

Comparative Analysis of Kinetic Models for Simulation of Biogas Production from Cow Dung and Fruit Waste via Anaerobic Digestion

Abdulwadud Yusuf Abdulkarim^{1*}, Hashim Adamu², Akorode Abdulafeez Olayinka³, Oyinkan Ilemore², Kish Elisha Dokas³, Mohammed Yusuf Mamman⁴

Abstract

This study investigates the optimization of biogas and biofertilizer production from cow dung and fruit waste through anaerobic digestion, utilizing various microbial growth kinetic models. Simulations were conducted using the Monod, Moser, Contois, and Tessier models to predict biogas yield and assess model accuracy. Results indicated that the Tessier model provided the closest fit to experimental data, with a biogas yield of 0.45 m³/kg VS, while the Monod model overestimated yields at 0.65 m³/kg VS. The Moser and Contois models produced more realistic estimates of 0.50 m³/kg VS and 0.48 m³/kg VS, respectively. Additionally, the cow dung's C/N ratio of 30:1, higher than the optimal 20–25:1 range, suggested its greater suitability for biofertilizer production. This study underscores the importance of selecting appropriate kinetic models for accurate simulation and highlights the potential for optimizing feedstock composition to enhance biogas yields, thereby contributing valuable insights into the modeling and optimization of anaerobic digestion processes for sustainable energy production. These findings highlight the critical role of selecting appropriate kinetic models for accurately simulating anaerobic digestion processes and optimizing feedstock composition to enhance biogas yields.

Keywords: Anaerobic digestion, biogas production, biofertilizer, kinetic models, cow dung, fruit waste

INTRODUCTION

The process of anaerobic decomposition of organic material has been recognized since the 18th century. By the mid-19th century, the role of anaerobic bacteria in this process was established. Anaerobic digestion is a widely used technology used in treating organic waste and production of renewable energy in the form of biogas [1] and about a century later, anaerobic digestion was recognized as an effective method for treating sewage and offensive materials [2]. In the 19th century, French bacteriologist Louis Pasteur and German chemist Felix Hoppe-Seyler demonstrated the microbiological formation of methane and carbon dioxide from acetate. Pasteur even proposed using manure from Parisian horses to produce gas for street lighting [3]. A feasibility study was carried out that horse dung is a potential candidate for biogas production [4].

*Author for Correspondence

Abdulwadud Yusuf Abdulkarim
E-mail: aabdulwadud@atbu.edu.ng

¹Senior Lecturer, Department of Chemical Engineering, Abubakar Tafawa Balewa University, Bauchi, Nigeria

²Lecturer II, Armament Engineering Department, Air Force Institute of Technology, Kaduna, Nigeria

³Assistant Lecturer, Armament Engineering Department, Air Force Institute of Technology, Kaduna, Nigeria

⁴Lecturer I, Armament Engineering Department, Air Force Institute of Technology, Kaduna, Nigeria

Received Date: October 22, 2024

Accepted Date: December 02, 2024

Published Date: January 02, 2025

Citation: Abdulwadud Yusuf Abdulkarim, Hashim Adamu, Akorode Abdulafeez Olayinka, Oyinkan Ilemore, Kish Elisha Dokas, Mohammed Yusuf Mamman. Comparative Analysis of Kinetic Models for Simulation of Biogas Production from Cow Dung and Fruit Waste via Anaerobic Digestion. International Journal of Membranes. 2025; 2(1): 47–63p.

In the past two decades, the use of biogas plants for renewable energy production via anaerobic digestion (AD) has gained popularity due to its applicability to a wide range of organic materials, including energy crops, animal waste, and the

organic fraction of municipal solid waste (MSW). The relative simplicity of AD plant designs has further facilitated this trend [5, 6].

Anaerobic digestion is a multifaceted biological process that transforms organic materials into methane through three primary stages: hydrolysis, acetogenesis, and methanogenesis, all taking place in low or oxygen-free environments [7, 8]. Anaerobic digestion is a commonly used biological process for converting organic materials into biogas through several biochemical reactions, including hydrolysis, acidogenesis, acetogenesis, syntrophic acetate oxidation, homoacetogenesis, and methanogenesis, all occurring with minimal or no oxygen [9]. This controlled breakdown of organic waste in an oxygen-free environment, facilitated by anaerobic microorganisms, also produces biofertilizers as a byproduct [10]. Biogas and solid sludge being produced by microorganisms because of digesting organic waste are the most viable alternatives to fossil fuel and bio-fertilizer production [11].

AD processes can be classified into two major categories: those that utilize inoculum and those that do not. The use of inoculum involves cultivating a consortium of microorganisms that facilitate the anaerobic bacteria process, leading to faster startup and enhanced biogas production. However, the cost of preparing and culturing the inoculum can be a significant drawback [12, 13]. In contrast, AD without inoculum is more commonly applied due to its lower cost and ease of implementation, though it typically results in slower startup and longer retention periods.

The mechanism of AD consists of four general processes: hydrolysis, acidogenesis, acetogenesis, and methanogenesis, involving complex interactions of various microbial consortia and environmental factors [14, 15]. Understanding these mechanisms and kinetics is crucial for effective reactor design and operation, impacting methane production, biofertilizer quality, system stability, and effluent quality [1, 16].

Mathematical models have been developed to investigate and optimize AD processes, given their inherent instability due to complex microbial interactions and organic matter transformations [17]. One of the most widely recognized models is the Anaerobic Digestion Model No. 1 (ADM1), established by the International Water Association (IWA). ADM1 describes the biochemical and physicochemical processes in AD through differential and algebraic equations, allowing for detailed simulation studies of biogas plants [18, 19]. While ADM1 has shown promising results in simulation studies, its complexity restricts its widespread use, necessitating careful calibration and potential modifications, especially under co-digestion conditions [19].

Methodology

Raw Materials Collection

For this work, two raw materials were sourced and prepared. Dried cow dung was collected from Bauchi State College of Agriculture, it was dried, the straws and husks of rice that mixed with the dung were removed and the dung was packed in bags for analysis. The fruit waste was obtained from Muda Lawan and Wunti Markets of Bauchi State close to the fruit sellers division. The fruit waste was dried, crushed, and packaged for analysis. The fruit waste consists of orange peels (43 kg), banana peels (9 kg), watermelon peels (17 kg), and pineapple peels (4 kg) in a ratio of 30:20:40:10, respectively.

MATLAB, short for Matrix Laboratory, is a multi-paradigm numerical computing environment developed by MathWorks. As a proprietary programming language, it supports matrix manipulation, function and data plotting, algorithm implementation, user interface development, and interaction with programs written in languages, such as C, C++, C#, Java, Fortran, and Python [20]. Although MATLAB is primarily designed for numerical computing, an optional toolbox utilizes the MuPAD symbolic engine for symbolic computations. Its built-in graphics facilitate data visualization and insight, while the desktop environment encourages experimentation and exploration. All MATLAB tools are

extensively tested and designed for seamless integration. Additionally, Simulink provides graphical, multi-domain simulation, and model-based design capabilities for dynamic and embedded systems [20–21].

Model Adoption and Modification

Numerous models have been developed by various researchers aiming either to directly apply the principles of Anaerobic Digestion Model 1 (ADM1) or to modify ADM1 principles prior to implementation. These models focus on simulating biokinetics in anaerobic digestion, addressing aspects, such as microbial activity, substrate breakdown, and methane production. For this work, a mathematical model was developed based on mass balances on biomass, the organic substrate, and biogas just as applied by [22, 23]. Further modifications were made to this model, which include the addition of a relationship that will account for the biofertilizer and the yield of the biofertilizer using an overall mass balance and some changes to the parameters defined. Another important modification to the model was the assumption that the biodigester is a semi-batch type instead of the initial assumption of it being a continuous stirred tank reactor (CSTR). Also, various microbial growth kinetic models were used in place of the Briggs-Haldane equation (that was used in the model). These include the Monod, the Moser, the Tessier, and the Contois. The assumptions for the feasibility of the model are listed below:

- The digester is a sealed reactor.
- A perfect agitation within the reactor.
- A biochemical reaction in the reactor.
- Uniformity in the reactor.
- An established transitional arrangement.
- The factor limiting bacterial growth is the organic substrate.
- The suspended biomass contributes to the biodegradation of the substrate [22, 24–25].

Solution of Adopted Model (with Simulink in MATLAB R2020)

This section shows the step-by-step development and modification of the adopted model. The block flow diagram of a biodigester is shown in Figure 1.

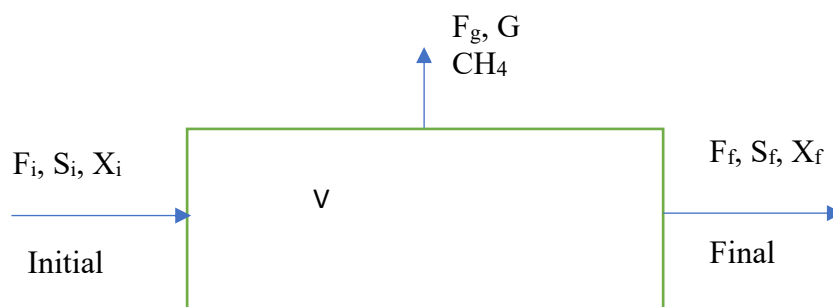


Figure 1. Block flow diagram of a biodigester.

where:

F_i , F_f , and F_g : denote the input and output flows of the liquid and the flow of the biogas produced, respectively.

S_i , S_f : Concentrations of the initial and final substrates (g/L).

X_i , X_f : Initial and final concentrations of biomass (g/L).

G : Concentration of methane in the biogas (g/L).

V : biodigester volume (L).

Mass Balance for the Biomass

$$\text{Input} + \text{growth} = \text{output} + \text{Accumulation} + \text{Detachment} \quad (1)$$

$$F_i X_i + \mu X V_t = F_f X_f + V_t \frac{dX}{dt} + K_d X V_t \quad (2)$$

Dividing through by the substrate volume and going by the assumption that the biodigester operation is semi-batch type. Equation 2 becomes

$$\frac{dX}{dt} = \mu X - k_d X \quad (3)$$

μ is given by the four replacements of the Briggs-Haldane equation, they are shown below:

The Monod Equation

$$\mu = \mu_{max} \left(\frac{S}{K_s + S} \right) \quad (4)$$

where:

K_d : Death rate of microorganisms (day^{-1})

μ : The rate of growth of anaerobic microorganisms (day^{-1})

μ_{max} : Maximum specific growth rate (day^{-1})

The Tessier Equation [24]

$$\mu = \mu_{max} (1 - e^{-SK_s}) \quad (5)$$

The Moser [26]

$$\mu = \mu_{max} \left(\frac{S^n}{K_s + S^n} \right) \quad (6)$$

where n is the fitting constant given as 2.9098

The Contois Equation [26]

$$\mu = \frac{\mu_{max} S}{K_{sx} X + S} \quad (7)$$

where K_{sx} is the substrate degradation rate necessary for the growth of microorganisms (g/g).

Mass Balance for the Substrate

During anaerobic digestion, some of the substrate is used in the formation of new cells and to provide the energy required for cellular activities and maintenance [22].

$$\text{Input} - \text{New cells Production} - \text{Growth} - \text{Maintenance} = \text{Output} + \text{Accumulation} + \text{Biogas (Methane + Other gases)} \quad (8)$$

But for batch operations, Input and Output rates are zero.

Thus, we have the equation:

$$\frac{dS}{dt} = -\frac{\mu X}{Y_x} - K_{sx} X \mu - K_{mx} X \mu - \frac{1}{Y_s} \left(\frac{dG}{dt} + \frac{dC_{CO_2}}{dt} + \frac{dC_{H_2}}{dt} + \frac{dC_{NH_3}}{dt} \right) \quad (9)$$

where

S = Substrate concentration (g/L).

Y_x = Biomass yield (g/g).

K_{sx} = Substrate degradation rate necessary for the growth of microorganisms (g/g).

K_{mx} = Substrate degradation rate necessary for cell maintenance (g/g).

Y_s = Biogas yield (g/g).

Mass Balance of Biogas

$$Input + Production = Output + Accumulation \quad (10)$$

For semi-batch process, input and output equal zero,

$$Q_i G_i = Q_f G_f - K V_t + V_t \frac{dG}{dt} \quad (11)$$

Dividing through Equation 11 by V_t , we have

$$K - \frac{Q_f}{V_t} G_f = \frac{dG}{dt} \quad (12)$$

$$\frac{Q_f}{V_t} = D \quad (13)$$

where K is the organic substrate conversion of methane and D is the dilution rate.

$$\frac{dG}{dt} = K - DG_f \quad \text{But } K = Y_{CH_4} \mu X \quad (14)$$

Therefore, Equation 14 becomes

$$\frac{dG}{dt} = Y_{CH_4} \mu X - DG \quad (15)$$

where Y_{CH_4} is the methane yield

A similar balance, can be taken for other gases in the biogas (i.e. CO_2 , H_2 , and NH_3), thus we have:

$$\frac{dC_{CO_2}}{dt} = Y_{CO_2} \mu X - DC_{CO_2} \quad (16)$$

$$\frac{dC_{H_2}}{dt} = Y_{H_2} \mu X - DC_{H_2} \quad (17)$$

$$\frac{dC_{NH_3}}{dt} = Y_{NH_3} \mu X - DC_{NH_3} \quad (18)$$

Overall, Balance Around the Digester

Taking the overall balance for the biodigester shown in Figure 2, a basis of 100 kg of feed was taken. 25 kg shredded waste sample and 75 liters of water (Table 1) [27].

$$Substrate (\% \text{ by vol.}) = Biogas (\% \text{ by vol.}) + Biofertilizer (\% \text{ by vol.}) \quad (19)$$

Prediction of Biofertilizer Yield

The biofertilizer yield was predicted with the assumption that 60% of the volatile solids were converted to biogas [28]. Then Equation 18 as stated by Ngumah et al. (2013) [29] can be used. With the value of biofertilizer produced estimated from Equation 18, biofertilizer yield can then be calculated.

$$Biofertilizer \text{ Produced} = (DM - VS) + (40\% \text{ of } VS) \quad (20)$$

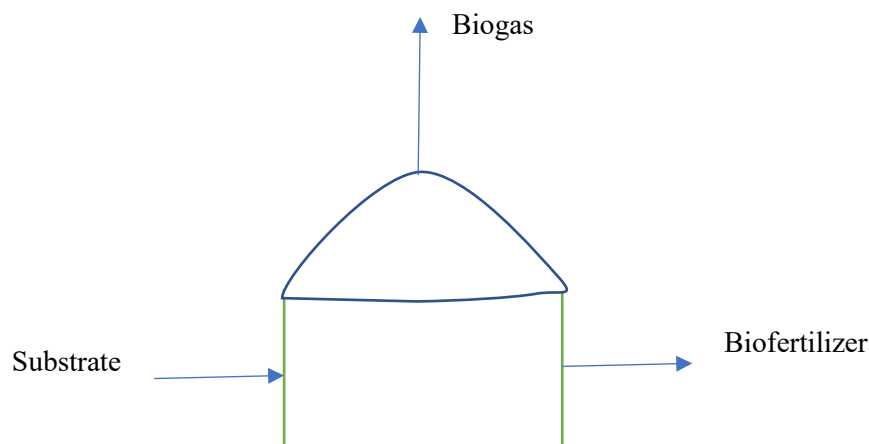


Figure 2. Mass balance around the digester.

Table 1. Parameter values for simulation of the model.

Parameter	Symbol	Value/Unit	Source
Initial Concentration of substrate	S_i	831.82 mg/L	[27]
Initial biomass concentration	X_i	57.69 mg/L	[27]
Maximum Specific growth rate	μ_{max}	0.053 day ⁻¹	[27]
Death rate of Microorganisms	K_d	0.038 day ⁻¹	[27]
Dissociation constant	K_s	21.23 mg/L	[27]
New cell production ratio	Y_x	0.082 g/g	[22]
Substrate degradation rate for microorganism growth	K_{sx}	0.983 g/g	[22]
Substrate degradation rate for microorganism maintenance	K_{mx}	0.4 g/g	[22]
Biogas production coefficient	Y_s	4.35 g/g	[22]
CH ₄ production coefficient	Y_{CH_4}	0.4 g/g	[22]
CO ₂ production coefficient	Y_{CO_2}	0.27 g/g	[22]
H ₂ production coefficient	Y_{H_2}	0.03 g/g	[22]
NH ₃ production coefficient	Y_{NH_3}	0.01 g/g	[22]

Execution and Validation of the Simulink Simulation

Simulink is a block diagram environment designed for multi-domain simulation and Model-Based Design, supporting system-level design, simulation, automatic code generation, and ongoing testing and verification of embedded systems. It offers a graphical editor, customizable block libraries, and solvers for modeling and simulating dynamic systems. Integrated with MATLAB, Simulink allows users to incorporate MATLAB algorithms into models and export simulation results for further analysis in MATLAB [21]. In this study, MATLAB 2020 was used to perform the simulation, beginning with the creation of a new project in the Simulink section of the MATLAB software. The library tab was clicked to display the list of blocks available in the version, and from the commonly used section, the integrator blocks were used. A total of six integrator blocks were initiated for use, each with initial values specified. For the substrates and biomass differential equation, the initial values stated in Table 1 were used, while as assumed, since there exists no gas at the beginning of the process, the initial values for all four gases were specified as zero. The MATLAB function box was used to link the expression of microbial growth rate to the Simulink model and a different m-file was used to specify the other parameters in Table 1. Other blocks used include: fifteen gain blocks, two mux blocks, two

sum blocks, three scope blocks, and three display blocks. The screenshot of the Simulink model is displayed as Figure 3.

To validate predictive models, parameter values determined experimentally were utilized to compare simulation outputs with experimental results, ensuring the reliability of model predictions when substituting for physical experiments [29, 30]. The physicochemical analysis of cow dung and food waste was conducted to characterize the feedstock before the digestion process. This analysis encompassed parameters, such as moisture content, sulfur content, organic carbon, ash content, biochemical oxygen demand (BOD), chemical oxygen demand (COD), total solids, total suspended solids (TSS), total nitrogen, and the carbon/nitrogen ratio.

For BOD, samples of cow dung and fruit waste were mixed with distilled water, and the initial dissolved oxygen (DO) levels were measured using a dissolved oxygen meter. These samples were incubated in darkness at 20°C for five days, after which the DO levels were measured again. The BOD was calculated by comparing the initial and final DO levels.

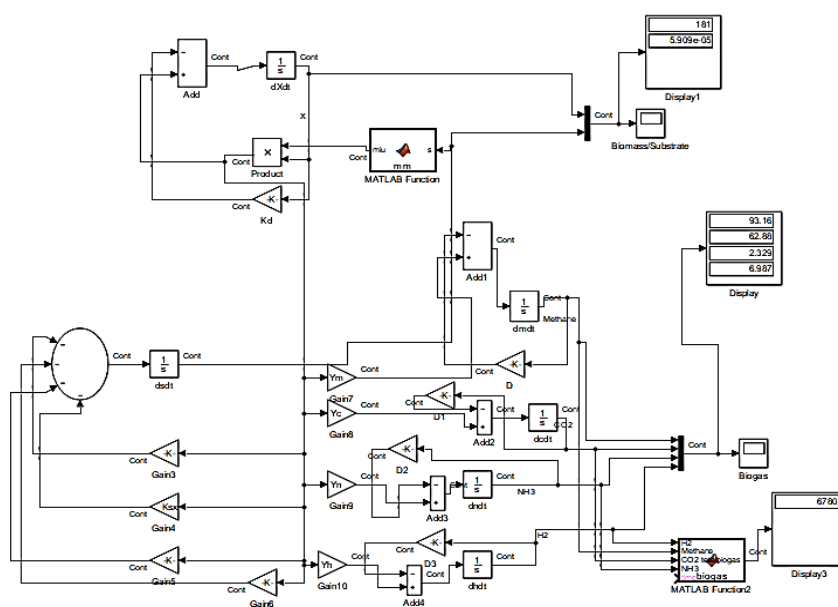


Figure 3. The Simulink block model.

The COD analysis was performed using the potassium permanganate consumption method. Samples were boiled with sulfuric acid and KMnO_4 , then titrated with oxalic acid and KMnO_4 . The COD was calculated based on the KMnO_4 consumption of the sample compared to the blank.

Total solids were measured using a moisture content analyzer set at 105°C, while total suspended solids were determined by filtering the sample, drying the residue at 105°C, and measuring the weight increase of the filter.

The organic carbon content was assessed through the Walkley-Black chromic acid wet oxidation method, which involves heating the sample with potassium dichromate and sulfuric acid, followed by titration with ferrous sulfate using a ferroin indicator. The organic carbon percentage was then derived from the titration results.

Total nitrogen was analyzed using a DR/890 Colorimeter and the Cadmium reduction method. The sample was treated with Nitraver 5 Nitrate reagent, mixed thoroughly, and nitrogen content was measured.

Potassium content was determined with atomic absorption spectroscopy (AAS). The samples were digested using nitric acid and hydrogen peroxide, filtered, and analyzed using the AAS machine to quantify potassium, following the method of Salman et al. (2014) [31].

Physicochemical Analysis

The results obtained after the characterization of fruit waste and cow dung are presented below in Table 2.

Table 2. Physiochemical analysis of cow dung and fruit waste.

Analysis	Value	
	Cow Dung	Fruit Waste
Biochemical oxygen demand (BOD ₅)	5.47 (mg/L)	5.06 (mg/L)
Chemical oxygen demand (COD)	3244.26 (mg/L)	426.6 (mg/L)
Moisture content	0.75%	3.95%
Organic carbon	472 ± 0.01	293.7 ± 0.01
Sulfur content	4.87%	1.77%
Nitrogen	12.21	13.2
C/N ratio	38.67	22.20
P ₂ O ₅	9.75	1.23
K ₂ O	8.63	1.15

The results obtained from the physicochemical analysis show a wide gap between BOD and COD. This is always expected as BOD only shows the oxygen demand needed for oxidation of organic matter, whereas COD indicates the total oxygen needed for chemically oxidizable matter. Most of the time, COD is preferred to BOD in process control applications since the results will be more reproducible and are available in a few minutes or hours rather than five days [32]. This is why the COD in most simulation cases, the COD is used as the substrate's quantity rather than the BOD. The COD and BOD values of the cow dung are higher than those of the fruit waste due to the complex nature of the organic waste, which has passed through various digestion processes within the ruminant digestion system.

The carbon-to-nitrogen (C/N) ratio reflects the balance between carbon and nitrogen in organic materials, serving as a critical indicator for managing biological treatment systems and optimizing biogas production [8, 33]. An imbalance in nutrients is a key factor limiting anaerobic digestion efficiency for organic wastes. To enhance nutrient levels and achieve optimal C/N ratios, co-digestion of organic mixtures is commonly applied [34, 35]. Ideal C/N ratios for anaerobic digesters range from 20 to 30, supporting efficient biogas production while minimizing excessive biofertilizer generation [33, 36]. A high C/N ratio indicates rapid nitrogen consumption by methanogens, leading to lower biogas yields. Conversely, a low C/N ratio can cause ammonia buildup and elevate pH levels above 8.5, which is harmful to methanogenic bacteria [33, 37]. Thus, with the C/N ratio of the cow dung greater than the limit of 20–30, it is expected that production of biofertilizer will be favored as against biogas.

Simulation Results

Model Solution

The results obtained from the Simulink model solution (auto ode45) were in two folds. The first is the graphical representation of the changes in the substrates and biomass concentration within the bioreactor, the results also show the peak-to-peak measurements, the total time for the simulation, and the time at which the peaks are attained. The second set of the result is displayed in the MATLAB command prompt where the total amount of biogas produced, the total amount of biofertilizer produced and the percentage yield of biofertilizer are all displayed. The summary of the results from the command prompt window for the model solution with respect to the kinetic equations applied is shown in Tables 3 and 4. Although Moser gave the highest biogas production followed by Tessier, however, Tessier fits

the best to the experimental data when the plots in Figures 4-9 are compared with the experimental outcome. This is a similar scenario to what was reported by Ain et al. (2016) [38] where the Tessier model fits best with the experimental data.

Table 3. Results from various kinetic models.

	Monod	Moser	Tessier	Contois
CH ₄ produced (kg)	0.0540	6.7500	3.8190	0.6140
CO ₂ produced (kg)	0.0361	4.5540	2.5760	0.4100
H ₂ produced (kg)	0.0040	0.5000	0.2850	0.0460
NH ₃ Produced (kg)	0.0014	0.1760	0.1000	0.0160
Total biogas produced (kg)	0.095	11.9840	6.7800	1.0900

Table 4. Biogas.

Kinetic Model	Monod	Moser	Tessier	Contois
Mass of dry raw material (kg)	25.000	25.000	25.000	25.000
Biofertilizer Produced (kg)	24.910	13.016	18.220	23.911
Biofertilizer Yield (%)	99.620	52.060	72.880	95.650

Application of the Monod Equation

Figure 4 shows a sharp decrease in the blue line (S) which represents the consumption of the substrate as the anaerobic process progresses. At around the 31st day after the start of the process, all the substrate (831.82 mg/L) has been consumed by microorganisms, this is seen as the line maintains its zero position. On the other hand, the red line (X) representing the biomass (microorganism) is seen to have a sharp increase from its initial value of 57.69 mg/L to a peak of 228.3 mg/L after which the slow decline to the zero value is observed in the plot. The result can be said to be in tandem with the normal behavior of microorganisms as described by Maier (2009) [39]. The kinetics postulates four phases for bacterial growth in batch cultures: lag phase, exponential, stationary, and death phase [39, 40]. For this model, the lag phase can be seen to start from day 0 to day 11, after which the exponential phase followed from day 11 to day 31. The stationary phase was absent in Figure 4, this could be attributed to the fact that the model has ascribed the behavior of the microbes directly to the availability of substrates, thus the sharp edge seen at the tip of Figure 4 in place of a stationary phase as the organisms immediately start cannibalizing and also start to die as soon as the substrate is eaten up. The death phase follows the description by (Maier, 2009) [39], as the period for the death phase is larger than that of the summation of both the lag and exponential phases (Figure 5). The total time taken for all the processes to attain completion (including the complete destruction of the microorganism) was 160 days which is far greater than the 14–40 days period reported by David (2013) and Nwabanne et al. (2009) [27, 41].

The plot in Figure 6 shows the profile of four components of the biogas produced. The least is ammonia gas and the dominant gas is methane. As presented in Table 4, the cumulative biogas produced was 95 g. Figure 6 shows the increase in the production of the biogas to a maximum corresponding to 31 days after the commencement of the digestion process and a decline in the rate of biogas production is seen after this period. This was due to the seizure in the activities of the microorganisms due to the absence of nutrition which led to the death phase.

Application of the Moser's Equation

The output from Moser's model is presented in Figures 7 and 8 for the biomass/substrate profile and the biogas production, respectively. It is evident from the plot that the model using Moser's equation does not follow the usual path of the growth curve for the bacterial population. The simulation shows a straight-line inverse relationship between the biomass and substrate which deviates from the normal biogas/substrate profile, and the biogas production plot shows a continuous increase in production to day 56, after which there was a decline to day 100 of the simulation. Although, the Moser model has been reported to be the best fit in the work carried out by Momoh et al. (2013) [42]. However, this

simulation clearly deviates from the work of (Maier, 2009) [39] which serves as the basis of comparison in this work.

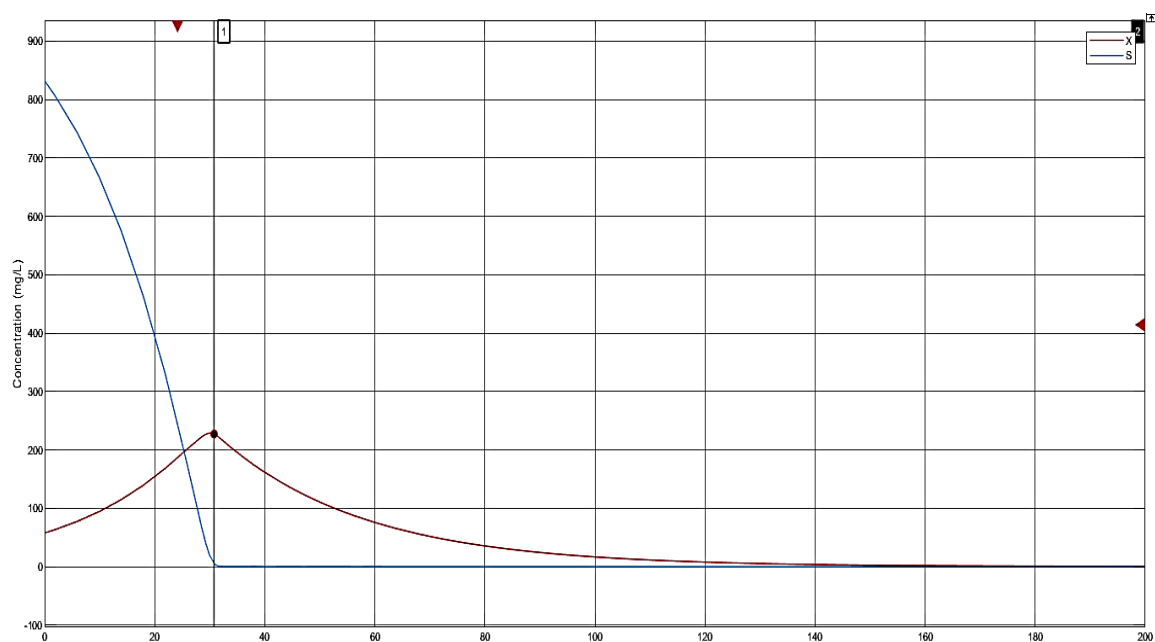


Figure 4. Plot of the substrate/biomass profile using the Monod Equation.

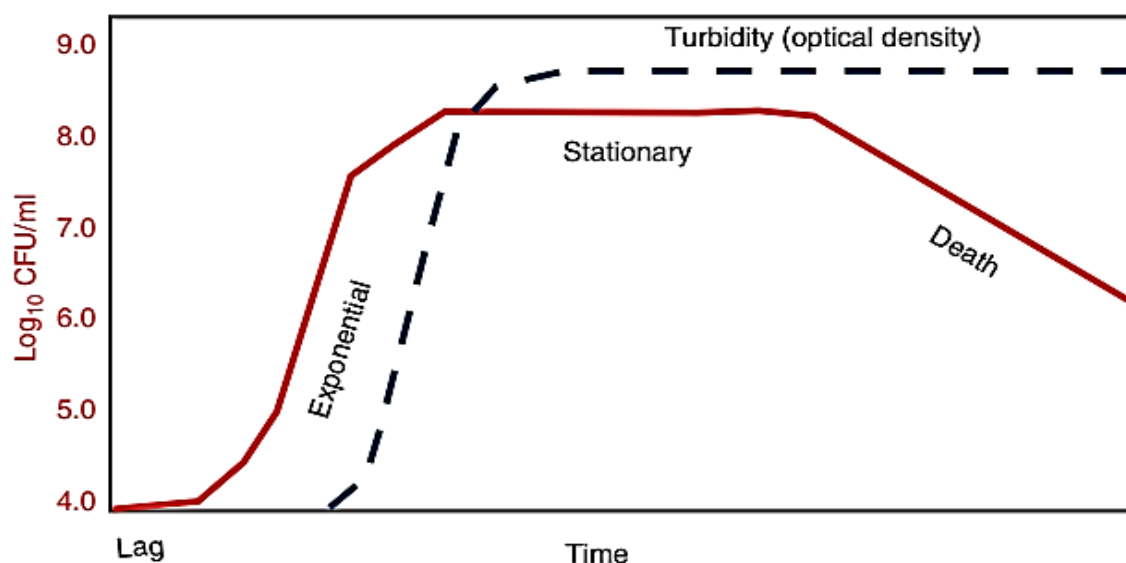


Figure 5. A typical growth curve for a bacterial population (Maier, 2009) [39].

Application of the Tessier's Equation

The biomass/substrate profile and the biogas production plot are shown in Figures 9 and 10, respectively. In this case, a very good plot of the biomass/substrate profile which compares closely to the microbial population growth curve was obtained. The lag phase although very short can be seen to come in between day 0 to day 3, after which the exponential phase starts between 3 days and ends at 6.76 days. The stationary phase is clearly seen between 6.76 days and 8.235 days after which the death phase started to the end of simulation time of 40 days. The simulation period is very close to the bio-digestion period that is mostly reported in the literature of 14–40 days [27, 41]. The biogas production also gave a good output as the production can be seen to proceed until 6.76 days after which the rate of biogas production declines to the end of the simulation period.

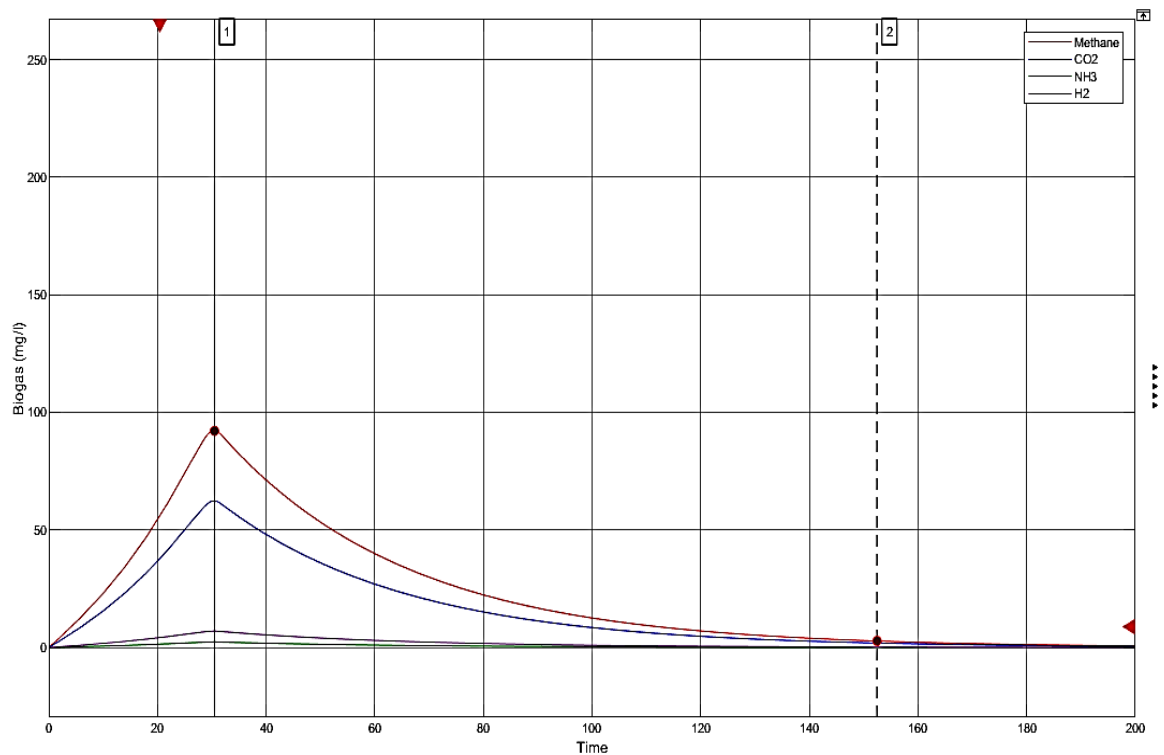


Figure 6. A plot of the biogas production progress using the Monod Equation.

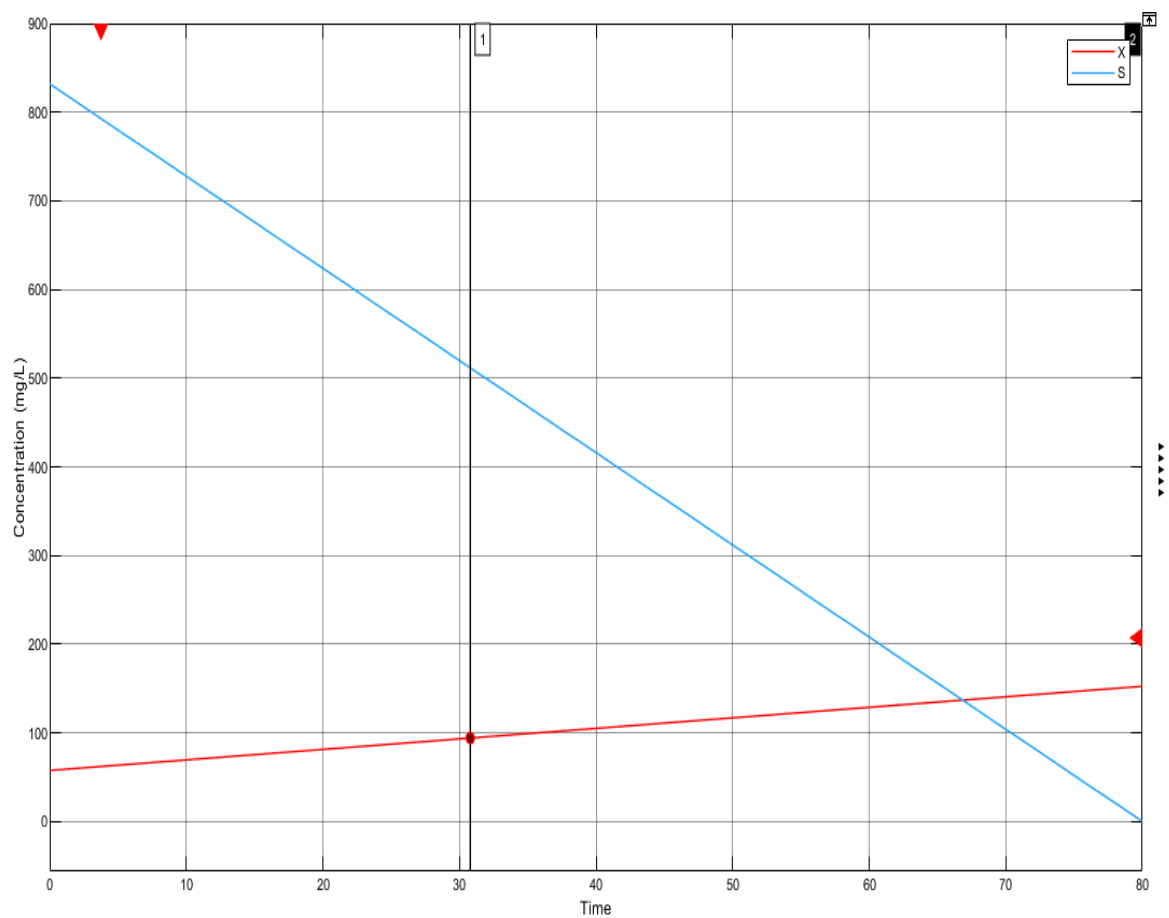


Figure 7. Plot of the substrate/biomass profile using the Moser's equation.

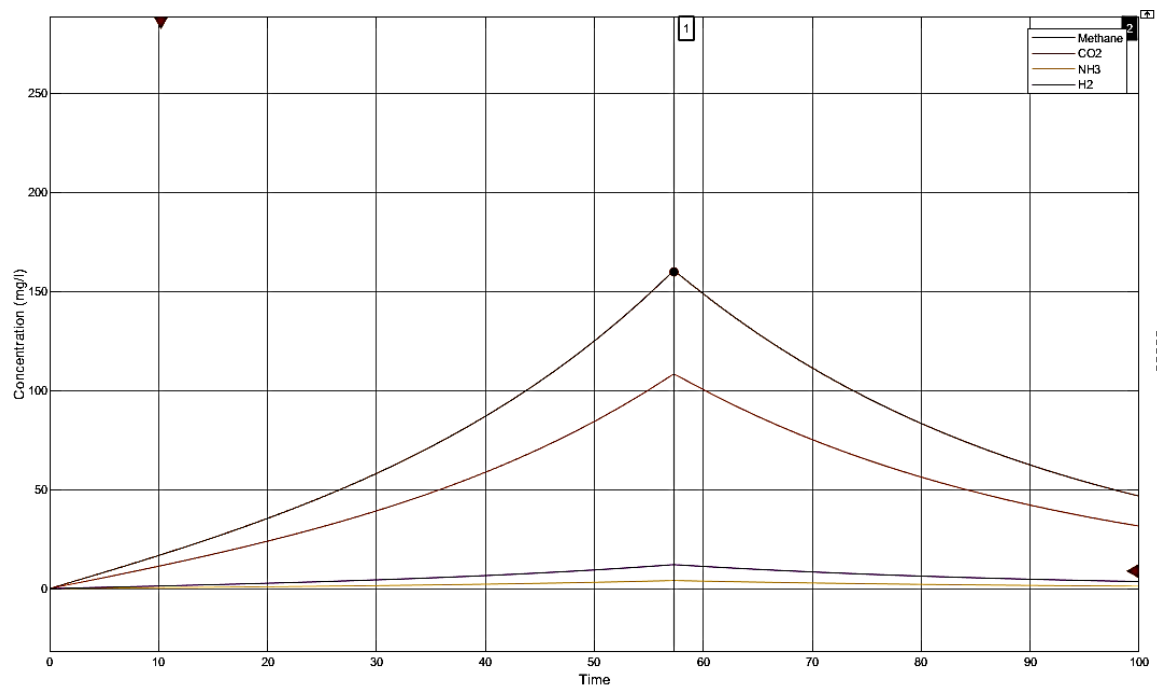


Figure 8. Screenshot of the biogas production progress using the Moser Equation.

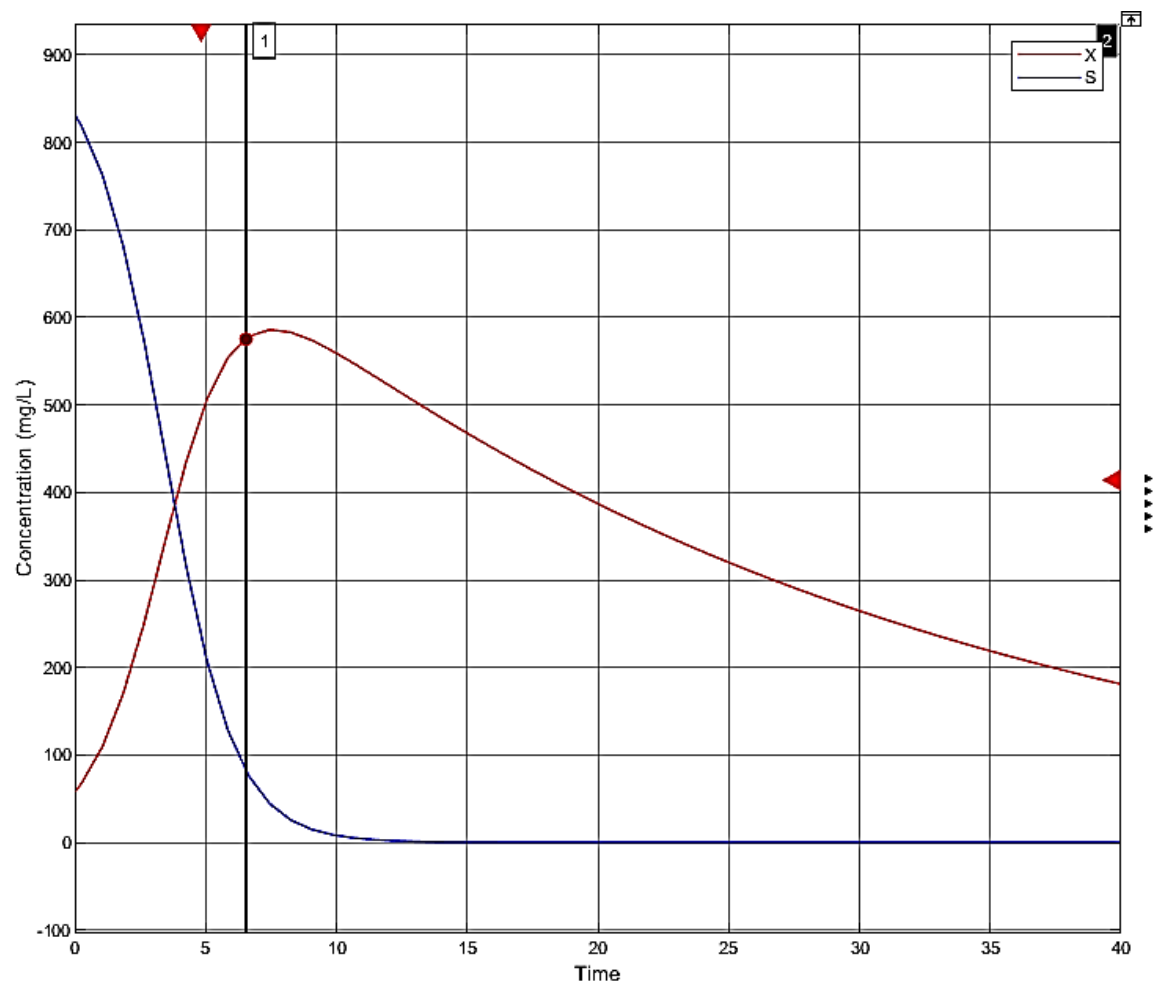


Figure 9. Plot of the substrate/biomass profile using the Tessier Equation.

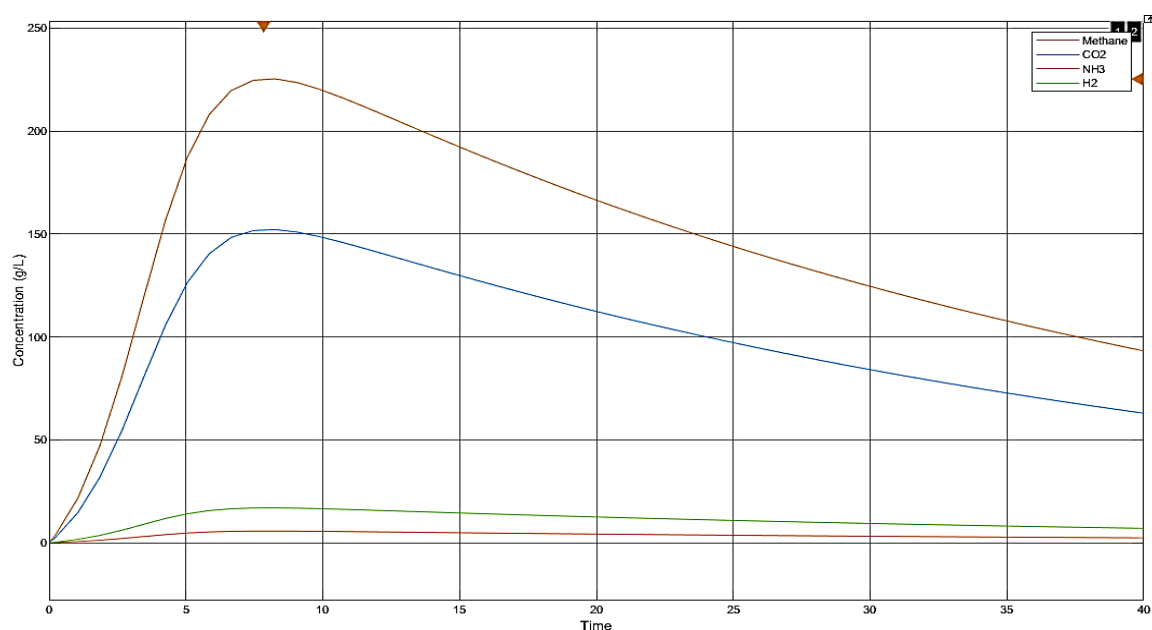


Figure 10. Screenshot of the Biogas production progress using the Tessier Equation.

Application of the Contois Equation

The output of the simulation using the Contois equation is shown in Figures 11 and 12 for biomass/substrate profile and biogas production, respectively. The plot in Figure 11 shows a visible deviation from the norm in microbial growth profile reported by Maier (2009) [39]. There is an absence of both the lag and stationary phase, but a very long time for microbial death period, also no meaningful deduction could be made from Figure 12 as the production started declining after 2.44 days of commencement of the process.

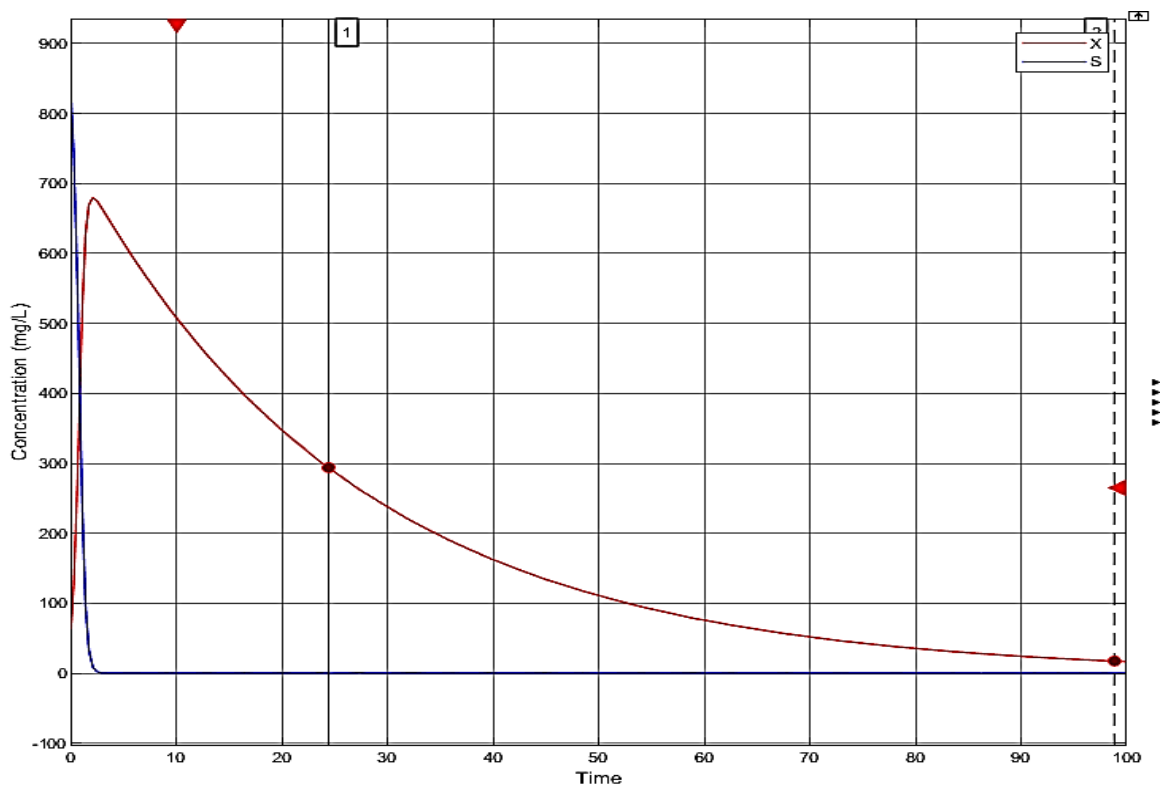


Figure 11. Plot of the substrate/biomass profile using the Contois Equation.

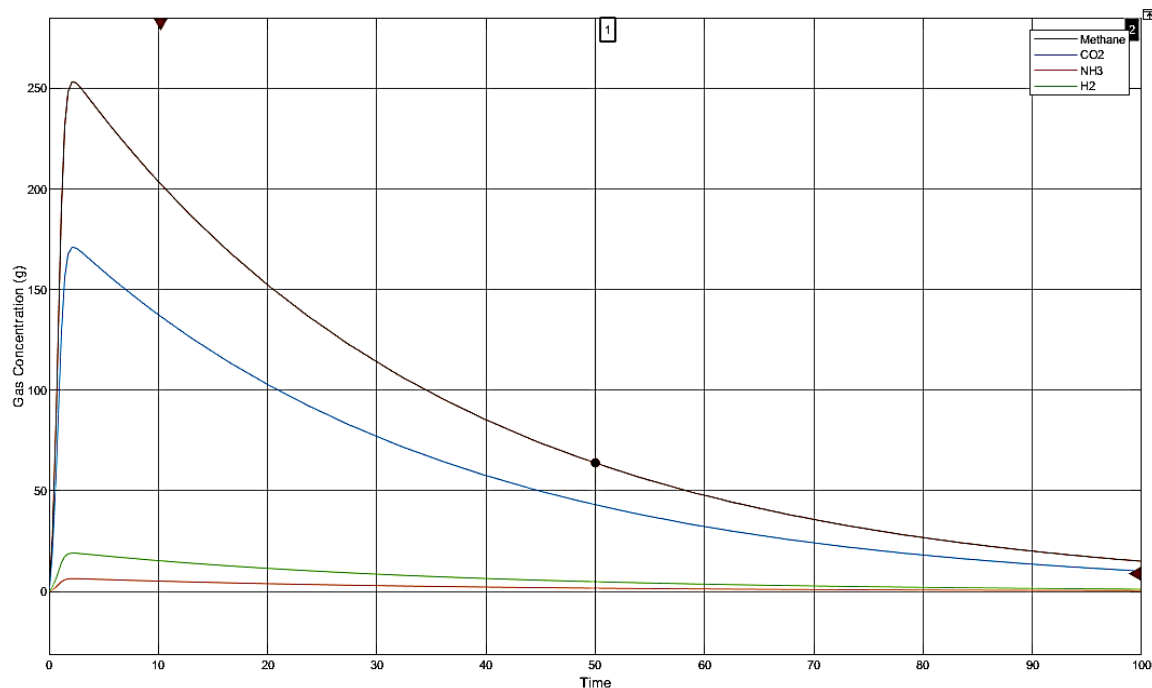


Figure 12. Screenshot of the biogas production progress using the Contois Equation.

Simulation Model Validation

The model was validated by comparing the results from the simulation to the practical results as reported by Nwabanne et al. (2009) [27]. The values reported were fitted in a curve as shown in Figure 13.

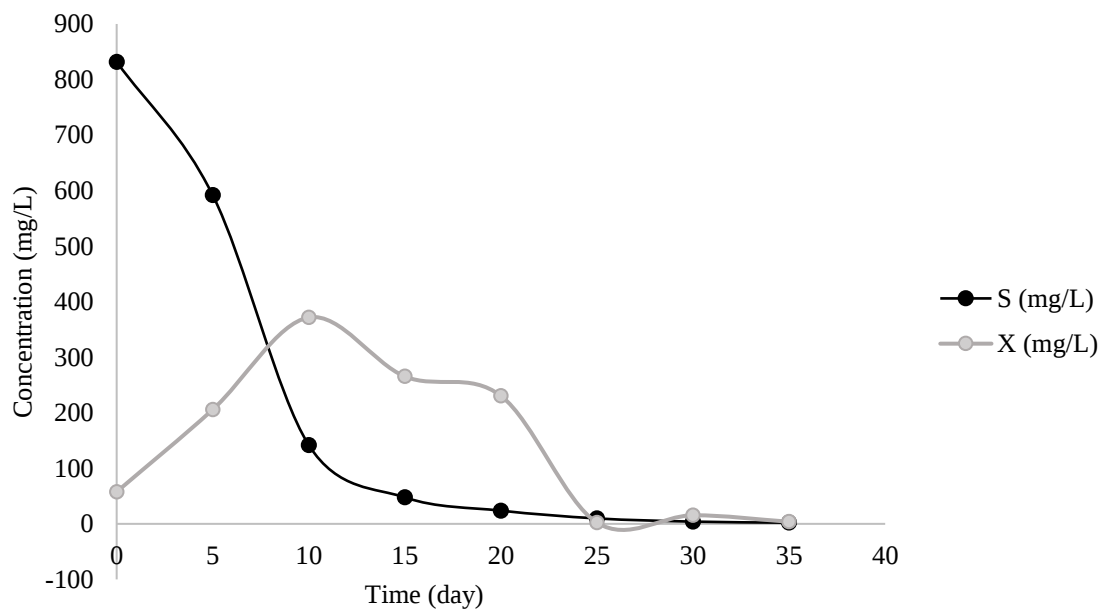


Figure 13. Variation of substrates and biomass with time (Nwabanne et al., 2009) [27].

Comparing Figure 13 and Figures 5, 7, 9, and 11, it is seen that the process attained completion within a span of 35 days as against the 160 days from the simulation results of Monod, 56.4 days for Moser, and 100 days for Contois. It can be seen that the closest of the models to the experimentally determined data is that to which the Tessier equation was applied, it has a period of around 40 days which is very

close to the 35 days determined experimentally although, the time to attain the stationary phase deferred with some days, that of the Tessier started around day 3 while from the experimental results, the plot shows the stationary phase around day 10. This can be related to the estimation of some of the parameters used for the simulation that were picked from the literature of some previous works. Some of these parameters are challenging to measure accurately, often requiring precise methods to minimize errors or avoid inaccuracies. However, the similitude of Figure 9 and Figure 13 and the visible phases of bacterial lifespan in anaerobic digestion as described by (Maier, 2009) [39] is an indication of the validity of the model.

CONCLUSIONS

This study provides a detailed simulation analysis of biogas and biofertilizer production from cow dung and fruit waste using various microbial growth kinetic models. The Tessier model exhibited the best fit with the experimental data, achieving a biogas yield of 0.45 m³/kg VS, compared to the Monod model's overestimation of 0.65 m³/kg VS. The Moser and Contois models produced more realistic estimates of 0.50 m³/kg VS and 0.48 m³/kg VS, respectively. The cow dung's C/N ratio was found to be 30:1, higher than the optimal range of 20-25:1 for biogas production, favoring biofertilizer production instead. These findings highlight the critical role of selecting appropriate kinetic models for accurately simulating anaerobic digestion processes and optimizing feedstock composition to enhance biogas yields.

REFERENCES

1. Amin FR, Khalid H, El-Mashad HM, Chen C, Liu G, Zhang R. Functions of bacteria and archaea participating in the bioconversion of organic waste for methane production. *Sci Total Environ.* 2021;763:143007.
2. Cyprowski M, Stobnicka-Kupiec A, Ławniczek-Wałczyk A, Bakal-Kijek A, Gołofit-Szymczak M, Górny RL. Anaerobic bacteria in wastewater treatment plant. *Int Arch Occupat Environ Health.* 2018;91:571–579.
3. Schön M. Numerical Modelling of Anaerobic Digestion Processes in Agricultural Biogas Plants. *BoD–Books on Demand*; 2010.
4. Owamah HI, Alfa MI, Onokwai AO. Preliminary evaluation of the effect of chicken feather with no major pre-treatment on biogas production from horse dung. *Environ Nanotech Monitor Manag.* 2020;14:100347.
5. Obileke K, Nwokolo N, Makaka G, Mukumba P, Onyeaka H. Anaerobic digestion: Technology for biogas production as a source of renewable energy—A review. *Energy Environ.* 2021;32(2):191–225.
6. Wolf, C. Simulation, optimization and instrumentation of agricultural biogas plants [Doctoral dissertation]. Ireland: National University of Ireland Maynooth; 2013.
7. Andriani D, Wresta A, Atmaja TD, Saepudin A. A review on optimization production and upgrading biogas through CO₂ removal using various techniques. *Appl Biochem Biotechnol.* 2014;172:1909–1928.
8. Zhang T, Liu L, Song Z, Ren G, Feng Y, Han X, Yang G. Biogas production by co-digestion of goat manure with three crop residues. *PloSOne.* 2013;8(6):e66845.
9. Nie E, He P, Zhang H, Hao L, Shao L, Lü F. How does temperature regulate anaerobic digestion? *Renew Sustain Energy Rev.* 2021;150:111453.
10. Alfa MI, Adie DB, Igboro SB, Oranusi US, Dahunsi SO, Akali DM. Assessment of biofertilizer quality and health implications of anaerobic digestion effluent of cow dung and chicken droppings. *Renew Energy.* 2014;63:681–686.
11. Kumar DJ, Mishra RK, Chinnam S, Binnal P, Dwivedi N. A comprehensive study on anaerobic digestion of organic solid waste: A review on configurations, operating parameters, techno-economic analysis and current trends. *Biotechnol Notes.* 2024.
12. Hossain MS, Karim TU, Onik MH, Kumar D, Rahman MA, Yousuf A, Uddin MR. Impact of temperature, inoculum flow pattern, inoculum type, and their ratio on dry anaerobic digestion for biogas production. *Sci Rep.* 2022;12(1):6162.

13. Lawal AA, Dzivama AU, Wasinda MK. Effect of inoculum to substrate ratio on biogas production of sheep paunch manure. *Res Agric Eng.* 2016;62(1):8–14.
14. Abdulkarim AY, Abdulsalam S, El-Nafaty UA, Muhammad IM. Bio-fertilizers via co-digestion: a review. *Traektoriâ Nauki= Path of Science.* 2019;5(6):3001–3011.
15. Lim JW, Park T, Tong YW, Yu Z. The microbiome driving anaerobic digestion and microbial analysis. *Adva Bioenergy.* 2020;5:1–61.
16. Yu L, Wensel PC, Ma J, Chen S. Mathematical modeling in anaerobic digestion (AD). *J Bioremed Biodeg S.* 2013;4(2).
17. Simeonov I, Karakashev D. Mathematical modelling of the anaerobic digestion including the syntrophic acetate oxidation. *IFAC Proc.* 2012;45(2):309–314.
18. Kunatsa T, Xia X. A review on anaerobic digestion with focus on the role of biomass co-digestion, modelling and optimisation on biogas production and enhancement. *Bioresour Technol.* 2022;344:126311.
19. Tejasen S, Taruyanon K. Modelling of two-stage anaerobic treating wastewater from a molasses-based ethanol distillery with the IWA anaerobic digestion model no. 1. *Eng J.* 2010;14(1):25–36.
20. Kujawski O, Steinmetz H. Anaerobic digestion model based on mass balances, development, implementation and validation. In *13th World Congress on Anaerobic Digestion.* 2013. pp. 1–4.
21. Sigmon K, Davis TA. *MATLAB Primer.* Boca Raton, Florida, USA: Taylor and Francis Group; 2004. pp. 1–232.
22. Fedailaine M, Moussi K, Khitous M, Abada S, Saber M, Tirichine N. Modeling of the anaerobic digestion of organic waste for biogas production. *Proc Comput Sci.* 2015;52:730–737.
23. Weinrich S, Nelles M. Systematic simplification of the Anaerobic Digestion Model No. 1 (ADM1)–Model development and stoichiometric analysis. *Bioresour Technol.* 2021;333:125124.
24. Menzel T, Neubauer P, Junne S. Role of microbial hydrolysis in anaerobic digestion. *Energies.* 2020;13(21):5555.
25. Annuar MS, Tan IK, Ibrahim S, Ramachandran KB. A kinetic model for growth and biosynthesis of medium-chain-length poly-(3-hydroxyalkanoates) in *Pseudomonas putida*. *Brazilian J Chem Eng.* 2008;25:217–228.
26. Zhi-Wu W, Yebo L. A theoretical derivation of the Contois equation for kinetic modeling of the microbial degradation of insoluble substrates. *Biochem Eng J.* 2014;82(1):134–138.
27. Nwabanne JT, Onukwuli OD, Ifeakandu CM. Biokinetics of anaerobic digestion of municipal waste. *Internat J Environ Res.* 2009;3(4):511–516.
28. Ngumah C, Ogbulie J, Orji J, Amadi E. Potential of organic waste for biogas and biofertilizer production in Nigeria. *Environ Res Eng Manag.* 2013;63(1):60–66.
29. Obileke K, Mamphweli S, Meyer EL, Makaka G, Nwokolo N. Development of a mathematical model and validation for methane production using cow dung as substrate in the underground biogas digester. *Processes.* 2021;9(4):643.
30. Urbina A, Hinnerichs TD, Hunter P, O’Gorman CC, Rutherford BM, Paez TL. Validation of mathematical models: an overview of the process. 2005.
31. Salman M, Abdel-Hameed ES, Bazaid SA, Al-Shamrani MG. Atomic absorption spectrometry and flame photometry for determination of minerals elements in fresh pomegranate fruit juice. *Der Pharma Chemica.* 2014;6(6):149–155.
32. Burhanuddin MF. Study of COD removal by anaerobic digestion using mixed culture from sewage [Bachelor thesis]. Malaysia: Universiti Malaysia Pahang; 2010.
33. Choi Y, Ryu J, Lee SR. Influence of carbon type and carbon to nitrogen ratio on the biochemical methane potential, pH, and ammonia nitrogen in anaerobic digestion. *J Animal Sci Technol.* 2020;62(1):74.
34. Gashaw A. Co-digestion of municipal organic wastes with night soil and cow dung for biogas production: A Review. *Afr J Biotechnol.* 2016;15(2):32–44.
35. Shahbaz M, Ammar M, Korai RM, Ahmad N, Ali A, Khalid MS, Zou D, Li X. Impact of C/N ratios and organic loading rates of paper, cardboard and tissue wastes in batch and CSTR anaerobic digestion with food waste on their biogas production and digester stability. *SN Appl Sci.* 2020;2:1–3.

36. Orhorhoro EK, Orhorhoro OW, Ebunilo PO. Analysis of the effect of carbon/nitrogen (C/N) ratio on the performance of biogas yields for non-uniform multiple feedstock availability and composition in Nigeria. *Int J Innov Sci Eng Technol*. 2016;3(5):119–126.
37. Kiyasudeen S K, Ibrahim MH, Quaik S, Ahmed Ismail S, Ibrahim MH, Quaik S, Ismail SA. An introduction to anaerobic digestion of organic wastes. Prospects of organic waste management and the significance of earthworms. 2016:23–44.
38. Abdullah NA, Nayan NA, Kamaludin NH, Idris ZM, Tompong MF. Cell growth kinetics of *Aspergillus Oryzae* in industrial natural rubber effluent serum. *ARPN J Eng Appl Sci*. 2006;11(4):2687–2692.
39. Maier RM, Pepper IL. Bacterial Growth. *Environmental Microbiology*. Cambridge, Massachusetts, USA: Academic Press; 2015. 37–56.
40. Muloiwa M, Nyende-Byakika S, Dinka M. Comparison of unstructured kinetic bacterial growth models. *South Afr J Chem Eng*. 2020;33:141–50.
41. David A. Science and Principles of Anaerobic Processing (Anaerobic digestion). In *Municipal Solid Waste Organics Processing Technical Document*. Gatineau, Quebec, Canada: Environment Canada; 2013. pp. 4–10.
42. Momoh OY, Anyata BU, Saroj DP. Development of simplified anaerobic digestion models (SADM's) for studying anaerobic biodegradability and kinetics of complex biomass. *Biochem Eng J*. 2013;79:84–93.