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Context of Different Graph Operations: Fibonacci Product Cordial Labeling of Herschel Graph

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Abstract: The function φ: V (G) → {F1, F2,..., Fn}, where Fj is the jth Fibonacci number (j = 1,..., n), is said to be Fibonacci product cordial labeling if the induced function φ*: E (G) → {0, 1} defined by $\varphi*\ (uv) = (\varphi(u)\varphi\ (v))(mod\ 2)$ meets the criterion $|e\varphi*(0) - e\varphi*(1)| \leq 1$. A graph known as the Fibonacci product cordial graph is one that permits Fibonacci product cordial labeling. The abstract summarizes a research paper that extends the study of cordial labeling to include Fibonacci product cordial labeling, focusing on the Herschel graph and its behavior under various graph operations. The Fibonacci product cordial labeling of the Herschel graph and several graph operations on it were examined in this work.

We established the following findings in this paper:

(1). The Herschel graph Hs is a Fibonacci product cordial graph.

(2). In a Herschel graph, a Fibonacci product cordial network is formed by fusing any

two neighboring vertices of degree

(3). Herschel graphs are Fibonacci product cordial graphs when any vertex in the graph is duplicated.

(4). The switching of a central vertex v in the Herschel graph Hs is a Fibonacci product cordial graph.

(5). The graph obtained by joint of two copies of Herschel graph Hs is a Fibonacci product cordial graph.

(6). DS(Hs) is Fibonacci product cordial graph.

Keywords: Fibonacci: Productcordiallabeling,fusion, duplication, switching, joint sum, degree splitting.

AMSMathematicsSubjectClassification(2010):05C78.

Introduction

If a finite undirected graph contains neither loops nor parallel edges, it is referred to as a graph. For standard notations and terminology in graph theory, see Harary [1]. A mapping from a set of numbers, usually a set of positive or non-negative integers, to the graph members is called a graph labeling.. Edge labeling is discussed if the domain is the set of edges. Labeling is referred to as total labeling if labels are applied to both vertices and edges.Fibonacci cordial labeling, prime cordial labeling, full cordial labeling,[2] and product cordial labeling are all included in the notion of cordial labeling.

For an engaging review of several graph labeling methods, we refer Gallian [3]. In 2019, Tessymol Abraham and Dr. Shiny Jose[4] introduced Fibonacci product cordial labeling, another variant of cordial labeling.Some special graphs with Fibonacci cordial labeling have been described by Rokad and Godasara [5]. The Fibonacci divisor cordial labeling of the Herschel graph with several graph operations was covered by Rokad [6].

Definition 1. A Herschel graph H_s is a bipartite undirected graph with 11 vertices and 18 edges.

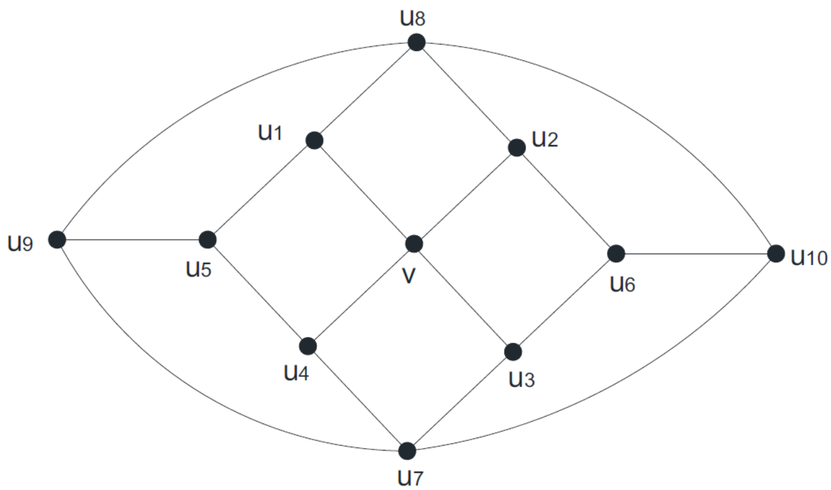

Figure (A)

In this paper, we always fix the position of vertices v, u_1, u_2, ..., u_{10} of H_s as indicated in the above Figure(A), unless or otherwise specified.[7]

Definition 2. Given a graph G, let u and v be its two different vertices. A single vertex, x, in G1 is used to fuse (identify) two vertices, u and v, creating a new graph G1, so that any edges that previously incident with u or v in G now incident with x in G1.

Definition 3. A new graph G1 is created when a vertex v_k' with N(v_k) = N(v_k') is added to a duplicate vertex (vk) of graph G. Put differently, if every vertex that was previously next to vk is now adjacent to v_k', then a vertex v_k' is considered a duplicate of vk.

Definition 4. The graph that results from joining the vertices of the first and second copies of a graph G is referred to as the joint sum of the two copies of G.[8]

Definition 5. Given a graph V = $S_1 \cup S_2 \cup S_3 \cup$ let G = (V(G), E(G))... For any Si, there are at least two vertices of the same degree, and T is equal to $V \backslash \cup S_i$. By adding vertices w_1, w_2, w_3,... w_t and linking to each vertex of Si for $1 \leq i \leq t$, one can construct the degree splitting graph of G, represented by DS(G), from G.[9]

Main Results

Theorem1. The Herschel graph H_S isa Fibonacci product cordial graph.

Proof: Let $G = H_S$ be a Herschel graph and let v be the central vertex and u_i ($1 \le i \le 10$) be the remaining vertices of the Herschel graph.Then $|V(G)| = 11$ and $|E(G)| = 18$.

We define the labeling f: $V(G) \rightarrow \{1,2,...,|V(G)|\}$ as follows.

$f(v) = F_1$, $f(u_1) = F_2$, $f(u_2) = F_3$,

$f(u_3) = F_6$, $f(u_4) = F_9$, $f(u_5) = F_4$,

$f(u_6) = F_5$, $f(u_7) = F_7$,$f(u_8) = F_8$,

$f(u_9) = F_{10}$, $f(u_{10}) = F_{11}$.

From the above labeling pattern, we have$e_f(0) = e_f(1) = 9$.

Hence $|e_f(0) – e_f(1)| \le 1$.

Thus Herschel graph H_S is a Fibonacci product cordial graph.[10]

Example1. Figure 1 depicts the cordial labeling of the Herschel graph Hs using the Fibonacci product.

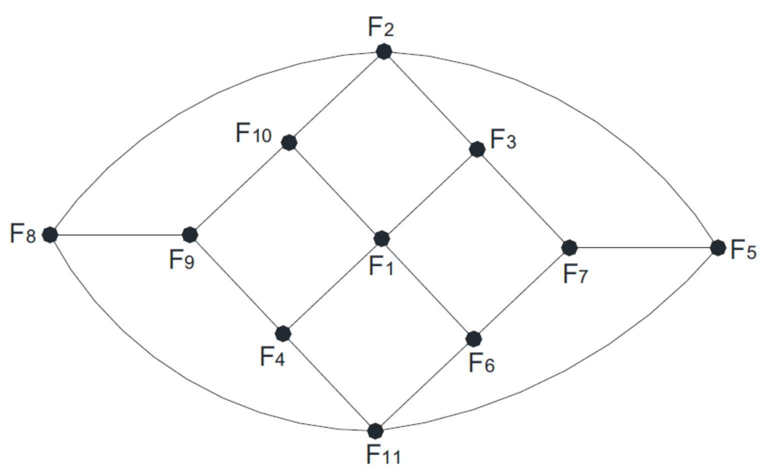

Figure1

Theorem 2.A Fibonacci product cordial graph is created when any two neighboring vertices of degree 3 in the Herschel graph fuse together.

Proof: Let H_S be the Herschel graph with $|V(H_S)| = 11$ and $|E(H_S)| = 18$.Let v be the central vertex of the Herschel graph and it has 3 vertices of degree 4 and 8 vertices of degree 3. Let G be the graph obtained by fusion of two adjacent vertices of degree 3 in the Herschel graph of H_S.

Then |V(G)| = 10 and |E(G)| = 17.

We define the vertex labeling $f:V(G)\rightarrow\{1,2,...,|V(G)|\}$ as follows.

Case1. Fusion of u_2 and u_6.

Suppose that u_2 and u_6 are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_{10}$,

$f(u_1) = F_3$, $f(u_3) = F_6$, $f(u_4) = F_9$,

$f(u_5) = F_2$, $f(u_7) = F_4$, $f(u_8) = F_5$,

$f(u_9) = F_7$, $f(u_{10}) = F_8$.

Case 2. Fusion of u_3 and u_6.

Suppose that u_3 and u_6 are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_{10}$,

$f(u_1) = F_3$, $f(u_2) = F_6$, $f(u_4) = F_9$,

$f(u_5) = F_2$, $f(u_6) = F_4$, $f(u_7) = F_5$,

$f(u_8) = F_7$, $f(u_9) = F_8$.

Case3. Fusion of u_6 and u_{10}.

Suppose that u_6 and u_{10} are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_{10}$,

$f(u_1) = F_3$, $f(u_2) = F_6$, $f(u_3) = F_9$,

$f(u_4) = F_2$, $f(u_5) = F_4$, $f(u_7) = F_5$,

$f(u_8) = F_7$, $f(u_9) = F_8$.

Case 4. Fusionof$u_5$andu_9.

Suppose that u_5 and u_9 are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_{10}$,

$f(u_1) = F_3$, $f(u_2) = F_6$, $f(u_3) = F_9$,

$f(u_4) = F_2$, $f(u_6) = F_4$, $f(u_7) = F_5$,

$f(u_8) = F_7$, $f(u_{10}) = F_8$.

Case 5. Fusion of u_4 and u_5.

Suppose that u_4 and u_5 are fused together as a single vertex u.

$f(v) = F_1$, $f(u) = F_{10}$,
$f(u_1) = F_3$, $f(u_2) = F_6$, $f(u_3) = F_9$,
$f(u_6) = F_2$, $f(u_7) = F_4$, $f(u_8) = F_5$,
$f(u_9) = F_7$, $f(u_{10}) = F_8$.

Case 6. Fusion of u_1 and u_5.

Suppose that u_5 and u_1 are fused together as a single vertex u.
$f(v) = F_1$, $f(u) = F_{10}$,
$f(u_2) = F_3$, $f(u_3) = F_6$, $f(u_4) = F_9$,
$f(u_6) = F_2$, $f(u_7) = F_4$, $f(u_8) = F_5$,
$f(u_9) = F_7$, $f(u_{10}) = F_8$.

Hence $|e_f(0) – e_f(1)| \leq 1$.
Hence, the fusion of any two adjacent vertices of degree 3 in the Herschel graph is a Fibonacci product cordial graph.[11]

Example2. Fibonacci product cordial labeling of fusion of u_6 and u_{10} in H_S is shown in Figure 2.

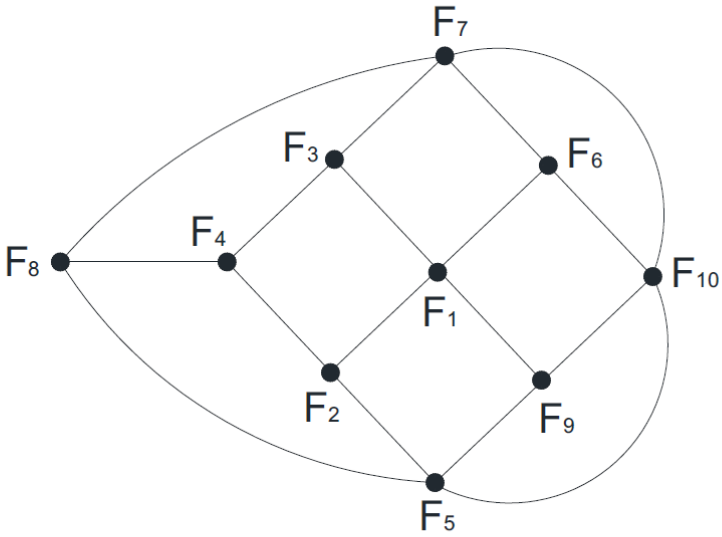

Figure 2

Theorem3. The duplication of any vertex in a Herschel graph is a Fibonacci product cordial graph.[12]

Proof: Consider the Herschel graph Hs, where |E(Hs)| = 18 and |V(Hs)| = 11.Let v be the central vertex in the Herschel graph Hs, and let u_k' be the vertex u_k 's duplicate. Let G be the graph that results from multiplying the single vertex of degrees 3 and 4 in Hs by two. [13]

Then $|V(G)| = 12$ and $|E(G)| = 21$.

We define the vertex labeling $f:V(G)\rightarrow\{1,2,...,|V(G)|\}$ as follows.

Case1. Duplication of vertex u_k, where k = 1, 3, 4, 5, 6, 9.

$f(v) = F_1$, $f(u_k')=F_{11}$,
$f(u_1) = F_2$, $f(u_2) = F_3$, $f(u_3) = F_4$,
$f(u_4) = F_5$, $f(u_5) = F_7$, $f(u_6) = F_6$,
$f(u_7) = F_8$, $f(u_8) = F_9$, $f(u_9) = F_{12}$,
$f(u_{10}) = F_{10}$.

Case 2. Duplication of vertex u_k,where k = 2, 10.

$f(v) = F_1$, $f(u_k')=F_{11}$,
$f(u_1) = F_3$, $f(u_2) = F_2$, $f(u_3) = F_4$,
$f(u_4) = F_5$, $f(u_5) = F_6$, $f(u_6) = F_7$,
$f(u_7) = F_8$, $f(u_8) = F_9$, $f(u_9) = F_{10}$,
$f(u_{10}) = F_{12}$.

From all the above cases we have $|e_f(0) – e_f(1)| \leq 1$.

Hence, the duplication of any vertex in a Herschel graph is a Fibonacci product cordial graph.[14]

Example3. Fibonacci product cordial labeling of the duplication of the vertex u_{10} in H_S is shown in Figure 3.

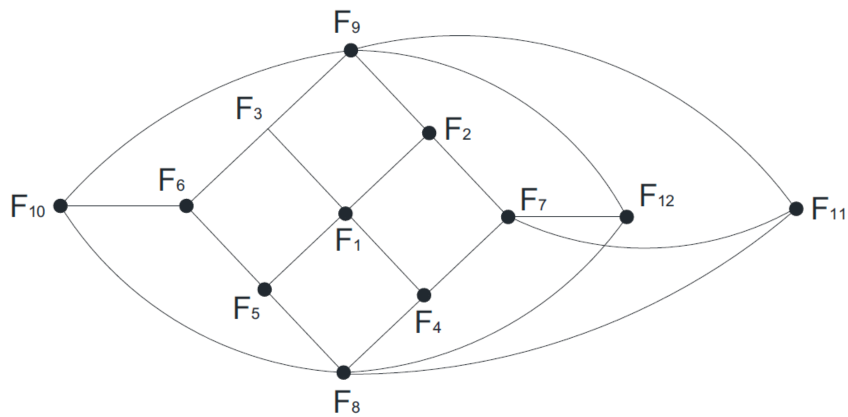

Figure 3

Theorem4. The switching of a central vertex *v* in the Herschel graph H_s is a Fibonacci product cordial graph.

Proof: Let H_S be the Herschel graph with $|V(H_S)| = 11$ and $|E(H_S)| = 18$.Let *v* be *t*he new graph created by swapping the core vertex, G, with the central vertex *v*. Then $|V(G)| = 1\ 1$ and $|E(G)| = 20$.

We define the labeling f:V(G)→{1,2,...,|V(G)|} as follows.

$f(v) = F_1$

$f(u_1) = F_3$, $f(u_2) = F_2$, $f(u_3) = F_4$,

$f(u_4) = F_5$, $f(u_5) = F_7$, $f(u_6) = F_8$,

$f(u_7) = F_{10}$, $f(u_8) = F_{11}$, $f(u_9) = F_6$,

$f(u_{10}) = F_9$.

From the above labeling pattern, we observe that $e_f(0) = e_f(1) = 9$.

Hence, $|e_f(0) – e_f(1)| \leq 1$.

Thus, the switching of a central vertex *v* in the Herschel graph H_s is a Fibonacci product cordial graph.

Example4. Fibonacci product cordial labeling of switching of a central vertex v in H_S is shown in Figure 4.

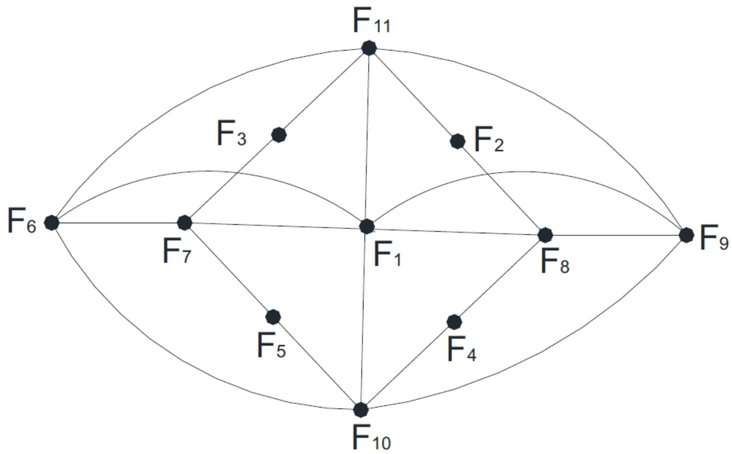

Figure 4

Theorem 5. The graph obtained by joint of two copies of Herschel graph H_s is a Fibonacci product cordial graph

Proof. Let G be the joint of two copies of Herschel graph H_s. Let$\{u,u1,u2,...,u10\}$ and $\{v,v1,v2,...,v10\}$ be the vertices of first and second copy of Herschel graph H_s respectively. Then $|V(G)| = 22$ and $E(G) = 37$.
We define the vertex labeling $f:V(G)\to\{1,2,...,|V(G)|\}$as follows.

$f(u) = F_{3,}$
$f(u_1) = F_1$, $f(u_2) = F_2$, $f(u_3) = F_4$,
$f(u_4) = F_5$, $f(u_5) = F_6$, $f(u_6) = F_9$,
$f(u_7) = F_{12}$, $f(u_8) = F_{15}$, $f(u_9) = F_{18}$,
$f(u_{10}) = F_{21}$.
and
$f(v) = F_{7,}$
$f(v_1) = F_8$, $f(v_2) = F_{10}$, $f(v_3) = F_{11}$,
$f(v_4) = F_{13}$, $f(v_5) = F_{14}$, $f(v_6) = F_{16}$,
$f(v_7) = F_{17}$, $f(v_8) = F_{19}$, $f(v_9) = F_{20}$,
$f(v_{10}) = F_{22}$.
From the above labeling pattern, we observe that $e_f(0) = 18$, $e_f(1) =19$.
Hence, $|e_f(0) – e_f(1) | \leq 1$.
Thus,the graph obtained by joint of two copies of Herschel graph H_s is a Fibonacciproduct cordial graph.
Example 5. Fibonacci product cordial labeling of the joint of two copies of Herschel graph

$H_s$is shown in Figure 5.

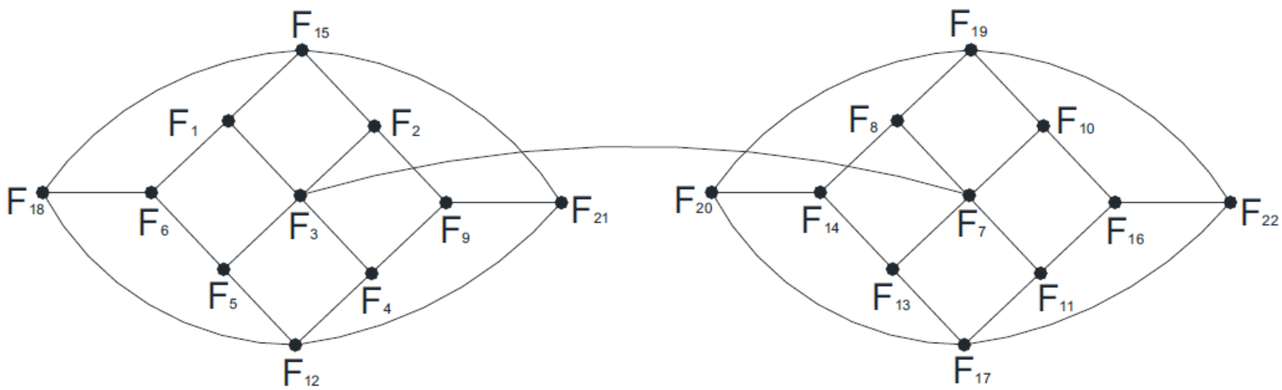

Figure 5

Theorem 6. $DS(H_s)$ is Fibonacciproduct cordial graph.

Proof . Consider $H_s$with $V(H_s) = \{v, u_i: 1 \leq i \leq 10\}$.Here $V(K_{1,n}) = V_1 \cup V_2$, where V_1= vertices of degree 3 and V_2= vertices of degree 4 . Now in order to obtain $DS(H_s)$ from G, we add $w_1$and w_2 corresponding to V_1 and V_2. Then $|V(DS(H_s))| = 13$ and $|E(DS(K_{1,n}))| = 28$.

We define the labeling $f:V(G)\to\{1,2,...,|V(G)|\}$asfollows.

$f(v) = F_8$, $f(w_1) = F_1$, $f(w_1) = F_2$.

$f(u_1) = F_{13}$, $f(u_2) = F_3$, $f(u_3) = F_9$,

$f(u_4) = F_{12}$, $f(u_5) = F_6$, $f(u_6) = F_4$,

$f(u_7) = F_{11}$, $f(u_8) = F_7$, $f(u_9) = F_5$,

$f(u_{10}) = F_{10}$.

From the above labeling pattern, we observe that $e_f(0) =15$, $e_f(1) = 14$.

Hence, $|e_f(0) – e_f(1) | \leq 1$.

Thus, $DS(H_s)$ is Fibonacciproduct cordial graph.

Example 6. Fibonacci product cordial labeling of $DS(H_s)$ is shown in Figure 6.

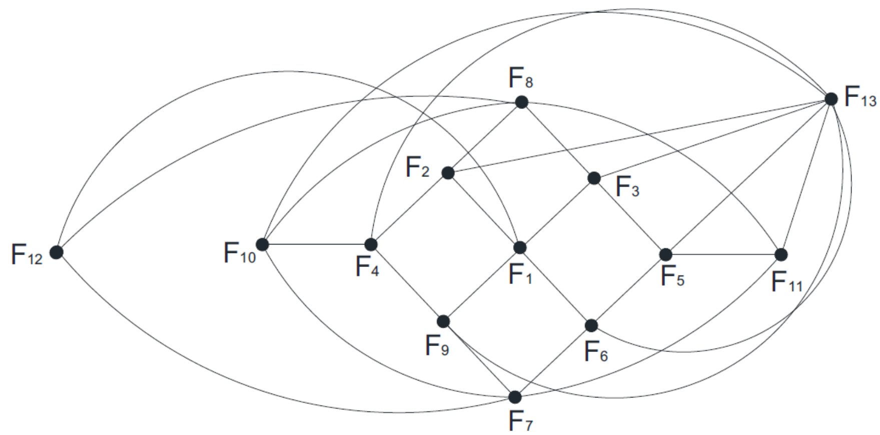

Figure 6

Conclusion

In this paper we have proved six new results of Herschel graph. The goal of the cordial graph is to investigate some new Fibonacci products. By proving six new results related to the Herschel graph, the paper contributes to the theoretical foundation of graph theory. Simultaneously, by exploring new Fibonacci products in the realm of cordial graphs, it aims to uncover novel interactions and applications that enrich our understanding of these mathematical structures.

REFERENCES

[1] Harary F. Graph theory, Addison Wesley, Reading. Massachusetts. 1969.
[2] Ghodasara GV, Rokad AH, Jadav II. Cordial labeling of grid related graphs. International Journal of Combinatorial Graph Theory and Applications. 2013;6(2):55-62.
[3] Gallian JA. A dynamic survey of graph labeling. Electronic Journal of combinatorics. 2018;1(DynamicSurveys):DS6.
[4] Abraham T, Jose S. Fibonacci product cordial graphs. Journal of Emerging Technologies and Innovative Research. 2019;6(1):58-63.
[5] Rokad AH, Ghodasara GV. Fibonacci cordial labeling of some special graphs. Annals of Pure and Applied Mathematics. 2016;11(1):133-44.
[6] Rokad AH. Fibonacci Divisor cordial Labeling of Herschel Graph in Context of

Various Graph Operations. International Journal of Advance and Innovative Research.2022; 9(1)

[7] Sugumaran A, Rajesh K. Sum divisor cordial labeling of Theta graph. Annals of pure and applied mathematics. 2017 Sep;14(2):313-20.

[8] Rokad AH. Difference cordial labeling in context of joint sum of some graphs. Research & Reviews: Discrete Mathematical Structures.2018;5(1):21-25.

[9] Sugumaran A, Rajesh K. Sum divisor cordial labeling of Herschel graph. Annals of pure and applied mathematics. 2017 Oct;14(3):465-72.

[10] Lourdusamy A, Patrick F. Sum divisor cordial graphs. Proyecciones (Antofagasta). 2016 Mar;35(1):119-36.

[11] Rokad AH. Product cordial labeling of double wheel and double fan related graphs. Kragujevac Journal of Mathematics.2019;43(1):7-13.

[12] Gondalia JT, Rokad AH. Multiply divisor cordial labeling. International Journal of Engineering and Advanced Technology (IJEAT). 2019;9(2).

[13] Varatharajan R, Navanaeethakrishnan S, Nagarajan K. Divisor cordial graphs. International journal of mathematical combinatorics. 2011 Dec 1;4:15.

[14] Vaidya SK, Shah NH. Some star and bistar related divisor cordial graphs. Annals of Pure and Applied Mathematics. 2013;3(1):67-77.