

Fracture Analysis of FRP Composites under Thermo-Mechanical Loads for Different Geometry Cutouts

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Abstract

Fiber-reinforced composites (FRPs) are used extensively in structural and non-structural components of the aerospace and automotive industries. To utilize these materials for structural applications, it is necessary to understand the fracture behavior of the material. In the present investigation of carbon fiber laminates, studies were carried out to understand the fracture toughness characteristics of the carbon fiber laminates with mechanical, thermal, and thermo-mechanical loadings of modes I, II, and III. The carbon composite laminate with four layers and different stacking sequences of angle-ply and cross-ply cases was investigated by considering three kinds of loadings: mechanical, thermal, and coupled thermo-mechanical loading. The finite element method was used to model the carbon composite laminate and to estimate the strain energy release rate (SERR) subjected to simply supported boundary conditions. The FE model predicted results are validated with analytical results and experimental predictions. Finite element models of laminated composite rectangular plates were modeled to determine the fracture behavior under mechanical, thermal, and combined thermo-mechanical loading. The strain energy release rate was estimated to characterize whether the crack is open or closed. Experimental results were produced by creating a composite with chopped carbon fibers and an epoxy matrix using the hand lay-up technique. The composite material with a pre-existing crack was tested for fracture toughness, and the same was simulated using ANSYS for validation. A close agreement was observed between the experimental and analytical results.

Keywords: Carbon fiber reinforced polymer (CFRP), fracture toughness, finite element analysis (FEA), strain energy release rate (SERR), thermo-mechanical loading

INTRODUCTION

A composite is a multiphase material that contains two or more distinct phase materials. The properties of these materials are superior to those of the individual materials. The main objective of the composite material is to replace conventional materials with high specific strength and high specific stiffness materials. Among the different categories of composite materials, the continuous fiber-reinforced composite material shows better mechanical properties. To meet the required properties of composite materials, the designer must arrange the continuous fibers in the required direction.

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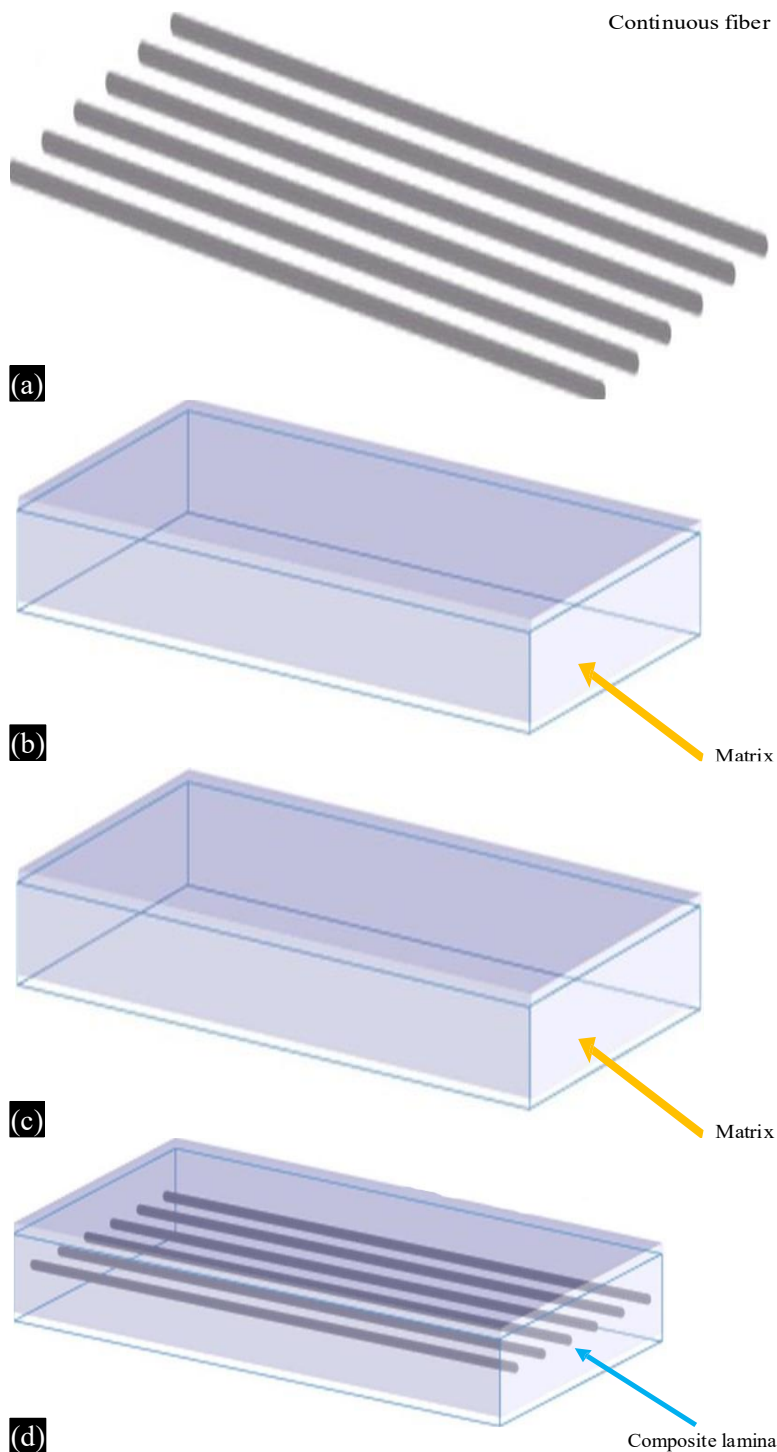
Composites are frequently used in two forms: the lamina and the laminate. The lamina is a flat or curved arrangement of unidirectional or woven fibers inserted in a matrix material. These laminas are stacked together to form a laminate with the desired fiber angle.

Lamina and Laminate

A layer of a thin sheet of material is called a lamina and is generally treated as a plate. When two or more sheets are adhered or glued together, the newly formed material is known as a laminate. A laminate consists of a number of layers or sheets of

lamina. The layers in the laminate may be of different materials, and the resulting material type can be considered a hybrid composite material, though the materials formed together may not be integrated in the same manner as the constituent materials present in the composites. The layers in the laminate should always be considered and must be bonded together permanently by using an adhesive so that they behave as one material; the details are presented in [Figure 1]. The fibers, matrix, and composite lamina independently contain all components as depicted in the figure.

Composite materials can withstand high loads, including fatigue and fracture loadings. The fracture behavior of the composites is discussed below.



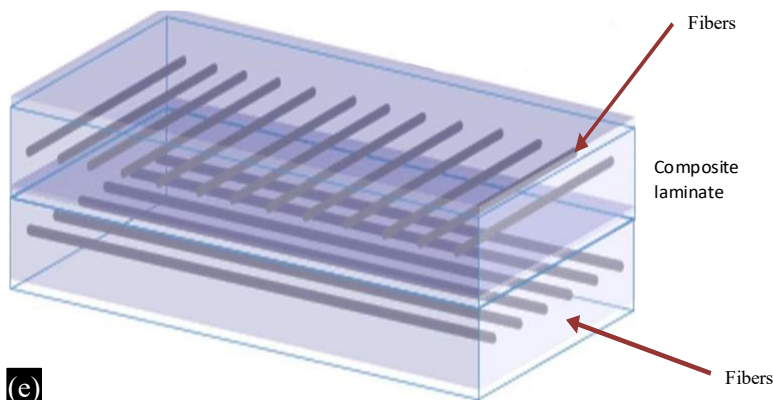


Figure 1. (a–e) Pictorial representation of lamina and laminate.

Importance of Fracture Mechanics

Composite materials are orthotropic materials, and the failure behavior of isotropic materials cannot be applied to composite materials. Composite material failure depends on many design parameters, such as stacking sequence, uncertainties in the structures (such as cutouts), and loading and boundary conditions.

Fracture mechanics is the branch of science that provides complete knowledge of the failure or fracture behavior of materials. Compared to ordinary materials, composite material fracture behavior depends on many factors; to unleash such factors, one must know the strain energy release rate, stress intensity factor, J-integral, or crack tip opening displacement for different loading conditions.

Modes of Fracture

Three types of fractures are considered in fracture mechanics. These types are termed modes of fracture [1].

- *Mode I fracture:* The tensile stress acts normal to the plane of the crack. It is also termed the opening mode, as it creates an opening for the crack to propagate further. This type of mode is critical as the load is applied in the crack plane. It has received great attention in research, structural design, and failure analysis.
- *Mode II fracture:* The shear stress acts parallel to the plane of the crack and perpendicular to the crack front. It is also termed the sliding mode due to the sliding action of the crack as it tends to propagate further from the opening.
- *Mode III fracture:* A shear stress acts parallel to the plane of the crack and parallel to the crack front. It is also termed the tearing mode, as the crack proceeds to tear the material while propagating further.
- *Mixed Mode fracture:* In general, structures are subjected not only to simple tension or compression loadings but also to shear and torsion loads. This combination of different loading patterns leads to a combination of three basic modes, known as mixed mode. Mixed modes can result from different types of interactions, such as mixed modes I/II, II/III, or I/III.

The Strain Energy Release Rate

According to the strain energy approach, existing crack extension occurs when the energy release available for crack growth is sufficient to overcome the material resistance. The amount of energy released is based on the direction of the load applied. Based on the mode of loading, the energy release rate is classified as mode I (opening mode), mode II (sliding mode), and mode III (tearing mode), as shown in [Figure 2].

Using analytical or experimental studies, researchers can find the energy release rate (G) depending on the type of load applied to the materials.

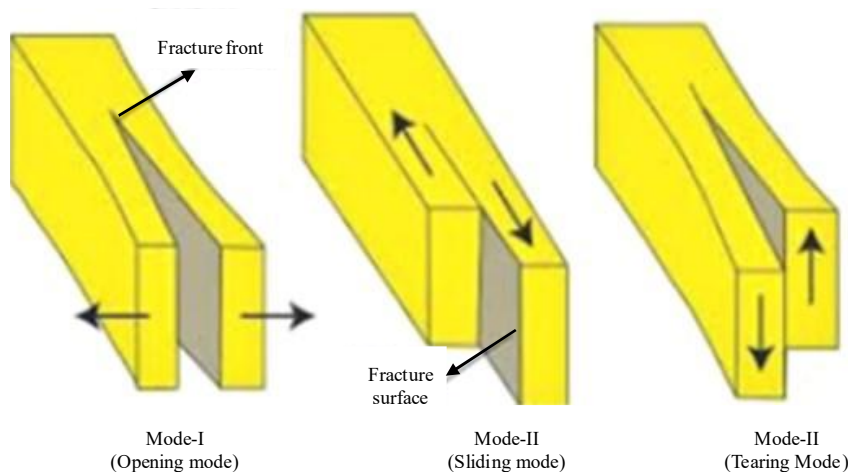


Figure 2. Fracture modes.

Loading Conditions

- (a) *Mechanical Load:* Mechanical loading refers to the loads applied to the specimen that are purely related to mechanical forces, such as force, pressure, thrust, tension, and compression. In the present study, a uniform pressure is applied to composite laminates with different cutouts.
- (b) *Thermal Load:* Thermal load refers to the loads applied to a material or structure resulting from the effects of temperature and its related factors. In the present study, a composite plate with various cutouts is subjected to different temperature zones with increasing factors.
- (c) *Thermo-Mechanical Load:* A combination of thermal and mechanical loads applied simultaneously is useful for studying combined effects or virtual representations of real-world models. In the present study, the combined effect of loading is considered to investigate fractures resulting from the aforementioned loading conditions on a composite plate with different cutouts.

The effects of mechanical, thermal, and thermo-mechanical loads on crack propagation with an edge crack in the composite laminate are studied. For all cases considered, the strain energy release rate (SERR) is estimated.

Composite materials generally offer improved specific strength and specific stiffness to achieve weight-saving objectives. In the development of composite structures, the study of fracture behavior is of great importance. The literature regarding thermo-mechanical loading and its impact on the fracture behavior of composite structures is specialized.

This chapter reviews published literature with notable observations from individual studies. For convenience, it has been arranged to cover analytical and experimental approaches to composite fractures.

1. *Analytical:* Relevant analytical work performed on the topic.
2. *Experimental and Analytical:* Literature pertaining to analytical work supported by experimentation.
3. *Analytical, FEM, and ANSYS:* Analytical work supported by numerical examples using the finite element method (FEM) and relevant work accomplished using ANSYS simulation software.

LITERATURE SURVEY

Analytical Model Investigations

Analytical approaches are more insightful and provide a wide range of conceptual acumen, which is helpful for emphasizing new phenomena, even with singularities. Helien and Cesari [1] studied a composite plate with a center crack subjected to a quadratic thermal gradient. A comparison between the analytical method, stress intensity factors (SIFs) obtained from elastic analysis, and the finite element method was made to correlate and validate their model. Wilson and Yu [2] presented methods

for using the J-line integral of the magnitude of crack-tip SIFs from finite element solutions under thermal stress crack problems. Wang and Yau [3] presented an analytical study of cracks arising from a circular hole in a unidirectional fiber-reinforced polymer (FRP) composite. The unidirectional fiber-reinforced composite with an off-axis pertaining to crack propagation forms a critical analysis. They found that the mixed-mode SIF and energy release rates for cracks emanating from a hole changed considerably with variations in fiber orientation within the composite.

Doblare et al. [4] utilized the boundary element method to compute crack propagation and the corresponding propagation angle. They also derived SIFs from the analysis to identify failure modes pertaining to orthotropic materials.

Liu and Nicholas [5] developed an efficient numerical analysis technique within the field of boundary element methods for determining steady-state fractures in thermo-elastic problems. Thermal SIFs and relative displacements for surface cracks were obtained. Tsai et al. [6] studied thermal weighted functions through an analytical approach to solve linear elastic problems.

Prasad et al. [7] studied the causes of crack growth in various structural failures. They found that structures could change the direction of crack growth due to temperature changes. Prasad et al. [8] presented a boundary element formulation for studying steady-state thermo-elastic cracks with a single-region analysis. Kokini and Choules [9] examined a plate made of functionally graded material with an edge crack. They used multi-layer beam theory to determine crack initiation resulting from tension in the distribution of thermal stresses. Giannakopoulos et al. [10] used an elasto-plastic analysis for thermocyclic loading on multifaceted composites with compositional gradients composed of metals and ceramics.

Jin and Batra [11] analyzed thermal stresses and SIFs under sudden cooling at cracked surfaces. The problem addressed an edge-cracked strip where the surface was suddenly cooled. The thermal stress intensity factor in a functionally graded strip with an edge crack was found to be insensitive to the shear modulus gradient as a result of the reduced thermal conductivity gradient.

Prasad et al. [12–13] presented thermo-elastic solutions for crack-tip stresses in bodies subjected to thermal loading and transient thermo-elastic crack problems. Noda [14] discussed functionally gradient plates as advanced high-temperature materials with and without edge cracks under thermal stress problems. Cheong and Kwon [15] investigated circular holes with cracks in cross-ply laminates subjected to uniaxial and biaxial loading. The effects of material orthotropy, loading conditions, and geometry on the crack tip were studied to obtain SIFs. Key observations indicated that SIFs for uniaxial tension were higher than those for biaxial tension. Duong and Yu [16] directly computed SIFs for a thermal crack problem. Dell'Erba et al. [17] presented the numerical implementation and formulation of a 3-D dual boundary element method for thermo-elastic problems with mixed-mode cracks. They found high accuracy in mode I and mode II thermal boundary conditions.

Rolfes et al. [18] presented a post-processing procedure for the evaluation of transverse thermal stresses in laminated plates. The distributions of transverse stresses were evaluated. They reported numerical results drawn from finite element analysis for four-layer and ten-layer cross-ply laminates subjected to thermo-mechanical loads.

Yuan and Kalkhof [19] examined the effects of temperature gradients on crack assessments using the deformation theory of plasticity. Choi et al. [20] investigated multi-layer thermal barrier coatings comprising an intermetallic bond coat with a thermally grown oxide layer and porous zirconia on a top coat for thermal protection. They plotted strain energy release rate versus E_1/E_2 , which indicated that the mode II interface toughness exceeded the elastic energy per unit area in the tri-layer G0.

METHODOLOGY

In this paper, a brief introduction is provided for various tools and techniques, such as regression analysis based on the statistical design of experiments (DoE) and finite element modeling used to model the welding process. The paper is divided into two parts: the first part details the steps of DoE, while the procedure used for element modeling is provided in part two [21].

Statistical Regression Analysis

Scope of Research

The fracture of composite laminates is critical for recommending these materials for structural applications. Carbon fiber-reinforced composite laminates are used in many fields, including the aerospace and transportation industries.

The fracture behavior of these carbon fiber-reinforced laminates is essential for estimating or predicting failure. To understand the fracture behavior of composite materials, virtual crack closure techniques (VCCT) are adopted by many researchers, as these methods indicate the failure tendency of the laminate.

The virtual crack closure technique (VCCT) is applied to composite materials with the finite element method (FEM). Rectangular-shaped composite plates are generally used in the above-mentioned applications. In the present study, rectangular composite laminates are modeled by using the FEM-based software ANSYS with four layers, and crack growth is evaluated by using the VCCT. The output obtained from the VCCT is the strain energy release rate (SERR). The SERR of rectangular plates is estimated in the presence of a crack by varying loading and boundary conditions. The effects of loading and boundary conditions on rectangular-shaped composite plates, achieved by varying the stacking sequence, are presented.

Research Gap

The influence of circular and elliptical cracks on stress distribution is reported by many authors under mechanical loading, but the effect of a cut-out and a virtual crack around the cut-out to estimate fracture behavior has not been addressed by researchers. From past work, it is observed that most studies focused on stress distribution around cracks in composite laminates to estimate fracture behavior. To estimate the fracture behavior of composite laminates with existing cracks, the SERR has not yet been discussed [1–4].

The fracture behavior of composite laminates has been studied for mechanical and thermal loads, but no work has been performed combining mechanical and thermal loads with cracks using the VCCT on composite laminates.

FINITE ELEMENT MODELING FOR FRACTURE BEHAVIOR OF COMPOSITES

Finite Element Analysis of FRP Composite Plate

This section presents the modeling of a rectangular plate [5] with dimensions of 100 mm in length, 10 mm in thickness, and a width varying from 60–300 mm for four-layered angle-ply and cross-ply laminates. This study is carried out to determine the effect of the laminate width related to the SERR using the finite element method with different fiber orientations.

Rectangular Plate

An angle-ply and cross-ply composite laminate with the aforementioned dimensions (l , b , t) of a rectangular plate with a delamination case is under study. Delamination and the crack front representation are shown in Figure 4 (a), and the meshed model is depicted in Figure 4(b). The type of element used for the analysis is a hexahedron element, and a structured mesh was created. The region where variation exists is considered a fine mesh, and other regions are taken as a coarse mesh. The delaminated region and the location where the crack is initiated are also shown in Figure 3.

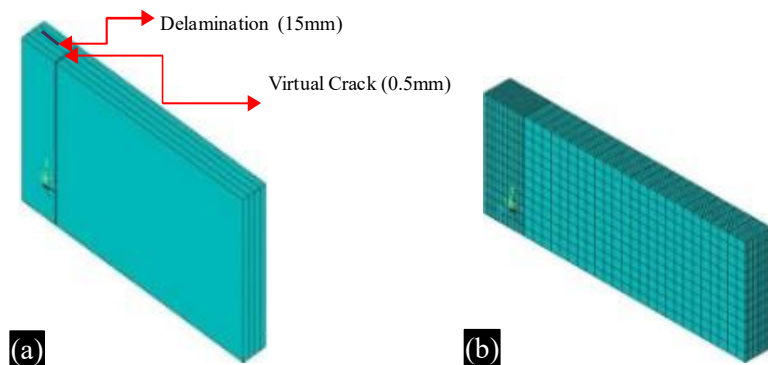


Figure 3. A laminate of four layers with delamination, (a) Solid Model (b) FE Model

Delamination

When a material tends to fail in layers when load is applied in an operating condition, the mode of failure is known as delamination. Here, the laminated material will separate or deform into layers of laminas or lamina deformation. Materials such as laminates, composites, and concrete can fail due to the delamination effect while in operation. Even coatings, such as surface coatings, paints, and films, can delaminate from the substrate to which they have been coated.

Delamination is a topic of research interest in the aerospace, automotive, and specialized industries where composites are utilized. In the delamination of a laminated composite, the adhesive between the layers of lamina may fail, leading to adhesion breaks, which cause the layers to outcast and separate from each other [6].

Angle-Ply and Cross-Ply

In composite laminates, several kinds of laminates exist based on factors such as fiber orientation and stacking sequence. Among them, angle-ply laminates and cross-ply laminates are types of laminates classified on the basis of internal fiber orientation in the laminas. In angle-ply laminates, each ply has its own orientation with alternating angles of $+\theta$ and $-\theta$. It can be either symmetric or anti-symmetric, where θ is fully populated. Conversely, cross-ply laminates are distributed with an arbitrary number of plies, each with a fiber angle orientation of either 0° or 90° , which can be either symmetric or anti-symmetric. These fiber angles for angle-ply and cross-ply are shown in Figure 4.

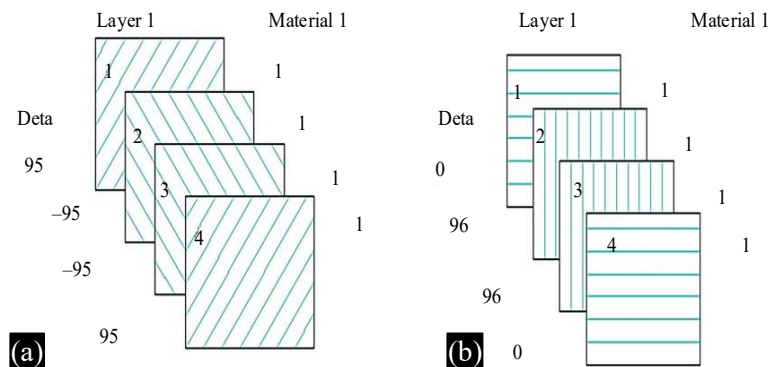


Figure 4. Angle-ply and cross-ply representation, (a) Angle-ply, (b) Cross-ply.

Calculation of SERR for Rectangular Plate

$$SERR = \frac{\text{Difference of energies}}{2 \times \text{Area of virtual crack } (l \times b)}$$

where Difference of Energies = (Strain energy when crack is open – Strain energy when crack is closed)

l = length of virtual crack,
 b = width of virtual crack

The composite laminate SERR is presented by considering the effects of laminate width, pressure acting on the laminate, thickness of the laminate, magnitude of loading effect on the considered laminate, and fiber angle effect along with the fiber position.

Effect of Width

The laminate geometry of the composite is modeled with dimensions of 100 mm in length, 10 mm in thickness, and varying widths (60 mm, 80 mm, 100 mm, 160 mm, 200 mm, 240 mm, and 300 mm). The 10 mm thickness of the laminate is maintained with four laminas (i.e., $10/4 = 2.5$ mm), meaning each layer has a thickness of 2.5 mm. The fiber angles of the laminate are considered to be $[\theta, -\theta, -\theta, \theta]$, having a symmetric layup sequence for modeling purposes. Pressure is applied at 5 MPa and temperature at 400°C.

$\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ,$ and 90° are considered in different configurations for the angle-ply laminate. A 15 mm edge crack with a 0.5 mm virtual crack is modeled for the considered configuration. The specimen geometry is shown in Figure 5.

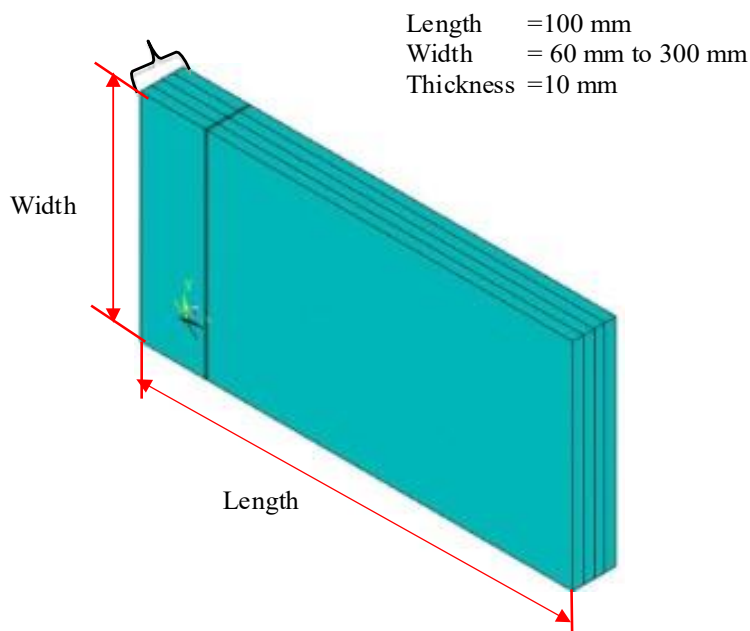


Figure 5. Geometry of the composite laminate.

Effect of Pressure

The laminate geometry of the composite is modeled with dimensions of 100 mm in length, 10 mm in thickness, and a width of 300 mm. The pressure is varied from 1–5 MPa. The 10 mm thickness of the laminate is maintained with four laminas (i.e., 2.5 mm) [7–16].

Varying $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ,$ and 90° in different configurations for the angle-ply laminate. A 15 mm edge crack with a 0.5 mm virtual crack is modeled for the considered configuration.

Effect of Thickness

The laminate geometry of the composite is modeled with dimensions of 100 mm in length, a width of 300 mm, and thicknesses of 8 mm, 10 mm, and 12 mm. The pressure is taken as 5 MPa and the temperature as 400°C. The thickness of the laminate is maintained with four laminas (i.e., $8/4 = 2$ mm, $10/4 = 2.5$ mm, and $12/4 = 3$ mm), meaning each layer has a thickness of 2 mm, 2.5 mm, or 3 mm.

For $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ,$ and 90° in different configurations for the angle-ply laminate, a 15 mm edge crack with a 0.5 mm virtual crack is modeled for the considered configuration [17–19].

CONCLUSION

In this thesis, three main situations regarding the use of cut-out types in composite laminates are discussed, specifically where servicing, routing, and other allied applications are critical. This study warrants attention in the fields of aerospace and automobile engineering. Although it focuses on thermal analysis, it also encompasses mechanical breakdowns. These are investigated using a numerical approach called finite element analysis. The major focus is on the failure mode in which crack initiation and propagation comprise a major part of the study.

The present work is directed toward the area of fracture analysis of FRP composites subjected to mechanical, thermal, and thermo-mechanical loads and respective boundary conditions.

REFERENCES

1. Wang ASD, Kishoret NN, Li CA. Crack development in graphite-epoxy cross-ply laminates under uniaxial tension. *Compos Sci Technol.* 1985;24:1–31.
2. Duflo M. The extended finite element method in thermoelastic fracture mechanics. *Int J Numer Methods Eng.* 2008;74:827–47.
3. Tsang DKL, Oyadiji SO, Leung AYT. Two-dimensional fractal-like finite element method for thermoelastic crack analysis. *Int J Solids Struct.* 2007;44:7862–76.
4. Dell’Erba DN, Aliabadi MH. BEM analysis of fracture problems in three-dimensional thermoelasticity using J-integral. *Int J Solids Struct.* 2001;38:4609–30.
5. Helien TK, Cesari F. On the solution of the centre cracked plate with a quadratic thermal gradient. *Eng Fract Mech.* 1979;12:469–78.
6. Wilson WK, Yu IW. The use of the J-integral in thermal stress crack problems. *Int J Fract.* 1979;15:377–87.
7. Doblare M, Espiga F, Gracia L, Alcantud M. Study of crack propagation in orthotropic materials using the boundary element method. *Eng Fract Mech.* 1990;37:953–67.
8. Liu N, Altiero NJ. A new boundary element method for plane steady-state thermoelastic fracture mechanics problems. *Appl Math Model.* 1992;16:618–29.
9. Prasad NNV, Aliabadi MH, Rooke DP. The dual boundary element method for thermoelastic crack problems. *Int J Fract.* 1994;66:255–72.
10. Prasad NNV, Aliabadi MH, Rooke DP. The dual boundary element method for transient thermoelastic crack problems. *Int J Solids Struct.* 1996;33:2695–718.
11. Wang SS, Yau IF. Analysis of cracks emanating from a circular hole in unidirectional fiber-reinforced composites. *Eng Fract Mech.* 1980;13:57–67.
12. Joshi SJ, Manepatil S. Stress intensity factors for cracks emanating from a circular hole in a finite orthotropic lamina. In: *IEEE Colloq Humanit Sci Eng Res*; 2011. p. 12–16.
13. Cheong SK, Kwon ON. Analysis of a crack approaching a circular hole in cross-ply laminates under biaxial loading. *KSME Int J.* 1998;12:41–49.
14. Prasad NNV, Aliabadi MH, Rooke DP. Effect of thermal singularities on stress intensity factors: edge crack in rectangular and circular plates. *Theor Appl Fract Mech.* 1996;24:203–15.
15. Tsai CH, et al. Thermal weight function of cracked bodies subjected to thermal loading. *Eng Fract Mech.* 1992;41:27–40.
16. Prasad NNV, Aliabadi MH, Rooke DP. Incremental crack growth in thermoelastic problems. *Int J Fract.* 1994;66:45–50.
17. Kokini K, Choules BD. Surface thermal fracture of functionally graded ceramic coatings: effect of architecture and materials. *Compos Eng.* 1995;5:865–77.
18. Giannakopoulos AE, Suresh S, Finot M, Olsson M. Elastoplastic analysis of thermal cycling layered materials with compositional gradients. *Acta Metall Mater.* 1995;43:1335–54.
19. Jin ZH, Batra RC. Stress intensity relaxation at the tip of an edge crack in a functionally graded material subjected to thermal shock. *J Therm Stresses.* 1996;19:317–39.

20. Noda N. Thermal stress intensity factor for a functionally graded plate with an edge crack. *J Therm Stresses*. 1997;20:373–87.
21. Duong CN, Yu J. The hybrid crack-tip element approach to thermoelastic cracks. *Int J Solids Struct*. 1998;35:5159–71.
22. Dell’Erba DN, Aliabadi MH, Rooke DP. Dual boundary element method for three-dimensional thermoelastic crack problems. *Int J Fract*. 1998;94:89–101.
23. Rolfes R, Noor AK, Sparr H. Evaluation of transverse thermal stresses in composite plates based on first-order shear deformation theory. *Comput Methods Appl Mech Eng*. 1998;167:355–68.
24. Yuan H, Kalkhof D. Effects of temperature gradients on crack characterisation under thermo-mechanical loading conditions. *Int J Fract*. 1999;100:355–77.
25. Choi SR, Hutchinson JW, Evans AG. Delamination of multilayer thermal barrier coatings. *Mech Mater*. 1999;31:431–47.
26. Abd-Alla AM, Abd-Alla AN, Zeidan NA. Thermal stresses in a non-homogeneous orthotropic elastic multilayered cylinder. *J Therm Stresses*. 2000;23:413–28.
27. Jin ZH, Paulino GH. Transient thermal stress analysis of an edge crack in a functionally graded material. *Int J Fract*. 2001;107:73–98.
28. Chen YZ, Hasebe N. Solution for a curvilinear crack in a thermoelastic medium. *J Therm Stresses*. 2003;26:245–59.
29. Kerezi B, Price JWH, Ibrahim R. A two-stage model for predicting crack growth due to repeated thermal shock. *Eng Fract Mech*. 2003;70:721–30.
30. Herrmann KP, Loboda VV, Kharun IV. Interface crack with a contact zone in an isotropic biomaterial under thermomechanical loading. *Theor Appl Fract Mech*. 2004;42:335–48.
31. Banks-Sills L, Dolev O. The conservative M-integral for thermoelastic problems. *Int J Fract*. 2004;125:149–70.
32. Shahani AR, Nabavi SM. Closed-form stress intensity factors for a semi-elliptical crack in a thick-walled cylinder under thermal stress. *Int J Fatigue*. 2006;28:926–33.
33. Tang XS. Mechanical and thermal stress intensification for mode-I crack tip: fracture initiation behaviour of steel, aluminium and titanium alloys. *Theor Appl Fract Mech*. 2008;50:92–104.
34. Nomura Y, Ikeda T, Miyazaki N. Stress intensity factor analysis at an interfacial corner between anisotropic biomaterials under thermal stress. *Eng Fract Mech*. 2009;76:221–35.
35. Kidane A, Chalivendra VB, Shukla A, Chona R. Mixed-mode dynamic crack propagation in graded materials under thermo-mechanical loading. *Eng Fract Mech*. 2010;77:2864–80.
36. Ranc N, Palin-Luc T, Paris PC. Thermal effect of plastic dissipation at the crack tip on the stress intensity factor under cyclic loading. *Eng Fract Mech*. 2011;78:961–72.
37. Bouhala L, Makradi A, Belouettar S. Thermal and thermo-mechanical influence on crack propagation using an extended mesh-free method. *Eng Fract Mech*. 2012;88:35–42.
38. Nagai M, Ikeda T, Miyazaki N. Stress intensity factor analysis of a three-dimensional interface crack between dissimilar anisotropic materials under thermal stress. *Eng Fract Mech*. 2012;91:14–36.
39. Li W, Deng X, Rosakis AJ. Determination of temperature field around a rapidly moving crack tip in an elastic-plastic solid. *Int J Heat Mass Transf*. 1996;39:677–90.