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Solving Optimization Problems Using the Graphical Method of Linear Programming

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Abstract

Linear programming is a mathematical optimization technique used for maximizing or minimizing a linear objective function while satisfying a set of linear constraints. This method plays a crucial role in various industries, including manufacturing, finance, transportation, and supply chain management. Among the different approaches to solving linear programming problems, the graphical method provides a simple and intuitive way to visualize and solve problems involving two decision variables. By plotting constraints on a Cartesian plane, the method allows for the identification of feasible regions and optimal solutions through geometric interpretation. Although limited to two-variable cases, it serves as an effective educational tool for understanding the fundamental principles of optimization. The graphical method follows a structured approach involving the formulation of an objective function, plotting constraints, determining the feasible region, and evaluating the objective function at the corner points. By analyzing these corner points, the optimal solution can be determined efficiently. This paper presents a step-by-step explanation of the graphical method, supplemented by a case study demonstrating its practical application in a real-world scenario. The study illustrates how businesses can utilize this method to maximize profits, minimize costs, and allocate resources efficiently. Despite its simplicity and effectiveness, the graphical method has limitations, such as its inability to handle problems with more than two decision variables. For larger and more complex problems, alternative approaches such as the simplex method or computational optimization tools are required. Nonetheless, the graphical method remains a valuable tool in operations research, providing foundational knowledge for more advanced linear programming techniques. This paper also discusses the advantages, disadvantages, and applications of the graphical method across various fields, reinforcing its importance in decision-making and optimization strategies.

Keywords: Linear programming, graphical method, optimization, feasible region, objective function, constraints, operations research.

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INTRODUCTION

Linear programming (LP) is a very useful mathematical technique which is applied to various mathematical problems such as production, distribution as well as investment analysis. It is used for optimization, where the goal is to either maximize or minimize a linear objective function which is subjected to a set of linear constraints. The key characteristic of linear programming is the linearity inherent in the problem. While limited in scope, this method provides valuable insights into key concepts such as feasible regions, constraints, and optimal solutions. This paper delves into the step-by-step application of the graphical method and its utility in solving real-world optimization problems [1].

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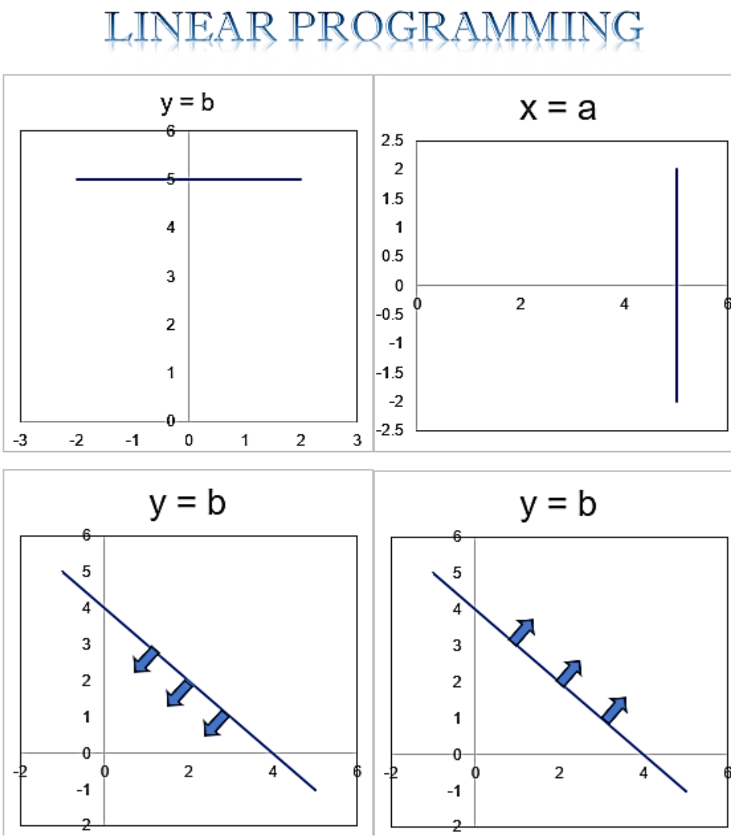

Figure 1.

LITERATURE REVIEW

The study of linear programming has a rich history, beginning with the pioneering work of George B. Dantzig in the 1940s. His development of the simplex algorithm revolutionized optimization by enabling the solution of high-dimensional linear programming problems. Early contributions by von Neumann introduced duality theory, a cornerstone of linear programming's theoretical framework [2].

The graphical method, while restricted to two-dimensional problems, serves as a crucial educational tool. Researchers such as Hillier and Lieberman (1995) emphasized its role in teaching the fundamentals of optimization, citing its effectiveness in visually demonstrating concepts such as constraints and feasible regions. More recent studies by Winston (2004) and Taha (2017) have highlighted the method's pedagogical value, providing numerous examples of real-world applications in production planning, resource allocation, and logistics.

Applications of linear programming have also been widely documented. In manufacturing, LP has been used to optimize production schedules (Srinivasan et al., 2012), while in agriculture, it has guided crop selection for maximum yield (Gupta & Sharma, 2019). A study by Basak et al. (2020) demonstrated the effectiveness of linear programming in optimizing supply chain logistics. Similarly, research by Chandrasekaran and Beavers (2019) explored the use of LP models in energy management systems. The graphical method, despite its limitations, remains a practical choice for small-scale problems and continues to inspire the development of visualization tools for higher-dimensional optimization [3].

FUNDAMENTALS OF LINEAR PROGRAMMING

Linear programming involves three key components:

1. *Objective Function:* A linear function to be maximized or minimized (e.g., profit, cost, or efficiency). The general form is: Px + Qy = R

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Where P, Q and R are the coefficients representing contributions to the objective function, and x and y are decision variables [4].

2. *Constraints:* A set of linear inequalities or equations that restrict the values of decision variables.
3. *Feasible Region:* The graphical representation of constraints forms a region within which all constraints are satisfied. This is the intersection of half-planes defined by inequalities.
4. *Optimal Solution:* The point in the feasible region where the objective function achieves its maximum or minimum value. It typically occurs at one of the vertices (corner points) of the feasible region [5].

OVERVIEW OF THE GRAPHICAL METHOD

The graphical method involves several key steps:

1. *Formulate the Objective Function:* The objective function is conveyed in terms of the decision variables.
   For example, if we want to maximize profit, it might be represented as $Z = ax + by$, where a and b are coefficients representing profit per unit of each variable x and y [6].
2. *Define Constraints:* Constraints are formulated as linear inequalities that define the limits within which the solution must lie. For example:
   $x + 5y \leq 12$
   $x - 6y \geq 0$
   $x + y \leq 20$
3. *Graph the Constraints:* Each constraint is represented on a Cartesian plane, with the equations of the inequalities plotted as straight lines. The feasible region is determined by shading the area that satisfies all the constraints simultaneously [7].
4. *Identify the Feasible Region:* The feasible region is the area where all constraints overlap over each other. This is the region which represents all the possible solutions that meet the criteria which are set by the constraints.
5. *Determine Corner Points:* The optimal solution will occur at one of the vertices or corner points of the feasible region. These points are calculated with the help of the intersection of the lines representing each constraint.
6. *Evaluate Objective Function at Corner Points:* The values of the objective function are then calculated at each of the corner points to identify which point gives the maximum or minimum value, depending on whether the goal is maximization or minimization [8].

Example Problem

Consider an example where we want to maximize $Z = 5x + 3y$ subject to the following constraints: Table 1.

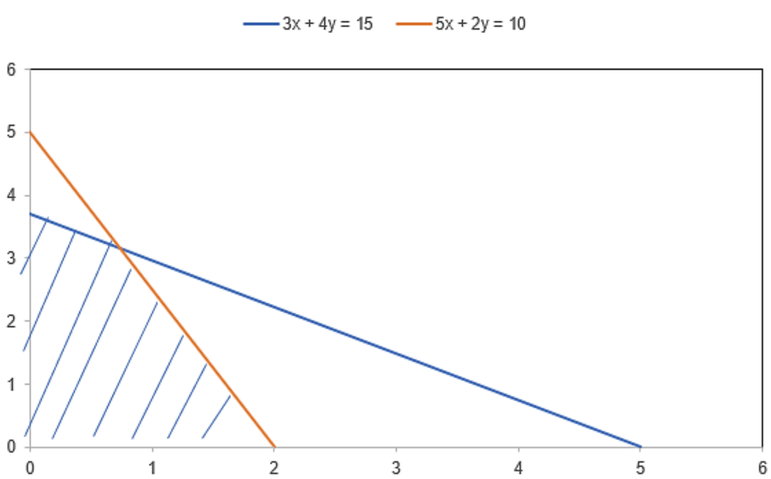

Figure 2.

i. $3x + 4y \leq 15$
ii. $5x + 2y \leq 10$
iii. $x \geq 0, y \geq 0$

Step-by-Step Solution

1. Graphing Constraints:
   Convert the inequalities into equations:
   $3x + 4y = 15$
   $5x + 2y = 10$

   Plot these lines on a graph (Figure 2)

Table 1. xy coordinates

	X	y	P(x,y)
1.	5	0	A(5,0)
	0	3.7	B(0,3.7)
2.	2	0	C(2,0)
	0	5	D(0,5)

2. *Shading Feasible Region:* Shade below $3x + 4y = 15$ and $5x + 2y = 10$. The feasible region is highlighted [9].
3. Identifying Corner Points:
   Calculate intersections:
   Intersections of $3x + 4y = 15$ and $5x + 2y = 10$.
4. Evaluating Objective Function:
   Now, calculate the value of Z at each point.
   Determine which point gives the highest value for maximization [10-12].

CASE STUDY: MAXIMIZING PROFIT

Problem Statement

One type of cake requires 200 g flour and 25 g of fat. Another type of cake requires 100 g flour and 50 g fat. Find out the maximum number of cakes which can be made from the 5 kg of flour and 1 kg of fat while assuming that there is no shortage of any other ingredients used in making the cake.

Solution

Let the first type of cake be **x** and the second type of cake be **y.** Table 2.

Table 2. Data

Cake	No. of units	Ingredient		Profit
		Flour	***Fat***	
I.	x	200	25	1
II.	y	100	50	1
Maximum	Availability	5000	1000	

No. of products are non negative.
First linear constraint is,
$200x + 100y \leq 5000$
$25x + 50y \leq 1000$
$x \geq 0 , y \geq 0$
Let $z = x + y$,

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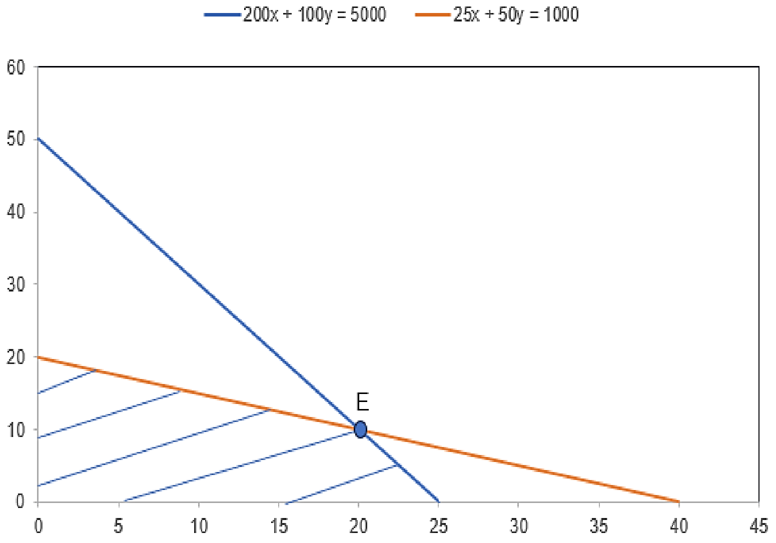

Figure 3.

Consider equality, Table 3

$$200x + 100y = 5000$$
$$25x + 50y = 1000$$

Table 3. xy coordinates.

	x	y	P(x,y)
1.	25	0	A(25,0)
	0	50	B(0,50)
2.	40	0	C(40,0)
	0	20	D(0,20)

Z at A = 25+0 = 25
Z at D = 0+20 = 20
Z at E (20,10) = 20+10 = 3

The maximum value of Z is 30.

CASE STUDY: MINIMIZING COST

Problem Statement

A dietician wishes to mix together 2 kinds of foods X and Y in such a way that the mixture contains at least 10 units of Vitamin A and 12 units of Vitamin B and 8 units of Vitamin C. Given are the Vitamin content of 1 Kg food. 1 Kg of food X costs Rs. 16 and 1 Kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet Table 4.

Table 4. Data

Food	Vitamin Content		
	A	B	C
X	1	2	3
Y	2	2	1

Solution

The first type of food is **x** and the second type of food is **y.** Table 5

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Table 5. Data

Food	No. of units	Vitamin Content			Cost
		A	***B***	***C***	
X	x	1	2	3	16
Y	y	2	2	1	20
Minimum requirement		10	12	8	

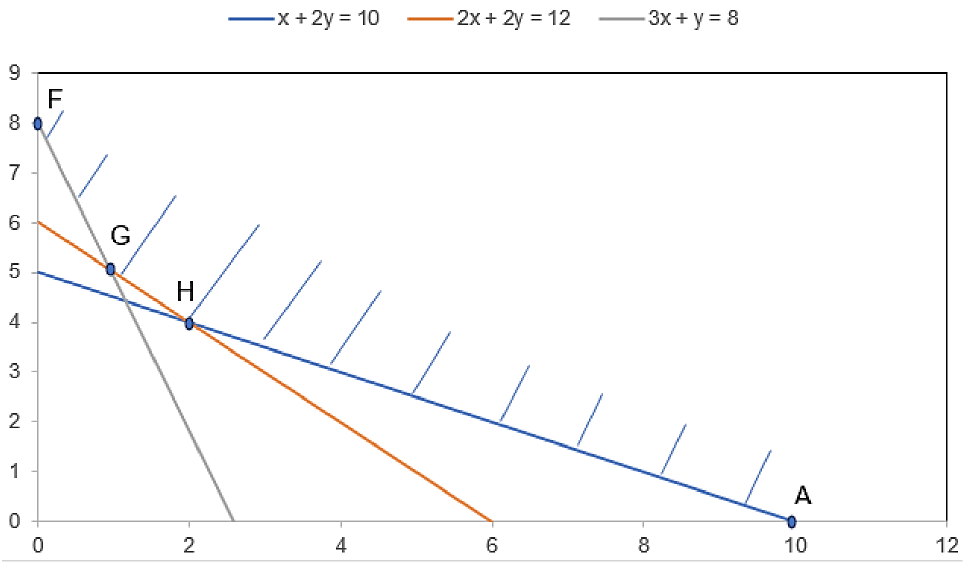

Figure 4.

No. of products are non negative.

First linear constraint is,

$x + 2y \geq 10$
$2x + 2y \geq 12$
$3x + y \geq 8$
$x \geq 0$, $y \geq 0$

Let z = 16 x +20 y,
Consider equality,

$x + 2y = 10$
$2x + 2y = 12$
$3x + y = 8$ Table 6

Table 6. xy coordinates

	x	y	P(x,y)
1.	10	0	A(10,0)
	0	5	B(0,5)
2.	6	0	C(6,0)
	0	6	D(0,6)
3.	2.6	0	E(2.6,0)
	0	8	F(0,8)

Points are F(0,8), G(1,5), H(2,4), A(10,0)

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Z at A = 16(10) + 20(0) = 160
Z at F = 16(0) + 20(8) = 160
Z at G = 16(1) + 20(5) = 116
Z at H = 16(2) + 20(4) = 112

The minimum value of Z is 112.

Applications

Linear programming, including the graphical method, has diverse applications across industries. Here are detailed examples:

1. Manufacturing
   - Optimizing production schedules to maximize profit or minimize costs.
   - Allocating resources such as labor, raw materials, and machinery efficiently.
   - Example: A study by Srinivasan et al. (2012) demonstrated how a manufacturing firm optimized its production mix using LP to achieve cost savings and higher productivity.
2. Agriculture
   - Assisting farmers in selecting crop combinations for maximum yield or profit based on available resources such as water, fertilizer, and labor.
   - Example: Gupta and Sharma (2019) used LP models to determine the optimal allocation of land for different crops, resulting in increased profitability.
3. Transportation and Logistics
   - Designing efficient delivery routes to minimize travel time and fuel costs.
   - Managing inventory and distribution in supply chains.
   - Example: Basak et al. (2020) applied LP to optimize logistics operations, significantly reducing operational costs.
4. Energy Management
   - Optimizing energy usage in industrial plants to reduce costs while meeting demand.
   - Managing renewable energy resources such as solar and wind to achieve sustainable energy goals.
   - Example: Chandrasekaran and Beavers (2019) employed LP to design energy-efficient management systems.
5. Finance
   - Portfolio optimization to achieve the best mix of investments for maximum returns or minimum risk.
   - Managing loan portfolios and asset allocations effectively.
6. Healthcare
   - Allocating medical staff, equipment, and facilities to meet patient needs efficiently.
   - Example: LP has been used in hospital management to optimize operating room schedules and resource allocation.

CONCLUSION

Linear programming is a very interesting topic of optimization techniques which has various applications in various fields such as management, aerospace, manufacturing, economics and more.

The graphical method of linear programming offers an effortless and visual approach to solving various optimization problems with two decision variables. It is an essential tool for fabricating the foundational knowledge of the linear programming concepts, including constraints, feasible regions, and optimization.

In some problems there may be more than one solution whereas some problems may have no finite solution.

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Despite its limitations, the linear programming has a diverse applicability and the graphical methods remain relevant in the educational contexts and provides a stepping stone to more advanced optimization techniques.

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