta t} Sin\Omega\mathrm{t}) \end{array}\right\} \qquad (26)$$

Taking the Laplace transform of Equation (26) with initial conditions Equation (5), after simplification and rearrangement, we obtain:

$$\bar{U}(m,s) = \frac{1}{r_1 - r_2}\left[A_4\left(\frac{\gamma_1}{S^2+\gamma_1^2} - \frac{\gamma_2}{S^2+\gamma_2^2}\right) + A_5\frac{\Omega}{S^2+\Omega^2} + A_6\frac{\Omega}{(s+\Omega)^2+\Omega^2} + A_7\frac{s+\beta}{(s+\beta)^2+\Omega^2} + A_8\frac{s}{S^2+\gamma^2}\right]\mathrm{x}\left[\frac{1}{s+r_2} - \frac{1}{s+r_1}\right] \tag{27}$$

Where

$$\left.\begin{aligned} &A_4 = \frac{Mg}{2\mu}, A_5 = C_{f1}(\Omega^2 - A_3), A_6 = -C_{f2}(\Omega^2\beta^2 - A_3) + A_1\beta,\\ &A_7 = C_{f2}(2\beta\Omega - A_1), A_8 = -A_1Cf_1\Omega\\ &r_1 = -\frac{1}{2}(A_1 + \sqrt{A_1^2 - 4A_1})\ r_2 = -\frac{1}{2}(A_1 - \sqrt{A_1^2 - 4A_1}) \end{aligned}\right\} \tag{28}$$

By adopting the following representation

$$F(S) = \frac{A_4\gamma_1}{S^2+\gamma_1^2} - \frac{A_4\gamma_2}{S^2+\gamma_2^2} + \frac{A_5\Omega}{S^2+\Omega^2} + \frac{A_6\Omega}{(S+\Omega)^2+\Omega^2} + \frac{A_7(S+\beta)}{(S+\beta)^2+\Omega^2} + \frac{A_8 S}{S^2+\Omega^2} \tag{29}$$

$$G_1(S) = \frac{1}{s+r_2}, G_2(S) = \frac{1}{s+r_1} \tag{30}$$

The Laplace inversion of (27) can be obtained as

$$U(m,t) = \emptyset\begin{bmatrix} A_4(J_1 - J_2) + A_5J_3 + A_6J_4 + A_7J_5 + A_6J_4 + A_7J_5 \\ +A_8J_6 - A_4(J_7 - J_8) + A_5J_9 + A_6J_{10} + A_7J_{11} + A_8J_{12}\end{bmatrix} \tag{31}$$

Where, after evaluation and simplification, the integrals $(J_1 - J_{12})$ becomes

$$J_1 = \frac{\gamma_1}{\gamma_1^2 + r_2^2}(e^{-r_2t} - Cos\gamma_1 t + \frac{r_2}{\gamma_1}Sin\gamma_1 t) \tag{32a}$$

$$J_2 = \frac{\gamma_2}{\gamma_2^2 + r_2^2}(e^{-r_2t} - Cos\gamma_2 t + \frac{r_2}{\gamma_2}Sin\gamma_2 t) \tag{32b}$$

$$J_3 = \frac{\gamma_2}{\Omega^2 + r_2^2}(e^{-r_2t} - Cos\Omega t + \frac{r_2}{\Omega}Sin\Omega t) \tag{32c}$$

$$J_4 = \frac{\Omega}{\Omega^2 + r_2^2(1-\Omega)^2}(e^{-r_2\Omega t}\ Cos\Omega t - e^{-r_2t} + e^{r_2\Omega t}\frac{r_2(1-\Omega)}{\Omega}Sin\Omega t) \tag{32d}$$

$$J_5 = \frac{\Omega}{\Omega^2 + r_2^2(1-\beta)^2}(e^{r_2\beta t} + \frac{r_2(1-\beta)}{\Omega}(e^{r_2\beta t}Cos\Omega t - e^{-r_2t})) \tag{32e}$$

$$J_6 = \frac{\Omega}{\Omega^2 + r_2^2}(Sin\Omega t + \frac{r_2(1-\beta)}{\Omega}(e^{r_2\beta t}Cos\Omega t - e^{-r_2t})) \tag{32f}$$

$$J_7 = \frac{\gamma_1}{\gamma_1^2 + r_1^2}(e^{-r_1t} - Cos\gamma_1 t + \frac{r_1}{\gamma_1}Sin\gamma_1 t) \tag{32g}$$

$$J_8 = \frac{\gamma_2}{\gamma_2^2 + r_1^2}(e^{-r_1t} - Cos\gamma_2 t + \frac{r_1}{\gamma_2}Sin\gamma_2 t) \tag{32h}$$

$$J_9 = \frac{\Omega}{\Omega^2 + r_1^2}(e^{-r_1t} - Cos\Omega t + \frac{r_1}{\Omega}Sin\Omega t) \tag{32i}$$

$$J_{10} = \frac{\Omega}{\Omega^2 + r_1^2(1-\Omega)^2}(e^{r_1\Omega t}Cos\Omega t - e^{r_1t} + \frac{r_1(1-\Omega)}{\Omega})e^{r_1\Omega t}Sin\Omega t \tag{32j}$$

$$J_{11} = \frac{\Omega}{\Omega^2 + r_1^2(1-\beta)^2}(e^{r_1\beta t} + \frac{r_1(1-\beta)}{\Omega}(e^{r_1\beta t}Cos\Omega t - e^{r_1t})) \tag{32k}$$

$$J_{12} = \frac{\Omega}{\Omega^2 + r_1^2}(Sin\Omega t + \frac{r_1}{\Omega}(Cos\Omega t - e^{r_1t})) \tag{32L}$$

Putting Equations (31) into Equation (18b) to obtain

$$U(x,t) = \frac{2}{L}\sum\sum_{m=1}^{\infty}\emptyset\begin{bmatrix} A_4(J_1 - J_2) + A_5J_3 + A_6J_4 + A_7J_5 + A_6J_4 + A_7J_5 \\ +A_8J_6 - A_4(J_7 - J_8) + A_5J_9 + A_6J_{10} + A_7J_{11} + A_8J_{12}\end{bmatrix}\sin\frac{m\pi x}{L} \tag{33}$$

Equation (33) represents the response amplitude of the time-dependent uniform elastic beam resting on a Pasternak foundation subjected to a harmonically varying moving load.

DISCUSSION OF RESULTS

The responses of a uniform time-dependent elastic beam resting on a two-parameter foundation and subjected to harmonically varying moving loads for various values of the Winkler foundation parameter (K), Pasternak foundation parameter (G), axial force (N), damping coefficient (alpha), and circular frequency (omega) of the moving load are shown in Figures 1–5.

Figure 1 depicts the decrease in the response amplitude of the beam subjected to a harmonic moving load as the values of the Pasternak foundation parameter increased. The deflection profile of a beam subjected to a harmonic moving load is shown in Figure 2, and it decreases as the values of the Winkler foundation parameter increase.

Figures 3, 4, and 5 show the response amplitudes of the beam subjected to harmonic moving loads for various values of damping coefficient, axial force, and circular frequency, respectively. From the

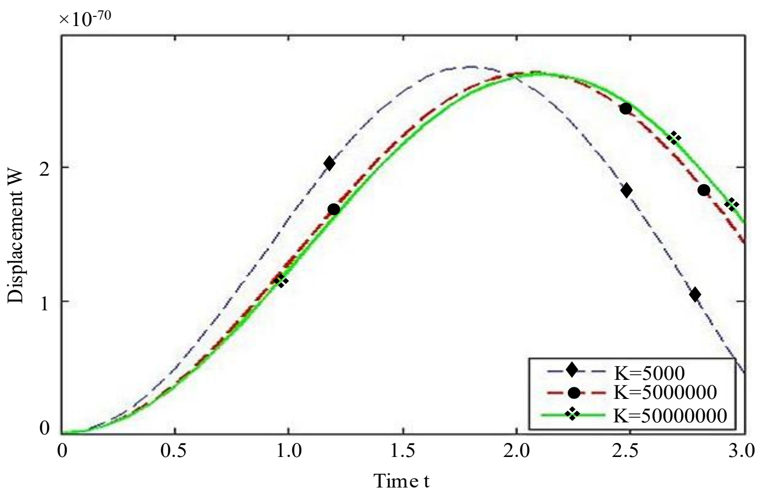

Figure 1. Response amplitude of time-dependent uniform elastic beam resting on a Pasternak foundation subjected to a harmonically varying moving load with varying foundation modulus (K).

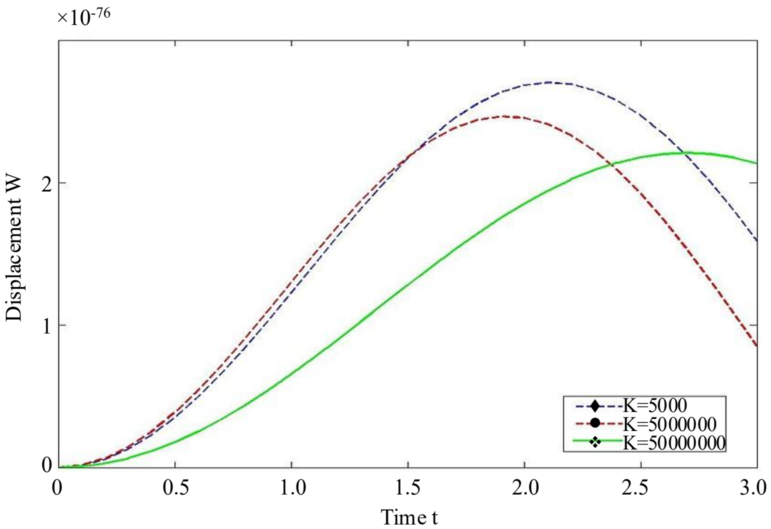

Figure 2. Response amplitude of time-dependent uniform elastic beam resting on a Pasternak foundation subjected to a harmonically varying moving load with varying shear modulus (G).

Figures 1 to 5, it can be observed that increasing the values of each parameter leads to a decrease in the response amplitudes of the beam.

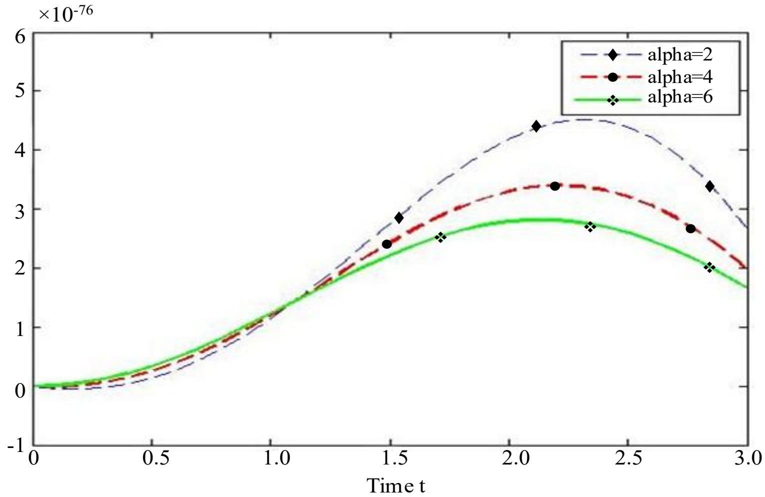

Figure 3. Response amplitude of time-dependent uniform elastic beam resting on a Pasternak foundation subjected to a harmonically varying moving load with varying damping coefficient (alpha).

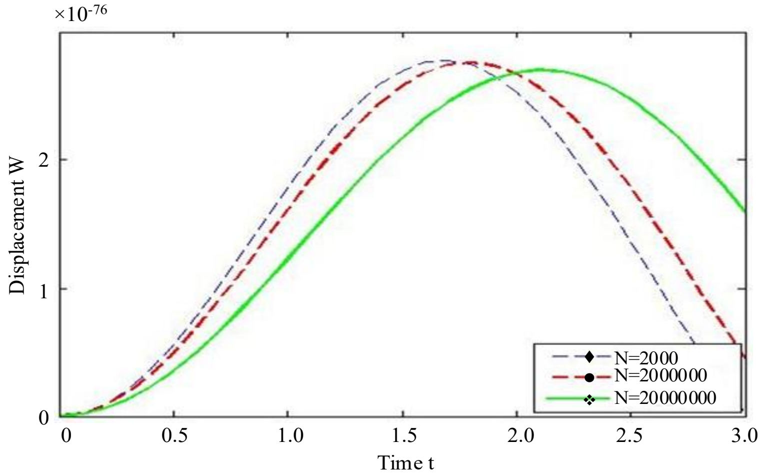

Figure 4. Response amplitude of time-dependent uniform elastic beam resting on a Pasternak foundation subjected to a harmonically varying moving load with varying axial force (N).

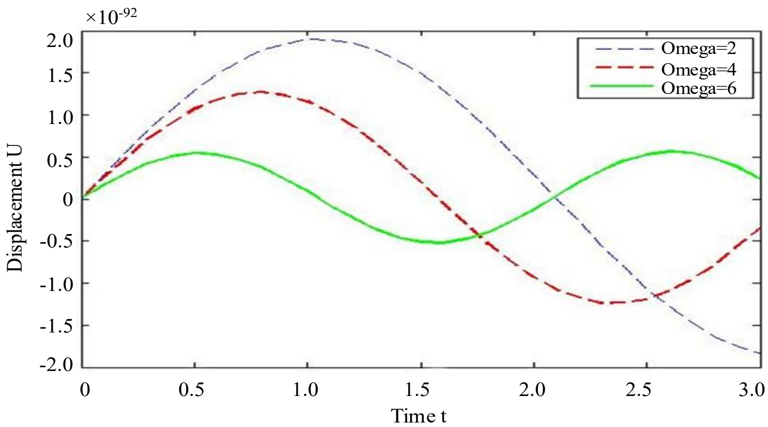

Figure 5. Response amplitude of time-dependent uniform elastic beam resting on a Pasternak foundation subjected to a harmonically varying moving load with varying circular frequency (omega).

CONCLUSION

The dynamic analysis of a uniform time-dependent elastic beam resting on a two-parameter foundation subjected to a harmonically varying moving load was investigated using the Mindlin–Goodman method, Fourier series transformation, Laplace transformation, and theory of convolution. In addition, taking into consideration the governing equation describing the dynamical system, the effect of damping is considered.

The effects of the structural parameters on the beam were evaluated, as shown in the graphs, and it was observed that, for a more noticeable effect, higher values of the Pasternak and Winkler foundation parameters are required. The results show that the damping coefficient and circular frequency of the moving load have more significant effects on the beam subjected to a harmonically varying moving load than the other structural parameters, as shown in Figures 3 and 5, respectively.

Finally, higher values of the structural parameters, especially the damping coefficient and circular frequency of the moving load, are required so that the resonance effects on the dynamical system can be drastically reduced, which could guarantee the safety of life and properties.

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