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A Critical Review of the Limitations of the Arithmetic Mean and the Robustness of the Median in Statistical Analysis

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Abstract

Arithmetic mean is an extremely popular measure of central tendency used in statistical analyses, primarily because it is so easy to calculate and has many positive mathematical characteristics. However, when there are outliers, skewed distributions or heterogeneous spread of data, the reliability of the arithmetic mean diminishes greatly. This paper includes a critical review of the limitations of the arithmetic mean and an assessment of the robustness of the median as an alternative measure. The methodology used in this study is both review-based and conceptual analytical methodology. The study synthesizes literature related to descriptive and robust statistics and examines the behavior of both measures under various data conditions. The results of the analysis indicate that the arithmetic mean is very sensitive to extreme values, which can lead to a misinterpretation of results when working with distributions that are not normal. By contrast, the median shows greater stability and is more resistant to the influence of outliers, which makes it better suited for highly skewed data and also for many real-world datasets such as income and performance data. The findings presented in this review suggest that the median is a more reliable measure of central tendency when the presence of Skewness and outliers exists. The mean is most appropriate for symmetric distributions. This study provides a conceptual framework to assist students in selecting an appropriate measure of central tendency by considering the relevant characteristics of the distribution.

Keywords: Arithmetic mean, descriptive analysis, median, outliers, robust statistics, skewness

INTRODUCTION

Data collection, organization, analysis, & interpretation is a fundamental discipline that provides an essential basis for any informed decision for any area of study (economics & related fields, education, healthcare, & business analytics). One fundamental concept of descriptive statistics is that of central tendency. It provides a single representative value from the entire data set (population or sample) to summarize that data set, providing a measure of central tendency.

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The three measures of central tendency are arithmetic mean, median, and mode. The arithmetic mean measurement statistically is the most widely used of the three measures, due to its ease of computation and strong algebraic properties that make it suitable for all sorts of further mathematical and statistical analyses. The calculation of the arithmetic mean requires the acquisition and incorporation of every observation in a population or sample being measured and thus provides a comprehensive measure of central value when all data are distributed ideally under normal conditions in relation to central value.

However, very few real-world data sets will satisfy these ideal statistical assumptions, and because of this, most will contain outliers, be skewed in one direction or another, and have unequal dispersion or spread characteristics. As a consequence, it will not be reliable to designate the arithmetic mean, median, or mode as the actual central tendency measurement for the entire data set when it contains a significant amount of outlier and/or extreme values. As an example, socio-economic data sets of income levels typically include a few high-incomes and exclude many low incomes; therefore, calculating an average individual income level in this circumstance will misrepresent the actual average income level experienced by most individuals.

To contrast the criterion termed as median, which can also be known as a center value within a set of observations, is less likely than a criterion referred to as mean, to be skewed by extreme values and is generally viewed as a more stable measure when analyzing central tendency for non-normal or non-symmetrical data sets. Another example of how the two differ is that median is based upon position while mean may change if an extreme value is placed into the distribution of the data point population (sometimes called outlier). Because of this characteristic, median has been shown to be beneficial in research studies where researchers have worked with data containing extreme values in today’s world.

There are also documented challenges when using median for analysis. Median does not consider all observations within the data set, and due to the nature of how it is calculated, is not suitable for advanced statistical analysis which requires properties of arithmetic. Because of this, median cannot replace or represent mean in every circumstance; therefore, there is a need to understand both measures’ properties and utilize them accordingly depending on specific situations. The purpose of this paper is to summarize the strengths/weaknesses of mean as compared to median for analyzing data in order to develop a clearer understanding of how to select appropriate central tendency for data depending on different circumstances such as amount of skew, amount of variability, and presence/ absence of outliers.

This study will provide a comparison of mean and median using existing literature and a general overview of how each of the two will help further advance the area of statistical analysis. The recommended framework should also help researchers better understand how to evaluate measures of central tendency based on types of data being utilized for analysis (such as skewness, variability, and whether an outlier exists).

LITERATURE REVIEW

The arithmetic mean is more responsive to extreme values in a dataset compared to the median. When analysing a skewed distribution, the arithmetic mean may fail to represent the true centre of the dataset, whereas the median provides a more reliable measure of central tendency because of its resistance to extreme observations [1]. This limitation of the mean can result in misleading interpretations in practical applications, such as income distribution analysis, where a small number of extremely high or low observations can significantly influence the average value. Therefore, the median often provides a more accurate representation of the central position of data in such situations [2, 3].

A new estimator based on the “median of the distribution of the mean” combines the advantages of both mean-based and median-based approaches. The study demonstrated that traditional mean estimators may lack robustness when datasets contain outliers, filtered observations, or incomplete information. This approach supports the importance of median-based techniques for improving reliability and accuracy in statistical analysis, particularly when data quality is uncertain [4].

Central tendency consists of different measures used to describe the typical or central value of a dataset. The arithmetic mean is highly sensitive to extreme observations and can shift toward the tails of non-normal distributions, reducing its ability to represent the dataset effectively. In contrast, the median remains stable under skewed conditions and provides a better indication of the central location of observations [5]. Similar conclusions regarding the influence of skewness on statistical measures have been discussed in statistical literature [6, 7].

The concept of robustness further explains the limitations of the arithmetic mean. The breakdown point defines the maximum proportion of contaminated data an estimator can tolerate before becoming unreliable [8, 9]. The arithmetic mean has a breakdown point of 0%, meaning that even a single extreme observation can significantly distort the result. Conversely, the median has a breakdown point of 50%, indicating strong resistance against outliers and demonstrating its robustness in the presence of extreme values [10, 11].

Several studies on robust estimation have confirmed the limitations of classical estimators under contaminated datasets. In the presence of outliers, traditional mean-based methods often produce less accurate results compared with robust alternatives such as the median and trimmed means. These findings highlight the importance of selecting appropriate measures of central tendency according to the characteristics of the dataset [12]. Similar observations regarding outlier influence and robust statistical methods have been reported in previous studies [13, 14].

The arithmetic mean is not always an appropriate measure for small datasets or datasets following skewed distributions. Relying only on the mean may produce misleading interpretations when substantial variability exists within the data. The median, due to its positional nature, provides a more representative summary of the dataset when observations are highly dispersed or asymmetrically distributed [15]. This is also supported by studies discussing exploratory data analysis and distributional properties [16, 17].

However, existing literature also recognizes certain limitations of the median. Unlike the arithmetic mean, the median does not utilize every observation in the dataset and does not possess several mathematical properties required for advanced statistical analysis and theoretical modelling [18–20]. Therefore, although the median is often preferred for skewed or contaminated datasets, the arithmetic mean remains valuable when data follow approximately symmetric distributions and when mathematical operations involving all observations are required [21, 22].

Overall, the literature indicates that no single measure of central tendency is universally suitable for all datasets. The selection between mean and median depends on important characteristics of data, including distribution shape, variability, skewness, and the presence of outliers [23–25]. The reviewed studies demonstrate that while the arithmetic mean is efficient under ideal conditions, the median provides greater reliability and robustness when dealing with real-world datasets containing extreme observations or non-normal distributions [26–28]. Therefore, careful consideration of data characteristics is essential for accurate statistical interpretation and decision-making (Table 1).

Table 1. Summary.

Author	Year	Focus area	Method used	Key finding	Limitation
Rakrak	2025	Data distribution	Conceptual analysis	Mean affected by outliers	Limited empirical data.
García-Pérez	2023	Estimators	Mathematical modeling	Median-based estimators are robust	Complex for beginners.
SAGE	2025	Central tendency	Theoretical review	Mean distorted in skewed data	Generalized discussion.
Tsamatsoulis	2023	Robust statistics	Comparative study	Median outperforms mean	Limited dataset scope.
PMC Study	2020	Data analysis	Empirical analysis	Mean unreliable in skewed data	Small sample focus.

GRAPHS AND ANALYSIS

Figure 1 represents a normal distribution (or symmetric distribution), where the arithmetic mean and the median occur at the same point along the X-axis of the distribution (middle). In addition, the arithmetic mean is an accurate measure of central tendency since the arithmetic mean and median are equivalent when there are no outliers affecting the distribution. Therefore, an arithmetic mean is the best measure of central tendency for normally distributed data since the data set is evenly distributed.

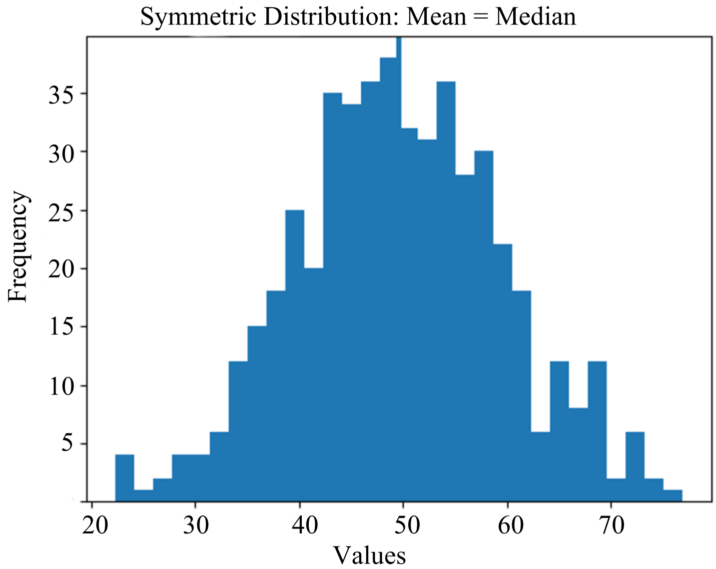

Figure 1. Symmetric distribution (mean ≈ median).

According to this figure, a skewed distribution is positively skewed by a few very large values in the right-hand meat of the distribution. As a result of the few very large values causing a positive bias in the arithmetic mean of the data, the median will tend to stay closer to the middle of the complete dataset. This emphasizes the constraint of the mean and demonstrates the robustness of the median in skewed data (Figure 2).

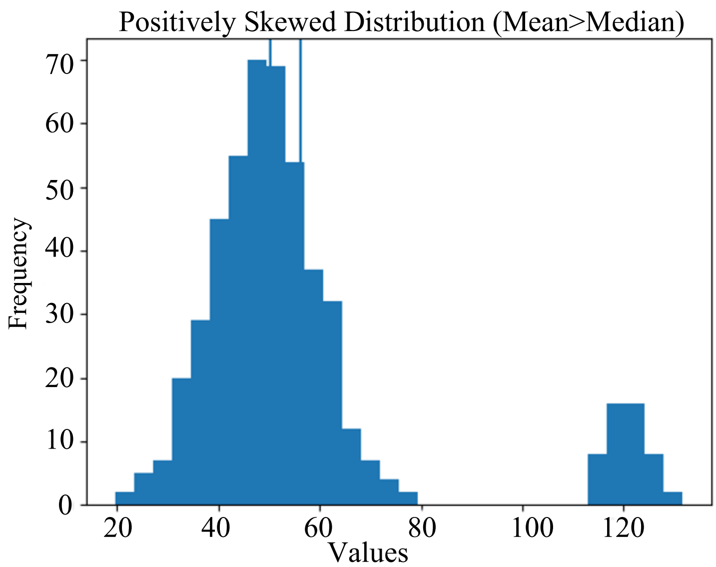

Figure 2. Positively skewed distribution (mean > median).

It is quite clear from the boxplot that the extreme values, or outliers, in the dataset are quite far from the rest of the data values, and this has a significant effect on the arithmetic mean, whereas it has a relatively minor effect on the median, which again illustrates its reliability (Figures 3 and 4).

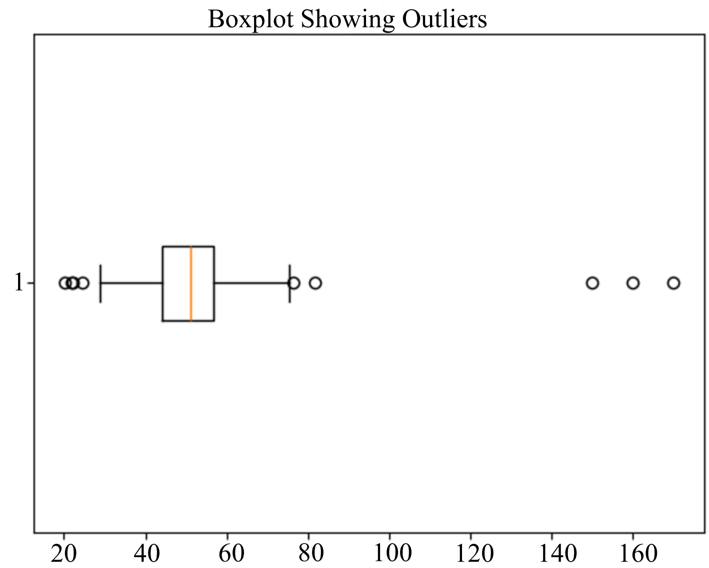

Figure 3. Boxplot with outliers.

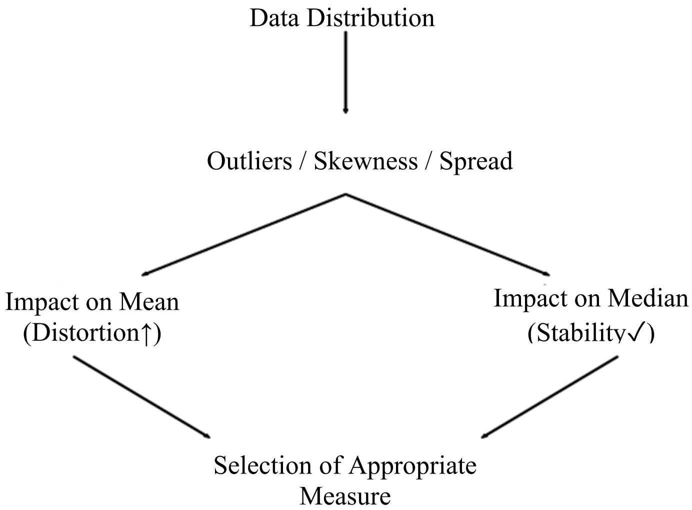

Figure 4. Conceptual framework.

The conceptual model illustrates how data characteristics and the choice of the appropriate central tendency are related. Outliers, Skewness, and the spread of the data directly impact the accuracy and reliability of the arithmetic mean, often leading to distortions of the arithmetic mean. However, the median is typically unaffected by the same data characteristics.

RESEARCH GAP

Many studies discuss the arithmetic mean's shortcomings and the median's strength; however, there are many areas that continue to be unexplained from either a practical or developmental reference standpoint. Most of the literature is either overly technical or only presents theoretical comparisons that do not provide a beginner or novice researcher with a simplified, structured description of the information they need. There also does not seem to be much emphasis on bridging the gap between conceptual understanding and practical application when using statistics to support decision-making processes.

In addition, there are few articles that provide any visual representation of the data or adequate instructional guidelines, making it difficult to see how using an inappropriate central tendency measure may affect the user's real-world results. The objective of this paper is to provide a clear conceptually structured framework based on a review of the literature, supporting both through examples as well as illustrations.

RESEARCH QUESTIONS

Four main research questions guide this research:

- What impact does outlier presence have on the reliability of arithmetic means?
- What causes arithmetic means to not represent central tendencies in skewed distributions?
- Under what conditions will the median be a more useful measure than the mean?
- How do characteristics of distributions influence our selection of measures of central tendency?

RESEARCH OBJECTIVES

The main objectives of this study are:

- To examine the limitations of the arithmetic mean in statistical analysis.
- To evaluate the robustness of the median in the presence of skewed data and outliers.
- To provide a comparative understanding of both measures.
- To develop a conceptual framework for selecting appropriate measures of central tendency.

RESEARCH METHODOLOGY

Using a qualitative review-based methodology, this study is based on secondary data obtained from reliable online sources, academic journals, books, and research articles related to descriptive and robust statistics.

To evaluate the behavior of the arithmetic mean and the median under varying data conditions (e.g., skewed or symmetrical distributions, presence of outliers, and varying levels of data dispersion), this study employs a comparative conceptual analysis approach. Illustrative examples and graphical representation are used to increase clarity and to simplify understanding. This method will provide the most effective means for synthesizing existing knowledge and presenting it in a format that is accessible to students, academics, and practitioners.

DATA ANALYSIS AND DISCUSSION

The graphical analysis presented above provides a clear basis for evaluating the performance of the arithmetic mean and the median under different data conditions.

Interpretation Across Distributions

In symmetric distributions the mean and median of a dataset are the same, verifying the arithmetic mean as an accurate reflection of central tendency and in keeping with what is suggested by conventional statistical theory and corroborated by studies published in the literature where the mean is shown to be effective in ideal conditions such as those where the distribution is normal.

However, in positively skewed distributions, the arithmetic mean becomes more skewed towards positive values because of the effect on it caused by extreme values. The results on this were found to be in line with the findings [1, 5] which both maintain that the mean becomes unreliable in instances where the distribution deviates from normality. In contrast to the arithmetic mean, the median is less impacted by extreme scores and, thus, is a more robust statistic for assessing positively skewed distributions.

Impact of Outliers

The results from the box plot analysis show that the presence of outliers has a dramatic effect on the value of the arithmetic mean but does not have nearly as much of an effect on that of the median. The results of these comparative analyses affirm the concept of breakdown points outlined in literature on robust statistics where the mean has a breakdown point of 0% while the median has a breakdown point of 50%. Furthermore, there are multiple studies [12], that point to the superiority of robust measures in cases where extreme values exist.

Real-World Implications

The results of this paper have serious practical implications. Many real-world datasets – like income distribution, businesses' revenues, housing prices, and academic performance – have skewed and/or outlier data. Using only the arithmetic mean as an indicator may, therefore, falsely misrepresent the conclusion and lead to poor decision making. For example, the average income calculated from the mean may be overstated (because a few people have very high incomes), thus misrepresenting the financial situation of the average person. Thus, the median provides a much better indication than the mean of what is typical for an individual.

Comparative Evaluation

The comparative analysis confirms that the effectiveness of a measure of central tendency is highly dependent on the structure of the dataset. While the arithmetic mean is mathematically advantageous and suitable for further statistical modeling, its sensitivity to skewness and outliers limits its applicability in non-ideal conditions. The median, although lacking strong algebraic properties, offers greater reliability in representing the central position of data in such scenarios (Table 2).

Table 2. Comparative summary of mean and median across data conditions.

Distribution type	Mean	Median	Best measure
Symmetric	Equal	Equal	Mean.
Positively Skewed	Mean > Median	Stable	Median.
With Outliers	Affected	Stable	Median.

The table summarizes the comparative behavior of the arithmetic mean and median across different data conditions. It reinforces that the selection of an appropriate measure of central tendency depends on the distribution characteristics of the dataset.

Critical Insights

Despite clear evidence of its limitations, the arithmetic mean continues to be widely used in practice, often without consideration of data distribution. This over-reliance on the mean indicates a disconnect between the theoretical statistical concepts of central tendency and their application in actual data analysis.

However, there are also limitations to relying exclusively on the median to represent the characteristics of a given dataset. In particular, using the median ignores the relationship and magnitude of values, and may result in overly simple/efficient datasets which do not provide complete and accurate portrayals of the underlying population.

Overall Discussion

"The above graphical analysis provides the basis for the following discussion."

In conclusion, the findings of this study demonstrate that there is no single measure/indicator of central tendency that can be used universally; the choice between mean and median should consider the characteristics of the data, including Skewness, variability, and outlier data. Combining graphical/visual representation of the data with conferencing the theoretical fundamentals of the data allows for the greatest degree of accuracy and better decision making in both the academic and practical realm.

FUTURE SCOPE

Further research opportunities arising from this study,

- Empirical research will be conducted utilizing authentic datasets.
- Comparing additional robust measures such as trimmed mean and mode.
- Applying these concepts in big data and machine learning contexts.
- Developing automated methods for selecting appropriate statistical measures.

CONCLUSION

This research included a review of the arithmetic mean's weaknesses and the evaluation of the median's strength as a definition of central tendency in statistical analysis. Results demonstrate the arithmetic mean is highly sensitive to extreme values (outliers) and skewed frequency distributions, therefore, it should not be utilized as a descriptive statistic for many real-world datasets. In contrast, the median is a more stable and reliable measure for the same dataset types.

Regardless of the reasons previously stated, either the arithmetic mean or the median can be used as appropriate measures of central tendency under various circumstances. The arithmetic mean is generally suitable for quantitative analysis of datasets which follow a normal (symmetrical) distribution; however, you can use the median when assessing the central tendency of those datasets which are asymmetrical and/or contain outlier scores. Due to the various characteristics and differences found in each type of data, it is essential to select your central tendency based on the attributes of your data instead of using some other standard in computing metrics.

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