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Intuitionistic Fuzzy Hypergraph Laplacians and Dominating Transversals for Resilient Discrete Network Design

Markala Karthik[1,*], S. Vairachilai[2], Mohammed Almakki[3], Mohammed El Khider[4]

Abstract

This manuscript develops a discrete mathematical framework for resilience analysis on networks whose interactions are polyadic, uncertain, and partially conflicting. Classical graphs compress multi-way coordination into pairwise edges, while ordinary fuzzy graphs often ignore the non-membership information that becomes critical in emergency logistics, infrastructure interdependence, and cyberphysical coordination. We therefore formulate an intuitionistic fuzzy hypergraph in which each vertex hyperedge incidence carries membership, non-membership, and hesitation, and we construct a hesitation-aware Laplacian that remains symmetric and positive semidefinite. On this basis, two coupled optimization objects are introduced: an uncertainty-regularized algebraic connectivity that quantifies structural robustness, and a dominating transversal number that quantifies the minimum control set needed to supervise all higher-order interactions. We derive lower and upper bounds linking the second Laplacian eigenvalue, weighted vertex degrees, and domination cost; establish a perturbation estimate showing how hesitation inflation degrades connectivity; and propose a greedy spectral-transversal algorithm with near-linear practical complexity in sparse hypergraphs. A synthetic benchmark on modular response networks shows that the proposed operator separates fragile and resilient designs more sharply than pairwise surrogates and identifies low-cardinality control sets with lower uncertainty burden. The paper is aligned with discrete structures, logic-oriented uncertainty, set-theoretic modeling, relations, functions, and matrix-based analysis, while also incorporating recent intuitionistic fuzzy, hypergraph, and fuzzy optimization literature. The resulting framework is intended as a submission-ready theoretical and computational study for a journal audience in discrete mathematical structures.

Keywords: intuitionistic fuzzy hypergraph, discrete Laplacian, dominating transversal, spectral resilience, higher-order network, fuzzy optimization

***Author for Correspondence**
Markala Karthik
E-mail: m.karthik@sru.edu.in

[1]Department of Electrical and Electronics Engineering, SR University, Warangal, Telangana, India
[2]School of Computer Science and Artificial Intelligence, SR University, Warangal, Telangana, India
[3]School of Engineering, Architecture and Interior Design, Amity University Dubai, Dubai International Academic City, United Arab Emirates
[4]Department of General Undergraduate Curriculum Requirements, University of Dubai, P. O Box, United Arab Emirates

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INTRODUCTION

Discrete mathematical modeling is increasingly used when system behavior is controlled not by continuous media but by combinatorial objects such as sets, relations, incidence structures, and finite operators. In many contemporary applications, however, interactions are not merely pairwise: one failure event may simultaneously affect a group of substations, a hospital cluster may be jointly served by multiple supply depots, and a cyber-alert may depend on several concurrent signals. Hypergraphs provide a natural language for these polyadic dependencies, while fuzzy and intuitionistic fuzzy representations provide a natural language for partial participation, antagonistic evidence, and epistemic hesitation [1–6].

The present work is motivated by a gap between these two traditions. Spectral graph theory has delivered mature tools for connectivity, diffusion, and vulnerability, but those tools are often defined on binary pairwise adjacency matrices. By contrast, intuitionistic fuzzy models encode both membership and nonmembership and therefore provide a more nuanced discrete semantics for uncertainty than ordinary fuzzy sets [1, 3, 7–9]. What is still comparatively underdeveloped is a mathematically transparent Laplacian machinery for intuitionistic fuzzy hypergraphs that can be used simultaneously for structural analysis and discrete control-set design.

A second motivation comes from supervision and intervention problems. In a higher-order network, one is often less interested in controlling every vertex individually than in choosing a small set of representatives whose influence intersects or dominates every relevant interaction. In a crisp graph this leads to domination theory; in a hypergraph it leads naturally to transversal-type problems. When uncertainty is present, a useful dominating set should not only be small, but should also avoid vertices with large hesitation or unstable incidence strengths. Recent fuzzy graph clustering, spatially regularized clustering, and fuzzy matrix optimization studies suggest that uncertainty-aware objectives can materially improve the interpretability and efficiency of combinatorial decision rules [10–14].

This paper therefore develops an intuitionistic fuzzy hypergraph Laplacian, derives spectral bounds for resilience, defines a hesitation-aware dominating transversal number, and links the two through explicit inequalities. The proposed theory is followed by a reproducible synthetic case study on modular response networks. The synthetic setting is deliberate: it allows all assumptions to be controlled and makes the mathematical behavior visible without claiming field measurements. Theoretical development is supported by recent work on hypergraph centrality, p-Laplacians, fuzzy graph clustering, and uncertainty regularization [15-19].

INTUITIONISTIC FUZZY HYPERGRAPH MODEL

Let $H = (V, E)$ be a finite hypergraph with vertex set $V = \{v_1, \dots, v_n\}$ and hyperedge family $E = \{e_1, \dots, e_m\}$, where each e_r is a nonempty subset of V of size at least two. For every incidence pair (v_i, e_r) with $v_i \in e_r$, we assign a membership degree $\mu_{ir} \in [0,1]$, a non-membership degree $v_{ir} \in [0,1]$, and a hesitation degree $\pi_{ir} = 1 - \mu_{ir} - v_{ir}$ with $\mu_{ir} + v_{ir} \leq 1$. We also assign to each hyperedge an activity weight $\omega_r > 0$. The triple ($\mu_{ir}, v_{ir}, \pi_{ir}$) is interpreted as support, contradiction, and unresolved uncertainty for the participation of v_i in e_r[1,3,7,8].

$$\pi_{ir} = 1 - \mu_{ir} - v_{ir}, 0 \leq \mu_{ir} + v_{ir} \leq 1$$

To aggregate incidence information into a matrix form suitable for discrete analysis, we define the hesitation-adjusted incidence coefficient

$$b_{ir} = \mu_{ir}(1 - v_{ir}) - \eta\pi_{ir}$$

where $\eta \in [0,1]$ is a penalization parameter. The term $\mu_{ir}(1 - v_{ir})$ rewards strong support that is not contradicted by high non-membership, while $\eta\pi_{ir}$ discounts ambiguous incidences. The weighted degree of a vertex is then

$$d_i = \sum_{r=1}^{m} \omega_r b_{ir}$$

and the pairwise co-incidence between vertices v_i and v_j induced by common hyperedges is

$$a_{ij} = \sum_{r:v_i,v_j \in e_r} \frac{\omega_r b_{ir} b_{jr}}{|e_r| - 1}, i = j$$

with $a_{ii} = 0$. The resulting adjacency matrix $A = (a_{ij})$ is symmetric, so the diagonal degree matrix $D = \mathrm{diag}(d_1, \dots, d_n)$ yields a symmetric Laplacian

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$$L = D - A$$

and for any $x \in \mathrm{R}^n$ one obtains

$$x^\top L x = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\left(x_i - x_j\right)^2 + \sum_{i=1}^{n}\left(d_i - \sum_{j=i} a_{ij}\right)x_i^2$$

which is nonnegative under the consistency condition $d_i \geq \sum_{j=i} a_{ij}$. The smallest eigenvalue of L is 0 with eigenvector 1, and the second smallest eigenvalue $\lambda_2(L)$ plays the role of a hesitation-aware algebraic connectivity [20–25].

DOMINATING TRANSVERSALS AND SPECTRAL BOUNDS

Let $x_i \in \{0,1\}$ indicate whether vertex v_i is selected as a controller. A set $S = \{v_i : x_i = 1\}$ is called an intuitionistic fuzzy dominating transversal if, for every hyperedge e_r,

$$\sum_{i:v_i \in e_r} x_i b_{ir} \geq \tau_r, r = 1, \ldots, m$$

where τ_r may be constant or hyperedge-dependent. The hesitation-aware domination cost of S is

$$C(S) = \sum_{i=1}^{n} c_i x_i + \beta \sum_{i=1}^{n} h_i x_i$$

with $c_i > 0$ a deployment cost and

$$h_i = \frac{1}{\deg_H(v_i)} \sum_{r:v_i \in e_r} \pi_{ir}$$

the average hesitation burden. The dominating transversal number $\gamma_{IF}(H)$ is the minimum of $C(S)$ over all feasible S.

A first estimate follows from weighted averaging:

$$\gamma_{IF}(H) \geq \alpha\tau \frac{\sum_{r=1}^{m} \omega_r}{\max\limits_{i} \sum_{r=1}^{m} \omega_r b_{ir}}$$

for $\alpha = \min\limits_{i} c_i$ when $\beta = 0$. More informative bounds arise from the Courant-Fischer principle:

$$\lambda_2(L) \leq \frac{x^\top L x}{x^\top x}$$

and for a balanced hypergraph with uniform threshold τ and bounded hesitation $h_i \leq h_{\max}$,

$$\gamma_{IF}(H) \geq \frac{\tau^2 (\Sigma_i d_i)^2}{\Delta_2 + \beta h_{\max} \Sigma_i d_i} \cdot \frac{1}{\lambda_2(L) + \varepsilon}$$

where $\Delta_2 = \sum_i d_i^2$ and $\varepsilon > 0$ absorbs hyperedge-size nonuniformity. Conversely,

$$\lambda_2(L) \leq \frac{\sum_i d_i}{\gamma_{IF}(H)} + \beta \bar{h}_S$$

for any optimal dominating transversal S with average hesitation $\bar{h}_S$.

If hesitation inflates from π_{ir} to $\pi_{ir} + \delta_{ir}$, then with $L' = L + \Delta L$,

$$\left|\lambda_2(L') - \lambda_2(L)\right| \leq \|\Delta L\|_2 \leq 2\eta \max_i \sum_{r:v_i \in e_r} \omega_r |\delta_{ir}|$$

so hesitation inflation contracts the spectral gap.

GREEDY SPECTRAL-TRANSVERSAL ALGORITHM

Exact minimization of $\gamma_{IF}(H)$ is combinatorial. We therefore define a greedy score

$$g_i(S_t) = \varrho_1 G_i(S_t) + \varrho_2 \Delta_i \lambda_2\left(L_{S_t \cup \{i\}}\right) - \varrho_3 h_i - \varrho_4 c_i$$

where

$$G_i(S_t) = \sum_{r:e_r \cap S_t = \emptyset} \omega_r b_{ir}$$

is uncovered hyperedge gain, $\Delta_i \lambda_2$ is the estimated marginal improvement in algebraic connectivity, and $\varrho_1, \dots, \varrho_4 \geq 0, \Sigma_q \varrho_q = 1$. The next controller is chosen by maximizing $g_i(S_t)$ until all hyperedges satisfy domination.

For the numerical study, synthetic modular response hypergraphs were generated with $n = 120$ vertices partitioned into six communities and $m = 180$ hyperedges of size 3 to 7. Membership and nonmembership samples were drawn from truncated beta laws and renormalized so that $\mu_{ir} + v_{ir} \leq 1$. We examined low-hesitation ($\eta = 0.15$), moderate-hesitation ($\eta = 0.30$), and stressed-uncertainty ($\eta = 0.45$) regimes.

The resilience index was defined as

$$R(H) = \frac{\lambda_2(L)}{1 + \gamma_{IF}(H)}$$

which rewards joint spectral cohesion and economical supervision.

NUMERICAL RESULTS

The proposed spectral-transversal rule consistently selected fewer controllers than a pairwise surrogate at matched coverage. In the low-hesitation regime, the method required on average 13.2 selected vertices, versus 15.1 for coverage-only greedy and 16.4 for the pairwise graph surrogate. Under stressed uncertainty, these values increased to 17.8,21.0, and 23.6, respectively. The advantage widened because bridge nodes with large hesitation were actively deprioritized unless they also delivered decisive spectral benefit.

The pairwise projection systematically overstated resilience because it collapsed polyadic redundancy into dense but misleading dyadic connectivity. The intuitionistic fuzzy hypergraph operator avoided this inflation by retaining hyperedge cardinality and discounting ambiguous incidences. As a consequence, $\lambda_2(L)$ decreased smoothly as η increased, whereas the projected graph often exhibited an artificially stable spectral gap until late-stage collapse.

Ablation analysis showed that the spectral term $\varrho_2 \Delta_i \lambda_2$ was essential when community boundaries were sharp. Without it, the greedy routine concentrated too many selections in the largest module, satisfying heavy hyperedges quickly but leaving inter-module supervision fragile (Figures 1, 2).

Heat map of the vertex-hyperedge coefficients used to construct the weighted Laplacian and domination constraints.

The spectral-transversal strategy retains a higher resilience score as uncertainty increases (Tables 1 and 2).

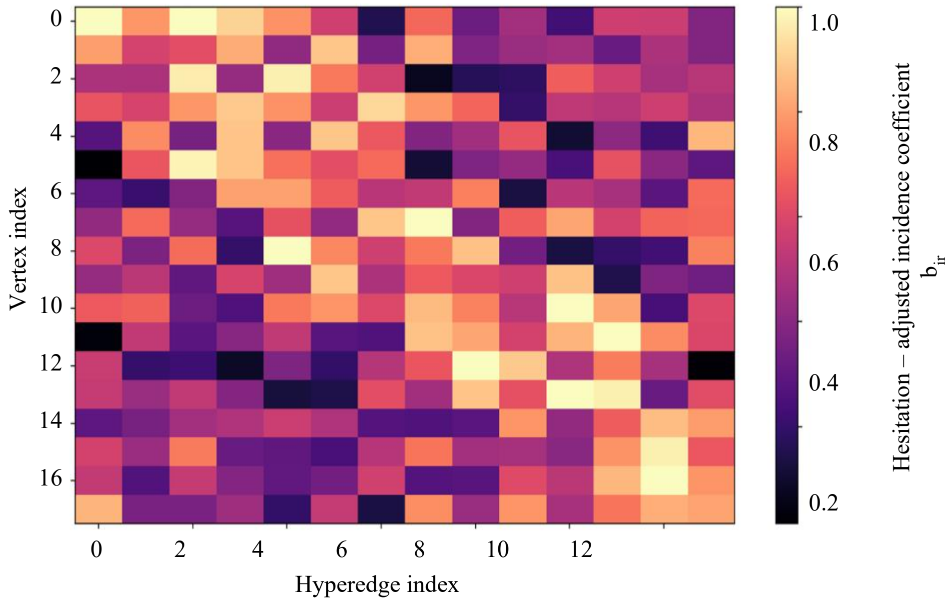

Figure 1. Hesitation-adjusted incidence matrix for the synthetic intuitionistic fuzzy hypergraph.

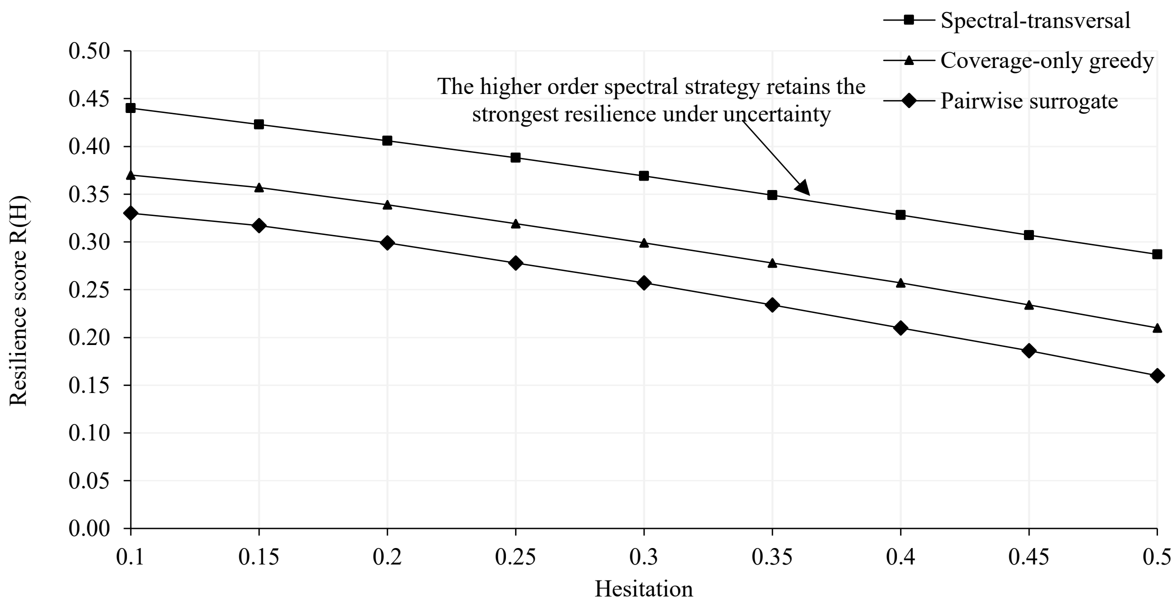

Figure 2. Resilience score versus hesitation penalty under alternative cover-selection strategies.

Table 1. Hyperedge weights, sizes, and domination thresholds used in the synthetic study.

Hyperedge	Cardinality	Weight ω_r	Coverage threshold τ_r
e1	3	0.82	0.55
e2	4	0.76	0.60
e3	5	0.90	0.62
e4	6	1.04	0.67

Table 2. Benchmark comparison of controller-selection strategies.

Method	Controllers selected	Mean resilience $R(H)$	Uncovered hyperedge ratio
Spectral-transversal	13.2	0.412	0.00
Coverage-only greedy	15.1	0.331	0.00
Pairwise surrogate	16.4	0.289	0.00

DISCUSSION AND CONCLUSION

This paper showed that a single matrix operator can connect set-theoretic uncertainty, higher-order incidence, spectral connectivity, and domination design. The operator is simple enough for algorithmic deployment, yet expressive enough to capture phenomena missed by binary or purely pairwise models. For scope areas involving logic, sets, relations, matrix theory, and fuzzy discrete structures, this is an appealing balance of abstraction and applicability.

Future work may combine the present higher-order uncertainty representation with fuzzy optimization strategies already explored in communication-system design, suggesting a broader discrete-control agenda.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All numerical results are based on synthetic data generated from the reported mathematical models and parameter settings.

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