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Automatic characteristic analysis method of multi-degrees-of-freedom planetary gear train based on graph theory

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Abstract

The purpose of this paper is to develop a computer-aided analysis method of the planetary gear train (**PGT**) based on graph theory. In this paper, a novel graph model is developed to represent the structure of the **PGTs**. Then, according to the gear shift strategy, the characteristic analysis models are deduced for each operating mode. The structure of the graph model is efficiently described through several variations of the incidence matrix. The speeds and the torques of all structural components are calculated by using matrix operations. The efficiency of the transmission system is calculated through the torque method in each operating mode. Furthermore, the power flow analysis model of each operating mode is obtained to analyse the power flow in the transmission system. The components that do not participate in the power flow are detected and the directions of the power flow through the components are assigned by using simple graph-based method. To verify the correctness and effectiveness of the proposed method, a novel hybrid transmission scheme is used. The results show that the proposed method can automatically calculate the characteristics of a **PGT** through efficient graph-based methods and matrix operations. Thus,

this method provides a basis for automatic selection of the optimal solution for a **PGT** among a large number of schemes.

Keywords

planetary gear train, characteristic analysis, graph-based method, optimal solution design, automation

Introduction

For its unique advantages, including high compactness and ease of power split and conflux, planetary gear trains are widely used in various vehicles, including conventional **ICE** vehicles as well as hybrid electric vehicles. However, it is very difficult to perform the conceptual design manually due to the large number of possible alternatives [1-5]. Such an excessive amount of manual computation often results in computational errors or the selection of local optimal alternatives. Therefore, automating the optimal design of the scheme is an important way to ensure both the accuracy of the scheme design and the computational efficiency.

The optimal design of a planetary gear train (**PGT**) structure can generally be divided into three stages, i.e. generation, evaluation and optimization stage of scheme [2]. In the generation stage of the scheme all possible gear train topologies are generated. Here, all pseudo-isomorphic, isomorphic and degenerate structures must be discarded. In the evaluation stage, it is determined whether the scheme is mechanically feasible and can realize the given gear ratios. In the final stage, the optimal structural scheme and design variables are identified based on the characteristic analysis and comparative evaluation of each scheme.

To perform these three stages automatically by a computer, first, **PGT** must be represented by a mathematical model that can be recognized by a computer. This mathematical model has a great influence on the performance and computational efficiency of each stage. Therefore, developing an excellent model that is available and computationally efficient at all stages of the scheme design offers a possibility to shorten the whole design cycle and increase the accuracy.

Relevant research work

Graph is an effective tool in the synthesis and analysis of mechanisms, including planetary gear mechanisms. So far, many methods using graph theory have been developed and utilized in different stages of the scheme design [3,5]. Problems and solutions associated with the synthesis of mechanisms during the scheme generation stage were described in various works [6-14]. Bucknor and Hwang and Huang described methodologies to check mechanical feasibility of gear train topologies through planarity check algorithms of graph theory [15, 16]. The other works depicted the kinematic, static, power flow and efficiency analysis methods of **PGTs** using graph theory [17].

The main problem to be solved in the scheme generation stage is to suppress the generation of pseudo-isomorphic, isomorphic and degenerate structures as well as enumeration of all possible alternatives. However, conventional graph-based methods did not effectively suppress the generation of such structures [7]. To solve the pseudo-isomorphism problem, different graphical representations of planetary gear mechanisms have been developed [8-10]. Sun et al. proposed a method to realize the isomorphism identification of **PGTs** using weighted adjacency matrix [11]. Yang and Ding and Ali et al. proposed the methods to suppress the generation of degenerate structures [12, 13]. Nafeh et al. proposed a method to reduce the generation of degenerate and isomorphic structures significantly while completely suppressing the generation of pseudo-isomorphic structures by using different link assortment arrays [14]. Here the planetary gear mechanism was represented by the canonical graph.

The goal of the characteristic analysis of the scheme in the optimization stage is to analyse the speeds and torques of components, the power flow and efficiency inside the **PGT** system, which are the basis of the comparative evaluation and the design of the next step.

The models used for characteristic analysis include lever analogy model, Fundamental Circuit (**FC**) diagram, topology graph and so on. Although the lever analogy model is a very intuitive and efficient analytical tool, we have to construct the model separately for each structure. In particular, the model has

the disadvantage of being very complex as the number of degrees of freedom and structural components increase [17]. Several researchers modelled **PGTs** using **FC** diagram and performed characteristic analyses based on **FC** equation [18-21]. These methods have to detect all **FCs** exist in the structure, and there is a disadvantage of the complex transformation of the model with the change of the structure. Gao and Hu proposed a new graphical representation of the **PGT** based on Wolf symbol and based on it analysed the relationship between topology and gear ratio [22]. Esmail and Hassan and Esmail proposed nomograph representation to analyse the structural characteristics of **PGTs** [23, 24]. Schulz and analysed the characteristics of hybrid transmission structures [25]. Cui et al. proposed the multi-attribute topological graph method in characteristic analysis of **PGTs** [26]. Yang et al. described a method to determine the efficiency of a multi-flow **PGT** by following the power flow through hypergraph and matrix operation [27]. Du and Yang proposed a model of **PGT** containing different components and presented two algorithms to determine the dynamic parameters of structural components [28]. However, these methods cannot determine the torque acting on the control components, which is an important evaluation index of the scheme. Xue and Li proposed a performance analysis method for multi-degree-of-freedom **PGT** that can solve this problem [29]. However, since the models used for kinematic, static and power flow analysis are all different, the transformation process between them is troublesome. Therefore, the computational efficiency cannot be achieved. Moreover, this method is limited to a single-input single-output system.

Research in this article

In order to solve above problems, we propose a novel graph model of **PGT** based on the canonical graph representation. This model reflects the information about planetary gear mechanism and control components, i.e. brakes and clutches. Then, according to the control strategy of the clutches, the characteristic analysis model and power flow analysis model for each stage are established, and the characteristic analysis of the gear train is carried out using graph theory. The flowchart of the method is shown in Figure 1.

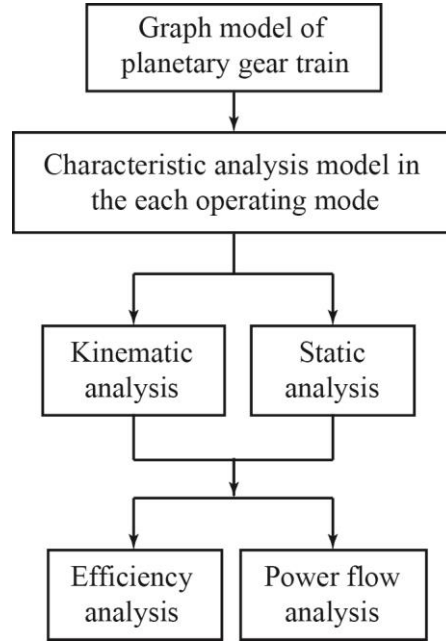


Figure 1. A flow chart of the characteristic analysis method proposed in this paper

The reminder of the paper is organized as follows.

1. Graph model of **PGT** and matrix representation: In this section, a graph model reflecting the overall structure of **PGT** is proposed. By placing the vertices in different layers and differentiating the weight of the edges, various components and the relationships between them are represented. Characteristic analysis model in each operating mode is deduced according to the control strategy of the **PGT**.
2. Characteristic analysis method of **PGT**: This section presents a characteristic analysis method based on the characteristic analysis model of **PGT**. In addition, the power flow analysis model was constructed by removing the components that do not participate in the power transfer in the characteristic analysis model, and the presence or absence of the power cycle and the magnitude of the circulating power were determined.
3. Results: This section demonstrates the effectiveness of the method through numerical examples. A new hybrid transmission scheme and its 3 operating modes are selected as examples.

Contributions

1. New modelling technique of **PGTs** is proposed. By differentiating the layer of the vertices and the

weight of the edges, **PGT** is represented in a systematic graph model. This contributes to the facilitation of further consideration of **PGT**, especially on the computer.

2. Characteristic analysis method based on the new graph model is proposed. Through several variations of incident matrix, kinematic and static coefficient matrices needed to calculate the speeds and torques are easily obtained. This facilitates the analysis of the characteristics of a great number of **PGTs**.
3. Components that do not participate in the power flow are easily detected through simple graph-based methods. Power flow characteristics can intuitively be seen through the power flow analysis model. This contributes to the next step of design process, such as the strength calculation of the components.
4. Proposed methods are implemented in **MATLAB**. This is essential to the automatic selection of optimal structures of **PGTs**. This can be utilized in the early stages of conceptual design of **PGTs**.

Graph model of PGT and matrix representation

In this section, we focus on developing a graph model that can facilitate automation of characteristic analysis of **PGTs**. We develop the concept of canonical graphs to propose a new graph model that represents the structure of **PGTs**.

Modified canonical graph of planetary gear mechanism

The planetary gear trains used in automatic transmissions consist of a combination of different kinds of planetary gear sets (**PGSs**), i.e. single planetary gear set (**SPGS**), double planetary gear set (**DPGS**), and complex compound planetary gear set (**CCPGS**). However, **CCPGSs** can be considered as a combination of the two previous types of **PGSs**. Therefore, **SPGS** and **DPGS** are two basic planetary gear set (**BPGS**)s of planetary gear mechanisms. Figure 2 shows two **BPGSs** and the decomposition of a Ravigneaux planetary gear train (**RPGS**) into **BPGSs**.

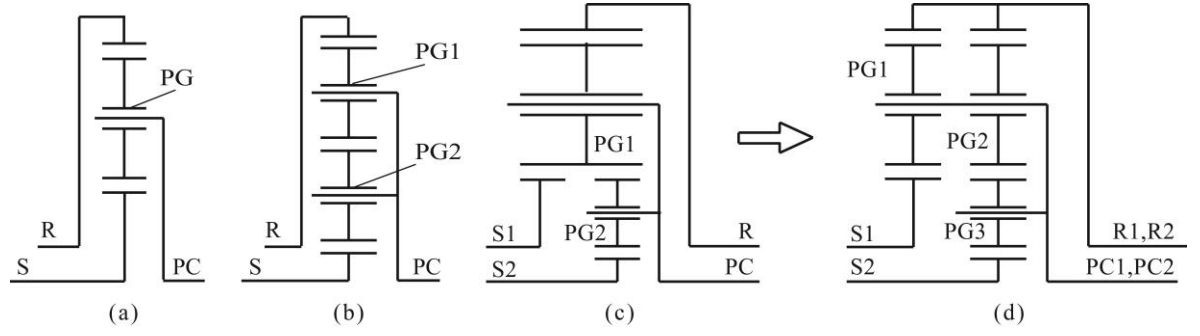


Figure 2. Kinematic diagram of two **BPGSs** and a **RPGS**: (a) **SPGS**; (b) **DPGS**; (c) **RPGS**; (d) decomposition of a **RPGS**.

S: sun gear; R: ring gear; PC: planet carrier; PG: planetary gear.

A **SPGS** contains one planetary gear in the structure, and a **DPGS** contains two planetary gears. In a canonical graph, a vertex represents a link and an edge represents a joint. The root vertex (the vertex on the ground layer) represents the fixed link, and the vertex of the first layer represents the central links, i.e., the sun gear, the ring gear, and the planet carrier. Also, according to the number of planetary gears, the corresponding number of vertices are located on the second layer. However, planetary gears do not transmit force and motion directly to the outside, but only serve as auxiliary components. Hence, the modified canonical graph contains only one vertex representing the planetary gear for each **BPGS**, regardless of the type of them. On the other hand, the weight of the edges are also modified in the modified canonical graph. The weights of the edges representing the revolute pair and the external gear pair are 1 and 2, respectively, as in the canonical graph, while the weights of the edges representing the internal gear pair are 3. Figure 3 shows the canonical and modified canonical graphs of the two **BPGSs** and the **RPGS**.

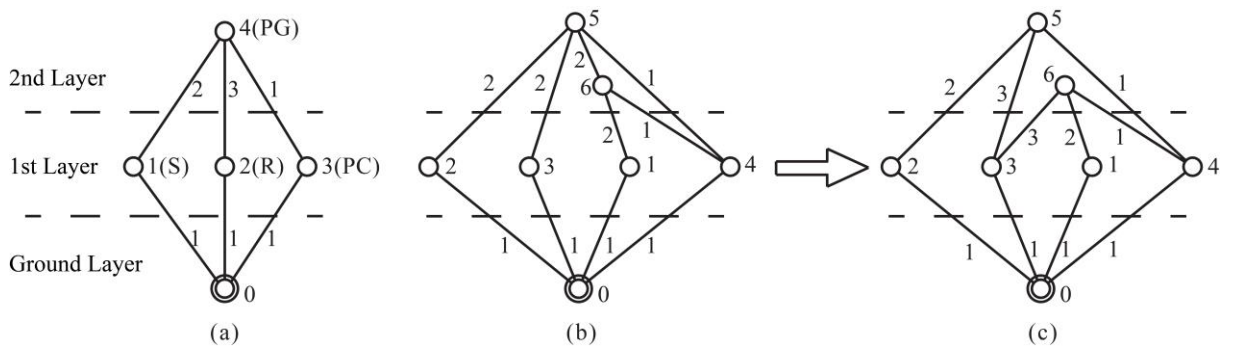


Figure 3. Modified canonical graph of **BPGS** and canonical and modified canonical graph of **RPGS**: (a) modified

canonical graph of a **BPGS**; (b) canonical graph of a **RPGS**; (c) modified canonical graph of **RPGS**. Edge labels indicate the weight of the edge.

*Graph model of **PGT***

The new graph model is based on the canonical graph, but its vertices and edges have different meanings with those of the previous graph model. In the conventional graph representation of the mechanism, vertices represent links and edges represent joints. In the new graph model, the edges represent different structural components of **PGT**. Then, through the weight of the edges, we can distinguish which component corresponds to the edges. Also, the vertices have different semantics according to the layers. The root vertex represents the outside (i.e. power sources and loads) of the gear train as well as the case. The vertex on the first layer represents the motion links of the planetary gear mechanism except for planetary gears, and the vertex on the second layer represents the **BPGS**.

The planetary gear mechanism is the most fundamental component of **PGT**. The motion links of planetary gear mechanism (except for planetary gears) can be divided into three families, i.e. power component, brake component and auxiliary connection component. A power component is a component that can be used as the input or output of power, and a brake component is one that can be locked with the case through the brake. Additionally, an auxiliary connection component is not capable of performing these two missions but only the transfer function of the power. Therefore, in the graph model of **PGT**, vertices on the first layer can be divided into three groups. We propose a method to implement this partitioning through the different weights of the edges connecting these vertices and the root vertex. The weight of the edge connecting the vertices on the first layer and the root vertex is 4, 5, and 6, respectively, depending on whether the vertex on the first layer is an input, an output or a brake component. In case of auxiliary connection component, the edge is deleted.

The weights of the edges connecting the vertex on the first layer with the one on the second layer are the same as in the modified canonical graph. These edges represent different central links of the **BPGS**, i.e.

the sun gear, the ring gear and the planet carrier.

Now, the remainder components which are needed to be represented in the graph model among the structural components of **PGT** are the clutches. A clutch is used to lock two motion links of **PGT**. Therefore, in the graph model, the clutch can be represented by adding a weight-7 edge between vertices representing two motion links being locked.

If a given gear train is a switched-input or switched-output one, the first layer contains an extra vertex representing the input or the output component, together with vertices representing the motion links of the pure planetary gear mechanism. Figure 4 shows the kinematic diagram and the graphical model of the AL4 automatic transmission. This gear train is a switched-input gear train, so there must be an extra vertex representing the input component. In Figure 4, vertex 5 indicates the input component.

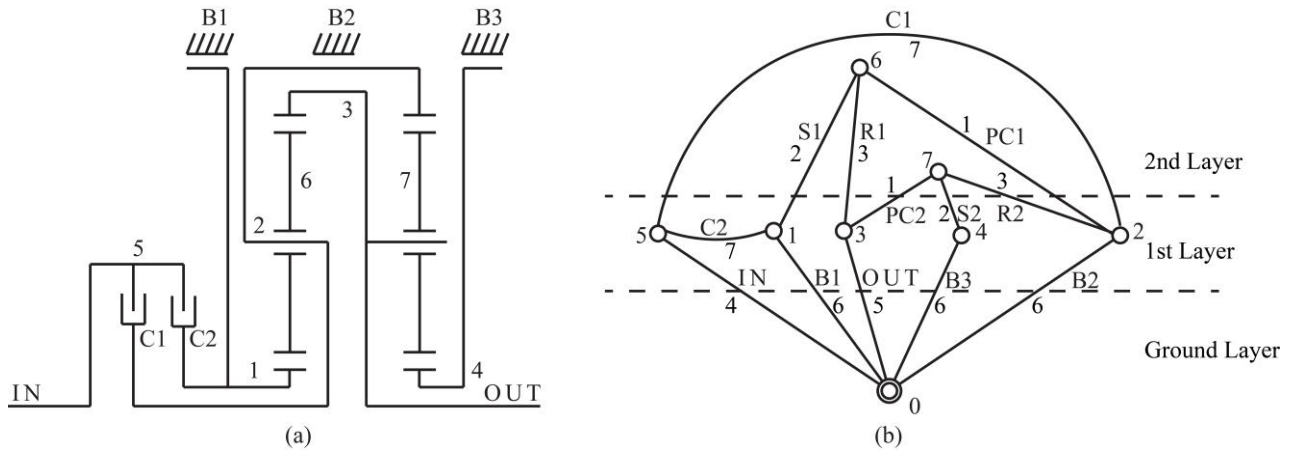


Figure 4. Kinematic diagram and graph model of AL4 automatic transmission: (a) kinematic diagram; (b): graph model.

IN: input component; OUT: output component; B: Brake; C: Clutch.

Characteristic analysis model for different operating modes

Whether the system is single or multiple input, **PGT** implements different operating modes using different control modes for the control components and power ones. Therefore, to perform the characteristic analysis in a given operating mode, the graph model of **PGT** has to be modified to accurately reflect the input-output relationship and the connectivity of control components. The following methods are used to

obtain the characteristic analysis model in the corresponding operating mode.

- (1) Identify the input-output relationship. That is, it is needed to be determined which power components are connected to the power source and which are connected to the load. Power components which are not connected to the power source or sink are removed. The weights of the edges are changed if necessary.
- (2) Then, the edges representing the unconnected control components are removed.

Figure 5 shows the characteristic analysis model of the AL4 automatic gear train at the forward first gear. This gear train connects the brake B3 and clutch C2 at the forward first gear.

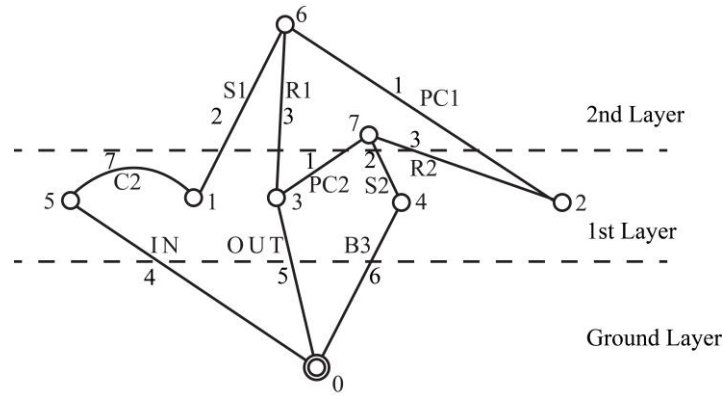


Figure 5. Characteristic analysis model of AL4 automatic analysis model at the forward first gear

Matrix representation

The structure of a graph can be conveniently represented by a matrix. The matrix representation of graphs facilitates the structural analysis of graphs by means of computers.

The incidence matrix is a matrix that expresses the incidence between vertices and edges. First, the vertices and edges are numbered appropriately. The numbers of vertices are increased one by one in the order of 1st and 2nd layers, with the root vertex as 0. The numbers of edges start from 1 and increase in the order of the components of the input, output, brake, clutch and **BPGS**. For the components of a **BPGS**, the number is increased according to the order of sun gear, ring gear, and planet carrier. In Figure 6, the results of numbering are shown for the graph model shown in Figure 5.

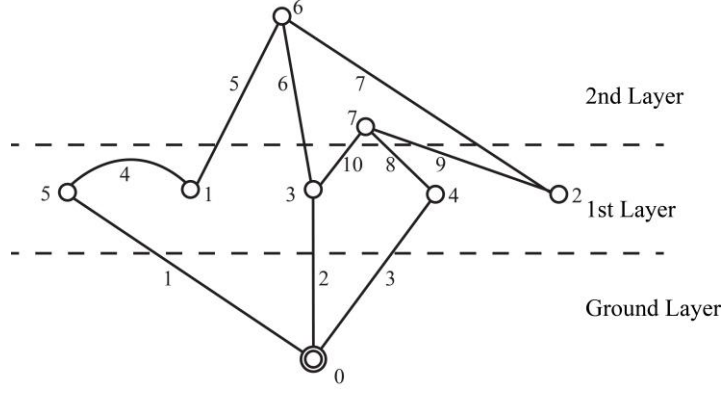


Figure 6. The numbering result for the graph shown in Figure 5.

When the number of vertices is v and the number of edges is e , the incidence matrix $\mathbf{B}$ is defined as a $v \times e$ matrix as follows:

$$b_{ij} = \begin{cases} 1 & \text{vertex } i \text{ is incident to edge } j \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where b_{ij} denotes (i, j) element of $\mathbf{B}$.

The incidence matrix of the graph $\mathbf{B}$ shown in Figure 6 is as follows:

$$\mathbf{B} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \quad (2)$$

The incidence matrix $\mathbf{B}$ does not specify which element a given edge represents. To solve this problem, we introduce a weighted incidence matrix $\hat{\mathbf{B}}$. This matrix is defined as follows:

$$\hat{b}_{ij} = \begin{cases} w_j & \text{vertex } i \text{ is incident to edge } j \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where $\hat{b}_{ij}$ denotes (i, j) element of $\hat{\mathbf{B}}$, and w_j is the weight of j -th edge. The weighted incidence matrix $\hat{\mathbf{B}}$ of the graph shown in Figure 5 is as follows:

$$\hat{\mathbf{B}} = \begin{pmatrix} 4 & 5 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 0 \\ 0 & 5 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 \\ 0 & 0 & 6 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 4 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 & 1 \end{pmatrix} \quad (4)$$

The matrix obtained by deleting the first row in $\mathbf{B}$, i.e., the row corresponding to the root vertex, is called the reduced incidence matrix and is denoted by $\tilde{\mathbf{B}}$. Then the reduced incidence matrix of the graph shown in Figure 6 is as follows:

$$\tilde{\mathbf{B}} = \begin{pmatrix} 0 & 0 & \vdots & 0 & 1 & \vdots & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 & \vdots & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & \vdots & 0 & 0 & \vdots & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & \vdots & 1 & 0 & \vdots & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & \vdots & 0 & 1 & \vdots & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \vdots & \dots & \dots & \vdots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \vdots & 0 & 0 & \vdots & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 & \vdots & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \quad (5)$$

The reduced incidence matrix $\tilde{\mathbf{B}}$ can be divided into six parts as follows:

$$\tilde{\mathbf{B}} = \begin{pmatrix} \tilde{\mathbf{B}}_{PC} & \tilde{\mathbf{B}}_{CC} & \tilde{\mathbf{B}}_{PGS1} \\ \mathbf{O} & \mathbf{O} & \tilde{\mathbf{B}}_{PGS2} \end{pmatrix} \quad (6)$$

where $\tilde{\mathbf{B}}_{PC}$, $\tilde{\mathbf{B}}_{CC}$ and $\tilde{\mathbf{B}}_{PGS1}$ represent the connectivity of the moving links with power components, control components and the components of the planetary gear train, respectively. $\mathbf{O}$ means zero matrix, and $\tilde{\mathbf{B}}_{PGS2}$ holds the information about the **BPGSs** and the central components belonging to them.

The weighted reduced incidence matrix $\bar{\mathbf{B}}$ can be defined and divided into submatrices similarly.

Characteristic analysis method of PGT

In this section, we describe the analysis method of the velocity, torque, system efficiency and circulating

power of different structural components of **PGT** in different operating modes using the graph model and matrix representation described in the previous section. To facilitate the calculation, we represent the velocity and torque of all structural components in vector form as follows:

$$\mathbf{T} = (T_1, T_2, \dots, T_i, \dots, T_e)', \quad \boldsymbol{\omega} = (\omega_1, \omega_2, \dots, \omega_i, \dots, \omega_e)' \quad (7)$$

where T_i and ω_i denotes the torque and velocity of the component which is corresponding to j -th edge.

If an edge refers to a connected clutch component in the characteristic analysis model, the speed of the component is assumed as the speed of components which are being locked, and if an edge refers to a connected brake, it is assumed to be the speed of braked component, i.e. 0.

These speed and torque values can be obtained by constructing the kinematic coefficient matrix (**KCM**) and static coefficient matrix (**SCM**) which represent the velocity and torque relationship of various structural components of **PGT**.

Kinematic analysis

The **KCM** $\mathbf{K}$ consists of **PGS** coefficient matrix $\mathbf{K}_1$, rigid connection coefficient matrix $\mathbf{K}_2$ and operating mode coefficient matrix $\mathbf{K}_3$ as follows:

$$\mathbf{K} = \begin{pmatrix} \mathbf{K}_1 \\ \mathbf{K}_2 \\ \mathbf{K}_3 \end{pmatrix} \quad (8)$$

1. **PGS** coefficient matrix

Three central links of a **BPGS** satisfy the following velocity relationship.

$$\omega_s + k\omega_r - (1+k)\omega_c = 0 \quad (9)$$

where ω_s , ω_r and ω_c represent the speed of sun gear, ring gear and planet carrier, respectively. k is the characteristic coefficient of the **BPGS**. For a **SPGS**, $k = Z_r / Z_s$, and $k = -Z_r / Z_s$ for a **DPGS**. Z_s and Z_r are the numbers of teeth of sun gear and ring gear, respectively.

Then, the **PGS** coefficient matrix can be easily obtained through the submatrix $\bar{\mathbf{B}}_{pGS2}$ of the weighted reduced incidence matrix of the corresponding characteristic analysis model. The **PGS** coefficient matrix corresponding to the example shown in Figure 5 is as follows:

$$\mathbf{K}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & k_1 & -(1+k_1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & k_2 & -(1+k_2) \end{pmatrix} \quad (10)$$

2. Rigid connection coefficient matrix

In a **PGT**, several structural components are rigidly connected to form a motion link. The velocities of these components are equal to each other. Thus, in the example shown in Figure 5, the following velocity relationships are satisfied:

$$\left\{ \begin{array}{l} \omega_4 = \omega_5 \\ \omega_7 = \omega_9 \\ \omega_2 = \omega_6 \\ \omega_2 = \omega_{10} \\ \omega_3 = \omega_8 \\ \omega_1 = \omega_4 \end{array} \right. \quad (11)$$

The rigid connection coefficient matrix represents the consistency of these velocities. For a **PGT** with n degrees of freedom and p number of **BPGSs**, information about the rigidly connected components can be found through the $1 \sim n+p$ -th rows of the reduced incidence matrix. That is, the velocities of the structural components corresponding to nonzero elements in the same row of this submatrix are identical. A row of the rigid connection coefficient matrix corresponds to one equation of Eq. (11). The element of the matrix for one structural component is 1, and the element of the matrix for the other one is -1 in the same row. The rigid connection coefficient matrix corresponding to the example shown in Figure 5 is as follows:

$$\mathbf{K}_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (12)$$

3. Operating mode coefficient matrix

The operating mode coefficient matrix gives information on which structural components are input and brake components in the given operating mode of the gear train. One row of the operating mode coefficient matrix represents a power component (input or output) with a given speed or a brake component. The corresponding matrix element is 1, which is the only non-zero value on the row. From the meaning of degrees of freedom, the number of rows of the operating mode coefficient matrix is equal to $n-1$. The operating mode coefficient matrix for the example shown in Figure 5 is as follows:

$$\mathbf{K}_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (13)$$

4. Kinematic analysis method

Kinematic analysis is insufficient with only **KCM** and the kinematic constant matrix is required. The kinematic constant matrix represents information about the output speeds of the power sources. Thus, for the example shown in Figure 5, when the output speed of the power source is ω_{IN} , the kinematic constant matrix is as follows:

$$\mathbf{V} = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \omega_{IN} \ 0)' \quad (14)$$

Then, the speed vector of the different structural components of **PGT** satisfies the following relationship:

$$\mathbf{K}\boldsymbol{\omega} = \mathbf{V} \quad (15)$$

Thus, the vector of all structural components can be obtained as follows:

$$\boldsymbol{\omega} = \mathbf{K}^{-1}\mathbf{V} \quad (16)$$

Static analysis

First, to simplify the calculation, we calculate the torque of all structural components without power loss. The static coefficient matrix $\mathbf{S}$ is composed of the **PGS** torque matrix $\mathbf{S}_1$, the torque equilibrium matrix $\mathbf{S}_2$, and the torque reference matrix $\mathbf{S}_3$.

$$\mathbf{S} = \begin{pmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \\ \mathbf{S}_3 \end{pmatrix} \quad (17)$$

1. **PGS** torque matrix

The following relationship is satisfied between the torque of three central components of a **BPGS**.

$$T_s = \frac{T_r}{k} = \frac{T_c}{1+k} \quad (18)$$

where T_s , T_r and T_c denote the torque acting on the sun gear, the ring gear and the planet carrier, respectively.

The **PGS** torque matrix represents the torque relationship between any two elements. For example, the relationship between the torque of the sun gear and the ring gear can be described. Which elements are belonging to the **PGS** can be easily obtained through the matrix $\bar{\mathbf{B}}_{PGS2}$. For the structure shown in Figure 5, the **PGS** torque matrix is written as follows:

$$\mathbf{S}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & -1/k_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1/k_2 & 0 \end{pmatrix} \quad (19)$$

2. Torque equilibrium matrix

In equilibrium, the torques acting on a **BPGS** or a motion link are balanced. Here, for convenience of calculation, the following assumptions about the direction of the torque are introduced.

- (1) The torques acting on the power and brake components are always positive.
- (2) The torque acting on the sun gear, the ring gear, and the planet carrier is positive for the **BPGS** and

negative for the motion link.

(3) The torque acting on a clutch is positive for one of the two motion links and negative for the other.

This relationship can be set arbitrarily.

In the characteristic analysis model, the **BPGSs** and the motion links are represented by the vertices on the first and second layer. So, the torque equilibrium matrix can be easily obtained through the reduced incidence matrix.

For the example shown in Figure 5, the transformed matrix $\tilde{\mathbf{B}}'_{CC}$ according to assumption (3) is as follows:

$$\tilde{\mathbf{B}}'_{CC} = \begin{pmatrix} 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (20)$$

Then torque equilibrium matrix $\mathbf{S}_2$ can be represented through submatrices of reduced incidence matrix as follows:

$$\mathbf{S}_2 = \left(\begin{array}{c|c|c} \tilde{\mathbf{B}}_{PC} & \tilde{\mathbf{B}}'_{CC} & -\tilde{\mathbf{B}}_{PGS1} \\ \mathbf{O} & \mathbf{O} & \tilde{\mathbf{B}}_{PGS2} \end{array} \right) \quad (21)$$

3. Torque reference matrix

The torque reference matrix represents which of the power sources is the reference for torque calculation. That is, when the torque of the input component is taken as the reference in the example shown in Figure 5, the torque reference matrix is as follows.

$$\mathbf{S}_3 = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) \quad (22)$$

4. Static analysis method

In order to calculate the torque of all structural components, it is necessary to know the torque of the reference component among the power sources or loads. For the example shown in Figure 5, the static constant matrix is as follows:

$$\mathbf{C} = (T_{IN} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)' \quad (23)$$

Torque vector satisfies the following equation.

$$\mathbf{S}\mathbf{T} = \mathbf{C} \quad (24)$$

Thus, the torque of all structural components is easily obtained using the following equation.

$$\mathbf{T} = \mathbf{S}^{-1}\mathbf{C} \quad (25)$$

Efficiency analysis

There are too many factors affecting the efficiency of **PGT**, so it is very difficult to determine the efficiency accurately. Therefore, for ease of calculation, only the gear meshing loss of the relative power is taken into account. And it is assumed to be 0.98 for the external meshing and 0.99 for the internal meshing. For a **SPGS**, the efficiency of the **BPGS** is $\eta_0 = 0.97$, and for a **DPGS**, $\eta_0 = 0.95$.

The efficiency of the **PGT** is calculated as follows:

$$\eta = \frac{\sum P_{OUT,i}}{\sum P_{IN,i}} = -\frac{\sum T'_{OUT,i}\omega_{OUT,i}}{\sum T_{IN,i}\omega_{IN,i}} \quad (26)$$

where $P_{IN,i}$, $T_{IN,i}$ and $\omega_{IN,i}$ indicate the input power, input torque and input velocity of i -th power source, while $P_{OUT,i}$, $T'_{OUT,i}$ and $\omega_{OUT,i}$ indicate the output power, output torque and output velocity.

Here $T'_{OUT,i}$ is the value when the power loss is considered.

As can be seen from Eq. (26), in order to calculate the efficiency of the **PGT**, it is necessary to determine the actual torque acting on each element when the power loss is taken into consideration. The method of determining the actual torque is almost the same as the ideal torque calculation method. Only the **PGS** torque matrix is established considering the power loss. Here, the following relationship is established between the torque of the sun gear and the ring gear:

$$T'_r = \eta_0^a k T'_s \quad (27)$$

where T'_s and T'_r denote the torque acting on the sun gear and ring gear, respectively, when the power loss is considered. α is an exponent which indicates the direction of the relative power and is calculated as follows.

$$\alpha = \text{sgn}(P_{rel}) \quad (28)$$

where P_{rel} denotes the relative power, and is calculated as $P_{rel} = T_s(\omega_s - \omega_c)$. sgn is a function that gives 1, 0 and -1 whether the value of the argument is positive, zero or negative. Eq. (27) is satisfied under the assumption that the direction of relative power is consistent whether the power loss is considered or not.

Therefore, based on the results of kinematic and ideal torque calculations, the actual torque of each structural component can be calculated by determining the flow direction of the relative power in each **BPGS** and modifying the **PGS** torque matrix. The efficiency of the **PGT** is obtained through Eq. (26).

Power flow analysis

In this section, a simple graph-based approach is introduced to the characteristic analysis model to determine the components involved in the power transfer and to determine the power flow inside the gear train. It is possible to determine the presence or absence of a power recirculation in the given operating mode and determine its magnitude when the power recirculation occurs.

1. Power flow analysis model

The power delivered by each component can be calculated by multiplying the torque and speed of the component. So, a component with no torque is applied or a braked component does not transmit power. Therefore, by deleting the vertices and edges representing these components, a detailed and intuitive analytical model for the power flow can be obtained.

The method of obtaining the power flow analysis model is as follows.

- (1) Delete the vertices representing the brake among the vertices in the first layer. The edges incident to such vertices are also deleted at the same time.
- (2) If there exists a vertex of degree 1 among the vertices in the first layer, we delete this vertex and the vertices in the second layer that are adjacent to it. These second layer vertices represent the unloaded **BPGSs**.
- (3) From the results of kinematic and static analysis, the flow direction of the power in each component is determined. When the power is greater than 0, it means that the power flows in, and when the power is less than 0, it means that the power flows out.
- (4) Determine the flow direction of the power in each component and distinguish the direction of the corresponding edges.
 - a) The direction of the edges except for the clutch edge is from the vertex on the lower layer to the one on the upper layer if the power flows in and vice versa.
 - b) In the case of clutch edge, one among two end vertices is selected. Then, except for the clutch edge, we find the sum of the power delivered through the other edges which are incident to the vertex. When the sum of the power is greater than 0, i.e., the power flows into the clutch, this vertex becomes the start point, and vice versa.

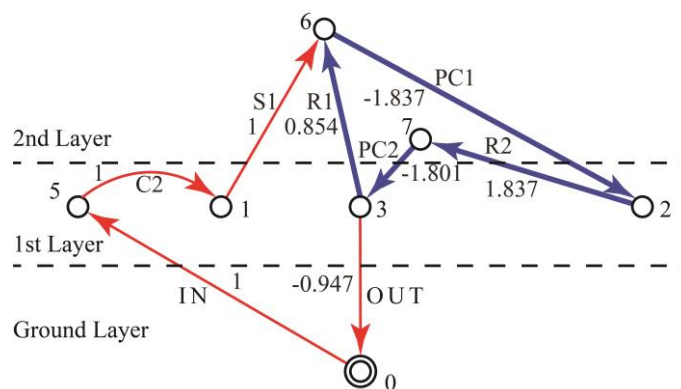


Figure 7. Power flow analysis model of AL4 automatic transmission at forward first gear: Edge labels indicate the ratio of transmitted power through the component to the input power. Bold edges in blue colour show the path of power recirculation.

2. Determination and calculation of power recirculation

Since the recirculating power is a virtual power, it is not available at the load. But it causes a large torque on the structural component and reduces the efficiency of the system. Therefore, the presence or absence of a recirculating power must be checked and its magnitude calculated. Check and calculation method of recirculating power are as follows:

- (1) Remove the root vertex from power flow analysis model.
- (2) Find all the circuits present in the resulting subgraph. If a circuit exists, then the power recirculation occurs, and vice versa.
- (3) When the power recirculation occurs, the magnitude of the recirculating power is the minimum among the power delivered by the different edges of the circuit.

As can be seen from Figure 7, there is a power recirculation in this case. The magnitude of the recirculating power is 0.854 times the input power. Here the characteristic coefficients of the two **BPGSs** are 2.45 and 2, respectively.

Results

To verify the accuracy of the proposed method, a **PGT** shown in Figure 8 is used. This **PGT** is for a hybrid electric vehicle and uses two inputs as a power source: an internal combustion engine and a motor. The motor can also operate in generator mode. Then, the performance analysis of the gear train is carried out for the three operating modes shown in Table 1.

The speed range of power source 1 is $1000 \sim 2500rpm$, and the speed range of power source 2 is $1000 \sim 3000rpm$. The torque of power source 1 is the reference value, and the range is $500 \sim 1000N \cdot m$, and the torque of power source 2 varies with the torque of power source 1. When power source 2 operates in generator mode, i.e., load, its speed range is $500 \sim 1000rpm$.

We also use **MATLAB** as an excellent computational tool to simplify the computation. Whole

computation process and algorithms are written in m-files. The characteristic coefficients of the two **BPGSs** in the structure are 2.45 and 2, respectively.

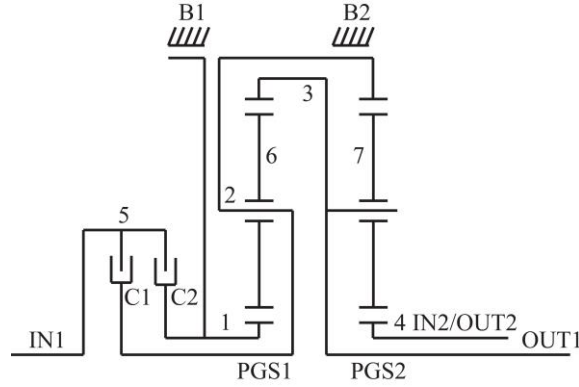


Figure 8. Kinematic diagram of a hybrid electric vehicle gear train

Table 1. Several operating modes of the given gear train

Operating mode	Power source	Power sink	Engaged control components
1	IN1	OUT1	B1, C1
2	IN1, IN2	OUT1	C2
3	IN1	OUT1, OUT2	C2

Graph model and characteristic analysis models

Figure 9 shows the graph model of the given gear train and Figure 10 shows the characteristic analysis model for the three operating modes described in Table 1.

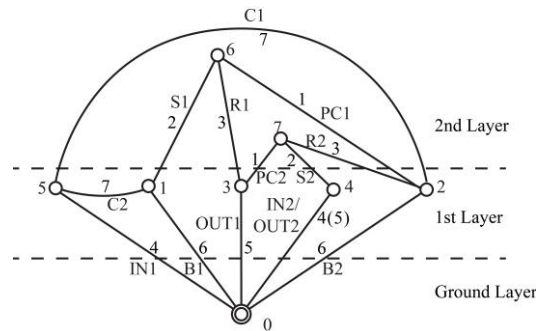


Figure 9. The graph model of the **PGT** shown in Figure 8.

The calculation of the efficiency and the recirculating power

1. First operating mode

In the first operating mode, the gear train operates as a single-input single-output system. The power flow analysis model is shown in Figure 11.

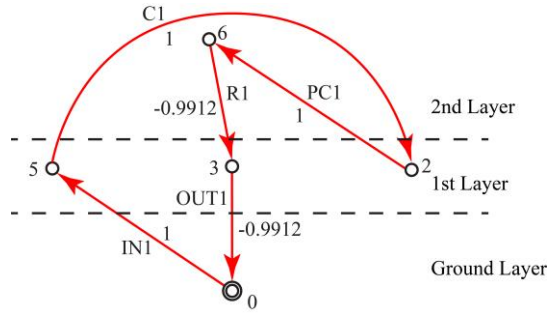


Figure 11. Power flow analysis model in the first operating mode

As can be seen from the figure 11, the efficiency of the gear train is 99.12% and the power recirculation does not exist.

2. Second operating mode

In the second operating mode, the gear train transfers power from two power sources to the load. This mode can be used when the load is large, including overcoming a steep slope. The corresponding power flow analysis model is shown in Figure 12. Then, the weight of the edge represents the ratio of the power delivered by the corresponding component to the power delivered by the power source 1 if the speeds of the power source 1 and 2 are $1000rpm$ and $1500rpm$, respectively. As can be seen from the Figure 12, in this case there exists a recirculating power. The efficiency of the gear train and the magnitude of the recirculating power with the variation of the speed of the power sources are shown in Figure 13.

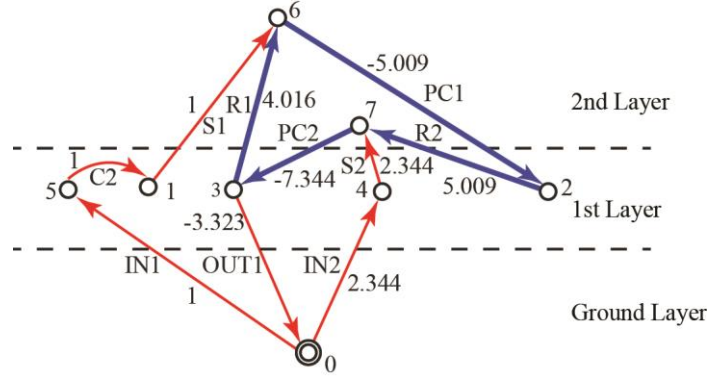


Figure 12. Power flow analysis model in second operating mode.

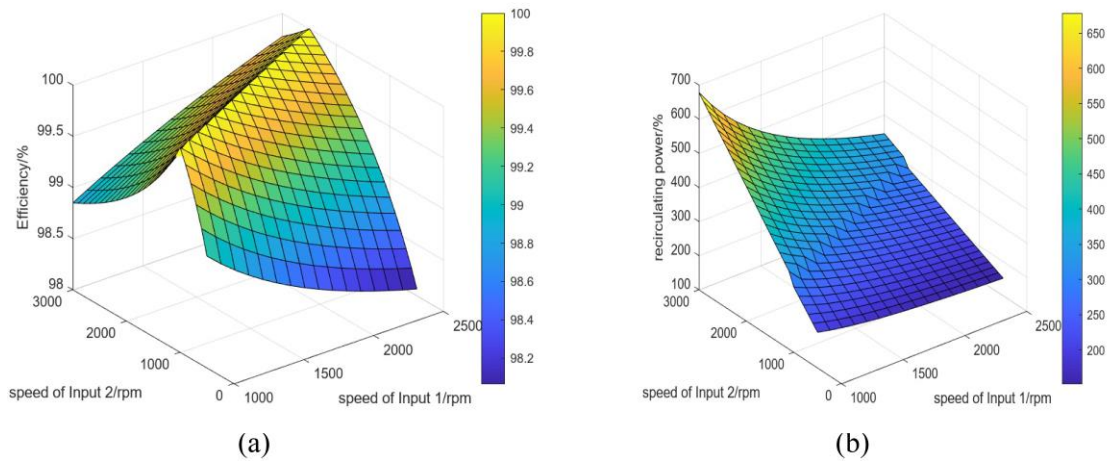


Figure 13. Efficiency and specific recirculating power in the second operating mode: (a) efficiency; (b) specific recirculating power.

3. Third operating mode;

The gear train operates as a single-input two-output system. That is, the power of the internal combustion engine will charge the battery through the generator, overcoming the road load. The power flow characteristic is shown in Figure 14, when the speeds of internal combustion engine and generator are 1500rpm and 1000rpm, respectively. At this time, the efficiency is 94.84% and there exists a recirculating power. The efficiency and the magnitude of the recirculating power at different speeds of the internal combustion engine and generator are shown in Figure 15. As can be seen from the figure, there exists power recirculation in this operating mode, but its magnitude is small enough so that no significant effect is applied to the efficiency of the system.

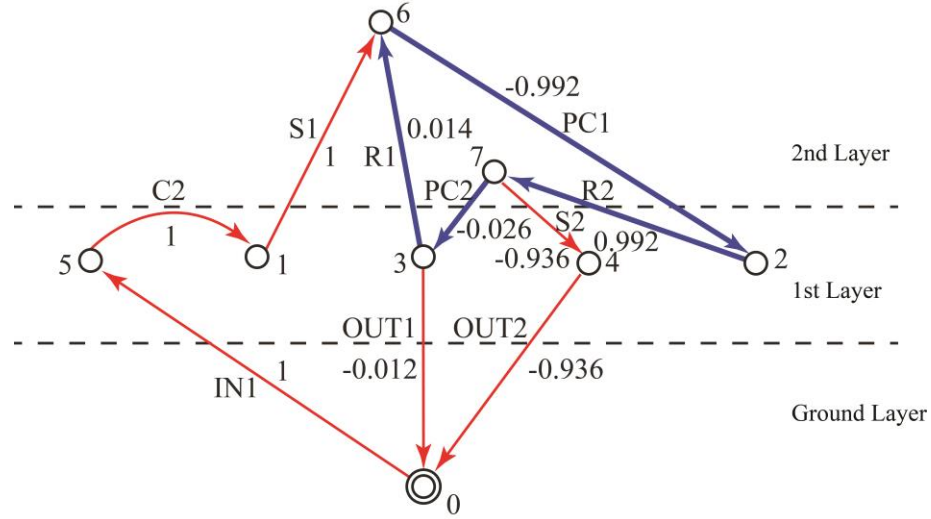


Figure 14. Power flow characteristics in the third operating mode.

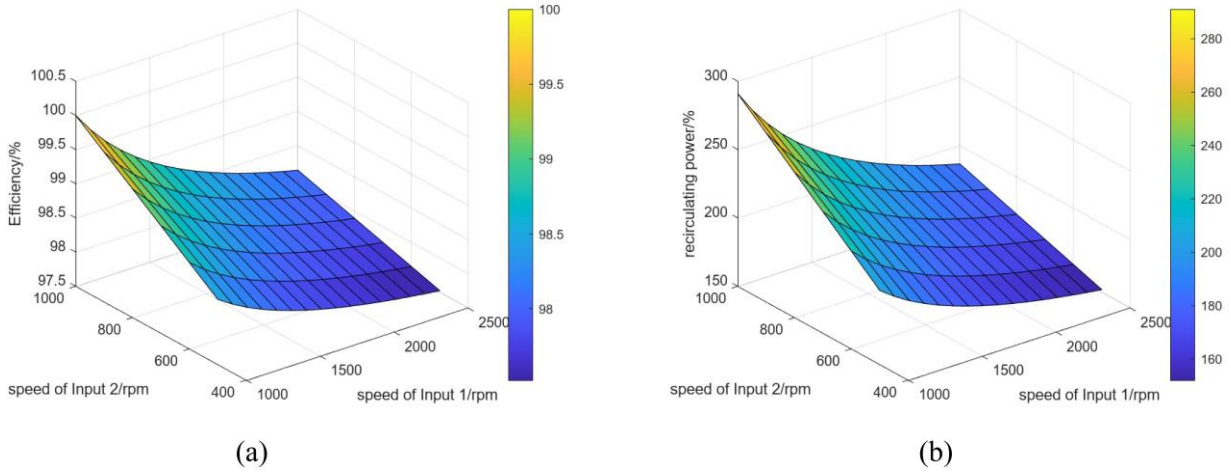


Figure 15. Efficiency and specific recirculating power in the third operating mode: (a) efficiency; (b) specific recirculating power.

The speed and torque of each component, the efficiency and recirculating power, and the detection of components that do not participate in the power transfer are essential for the next design calculation. It took an average of 0.203 s to calculate the overall characteristics of each system. Therefore, it can be used to select the optimal solution among a large number of candidate alternatives.

Conclusions

1. A graph model is developed that reflects the overall structure of **PGT** based on the effective canonical graph which is effective at the generation stage of the scheme. This enables less transformation

between models used in different stages of the scheme synthesis, accordingly, increased the design efficiency of **PGTs**.

2. In addition, a characteristic analysis model is developed for each operating mode using graph theory and clutch control rules, and a new method is proposed to automatically implement kinematic analysis, static analysis, power flow and efficiency analysis. This method doesn't need complex formulae derivation or model transformation. So is effective in comparative analysis of a great number of schemes. This method can also be applied to multi-input multi-output systems.
3. Furthermore, power flow analysis model has been proposed to check whether recirculating power exists in the system and, if present, to determine its magnitude. It can be seen intuitively through the power flow analysis model. This model and analysis method may contribute to the next stage of the scheme synthesis, such as the strength calculation of the components.
4. Especially, all the analysis process is implemented in **MATLAB** software. The proposed method provides a basis for automating the scheme synthesis of different **PGTs**.

Declaration of Conflicting Interests

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Data availability

Data will be made available on reasonable request.

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Appendix 1

Notation

B	brake
$\mathbf{B}$	incidence matrix
b_{ij}	(i, j) element of incidence matrix
$\hat{\mathbf{B}}$	weighted incidence matrix
$\hat{b}_{ij}$	(i, j) element of weighted incidence matrix
$\tilde{\mathbf{B}}$	reduced incidence matrix
$\tilde{b}_{ij}$	(i, j) element of reduced incidence matrix
$\overline{\mathbf{B}}$	weighted reduced incidence matrix
$\overline{b}_{ij}$	(i, j) element of weighted reduced incidence matrix

$\mathbf{B}_{CC}$	control component connectivity matrix
$\tilde{\mathbf{B}}'_{CC}$	modified control component connectivity matrix
$\mathbf{B}_{PC}$	power component connectivity matrix
$\mathbf{B}_{PGSi}$	i -th BPGS connectivity matrix
C	clutch
$\mathbf{C}$	static constant matrix
IN	input component
$\mathbf{K}$	kinematic coefficient matrix
$\mathbf{K}_1$	PGS coefficient matrix
$\mathbf{K}_2$	rigid connection coefficient matrix
$\mathbf{K}_3$	Operating mode coefficient matrix
k	characteristic coefficient of BPGS
k_1, k_2	characteristic coefficient of i -th BPGS
OUT	output component
PC	planet carrier
PG	Planetary gear
$P_{IN,i}$	power flow through input component
$P_{OUT,i}$	power flow through output component
R	ring gear
S	sun gear

$\mathbf{S}$	static coefficient matrix
$\mathbf{S}_1$	PGS torque matrix
$\mathbf{S}_2$	torque equilibrium matrix
$\mathbf{S}_3$	torque reference matrix
$\mathbf{T}$	torque vector
T_i	torque of i -th component
T_s, T_r, T_c	torque of sun gear, ring gear, planet carrier
$T_{IN,i}$	torque of i -th input component
$T_{OUT,i}$	torque of i -th output component not considering the loss
$T'_{OUT,i}$	torque of i -th output component considering the loss
$\mathbf{V}$	kinematic constant matrix
Z_r, Z_s	teeth number of ring gear and sun gear
α	exponent according to the direction of relative power flow in BPGS
$\boldsymbol{\omega}$	speed vector of components
ω_i	speed of i -th component
$\omega_{IN,i}$	speed of i -th input component
$\omega_{OUT,i}$	speed of i -th output component
$\omega_s, \omega_r, \omega_c$	speed of sun gear, ring gear and planet carrier
η	efficiency of PGT
η_0	efficiency of BPGS