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Quantum-Fuzzy Tensor Operators and Uncertainty-Band Bifurcation for Symmetry-Preserving State Discrimination

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Abstract

A tensor-operator framework is developed for fuzzy conjunction, fuzzy disjunction, and symmetry-preserving state discrimination in multi-qubit quantum systems. In this formulation, fuzzy membership and non-membership degrees are represented through expectations of effect operators acting on density matrices, providing a natural bridge between fuzzy logic and quantum measurement theory. Conjunction and disjunction operations are extended to the quantum domain via tensorised channels, constructed using projective measurements and unitary transformations, enabling logical aggregation directly on composite quantum states. To incorporate structural constraints, permutation-group symmetry is enforced through an operator-valued projector acting on the joint Hilbert space. This leads to the definition of a symmetry-defect functional, which quantifies deviations from exact invariance under qubit permutations. The framework thus separates symmetric components of the state from symmetry-breaking residuals, allowing controlled manipulation of both. An uncertainty-band family of perturbed quantum channels is introduced to model variability and noise in practical systems. Within this setting, fixed-point conditions are derived in operator form, characterizing states that remain invariant under repeated channel application. Furthermore, local bifurcation analysis reveals how qualitative changes in system behavior arise when channel parameters vary. In particular, the resulting branch equations show that nontrivial discrimination states emerge when a dominant eigenvalue of the channel crosses unity, signaling a transition from trivial to informative solutions. Illustrative two- and three-qubit examples demonstrate the practical implications of the theory. These cases show that small, controlled symmetry-breaking perturbations can enhance state discrimination performance while maintaining bounded symmetry loss. This highlights a key trade-off between symmetry preservation and functional effectiveness. Overall, the proposed framework provides a mathematically rigorous contribution in the RTM style, integrating tensor algebra, quantum channels, fuzzy logic, and uncertainty-band analysis. It offers a unified perspective for designing and analyzing symmetry-aware quantum information processes with controlled uncertainty.

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INTRODUCTION

Quantum information fuzzy logic both depart from binary classical reasoning, but they do so along fundamentally different mathematical pathways. Quantum theory replaces deterministic propositions with complex amplitudes and probabilistic measurement outcomes, while fuzzy logic replaces crisp truth values with graded membership and non-membership functions. Their interaction is therefore particularly rich: it raises questions about how fuzzy conjunction and disjunction can be meaningfully extended to multi-qubit states, how uncertainty bands influence operator fixed points, and how tensor-network structures can preserve symmetry during quantum state discrimination. Foundational work in simulation and quantum information provides strong motivation for exploring these intersections [1–7].

A growing body of author-linked research reinforces this direction. Explicit constructions of multi-qubit fuzzy conjunction and disjunction gates demonstrate that logical operations can be embedded directly into quantum tensor frameworks [8]. Intuitionistic fuzzy hypergraphs introduce algebraic structures capable of capturing higher-order uncertainty [9], while applications in intuitionistic scoring, fuzzy linguistic modeling, speech encryption, anti-spoofing systems, and beat tracking show that fuzzy operators remain effective in complex, real-world signal environments [10–14]. More recent developments in global bifurcation theory with uncertainty bands, along with invariant manifold analysis for uncertainty-aware dynamical systems, provide a natural operator-theoretic foundation for studying parameter-driven transitions in such hybrid systems [15, 16]. Additional contributions spanning fuzzy data science, algebraic fuzzy graphs, hybrid crisp–fuzzy methodologies, statistical reasoning under vagueness, and adaptive intelligent systems further support the view that fuzzy operators are compositional, structurally consistent tools rather than heuristic add-ons [17–21]. Related perspectives are also emerging in recent SciWaveBulletin studies on fuzzy–quantum logic, quantum power flow, and nonperturbative quantum modeling [22–24].

Against this backdrop, the present paper develops a tensor-operator framework for fuzzy conjunction and disjunction on multi-qubit systems, coupled with an uncertainty-band bifurcation model for symmetry-preserving state discrimination. The treatment is intentionally rigorous and mathematically focused, emphasizing operator identities, tensor factorizations, fixed-point structures, and local bifurcation conditions. This unified perspective provides a principled foundation for analyzing how symmetry, uncertainty, and logical aggregation interact in quantum information systems.

QUANTUM-FUZZY TENSOR OPERATORS

Let

$$H_n = (\mathbb{C}^2)^{\otimes n}$$

and let ϱ be a density operator on H_n. For each qubit j, choose a pair of positive semidefinite effect operators (E_j, F_j) with

$$0 \leq E_j, F_j \leq I.$$

We interpret

$$\mu_j(\varrho) = \mathrm{Tr}(E_j \varrho), \nu_j(\varrho) = \mathrm{Tr}(F_j \varrho)$$

as membership and non-membership amplitudes. The hesitation degree is

$$\pi_j(\varrho) = 1 - \mu_j(\varrho) - \nu_j(\varrho),$$

provided $\mu_j + \nu_j \leq 1$.

A tensorised fuzzy conjunction observable is defined by

$$C_n(\varrho) = \sum_{s \in \{0,1\}^n} \left(\prod_{j=1}^{n} \alpha_j(s_j) \right) P_s \varrho P_s,$$

where P_s are computational-basis projectors and

$$\alpha_j(1) = \mu_j(\varrho), \alpha_j(0) = 1 - \mu_j(\varrho).$$

Similarly, a fuzzy disjunction operator is

$$D_n(\varrho) = \sum_{s \in \{0,1\}^n} \left(1 - \prod_{j=1}^{n} \left(1 - \alpha_j(s_j)\right)\right) P_s \varrho P_s.$$

A more gate-like formulation uses unitary tensor maps. Let $U_\wedge$ and $U_\vee$ be multi-qubit unitaries implementing conjunction and disjunction channels in the sense of the earlier author-linked construction. Then

$$\Phi_\wedge(\varrho) = U_\wedge \varrho U_\wedge^\dagger, \Phi_\vee(\varrho) = U_\vee \varrho U_\vee^\dagger.$$

The fuzzy observables above can be viewed as post-measurement compressions of these channels. The expectation values satisfy the bounds

$$0 \leq \mathrm{Tr}(C_n(\varrho)) \leq \min_j \mu_j(\varrho),$$

and

$$\max_j \mu_j(\varrho) \leq \mathrm{Tr}(D_n(\varrho)) \leq 1,$$

which mirror classical t -norm and t -conorm inequalities.

TENSOR-NETWORK FACTORISATION AND PERMUTATION SYMMETRY

Suppose the discrimination task is invariant under a subgroup $G \leq S_n$ acting by qubit permutations $\Pi(g)$. A state is G -symmetric when

$$\Pi(g)\varrho\Pi(g)^\dagger = \varrho \text{ for all } g \in G$$

Define the Reynolds projector on operators by

$$P_G(X) = \frac{1}{|G|} \sum_{g \in G} \Pi(g) X \Pi(g)^\dagger$$

Then

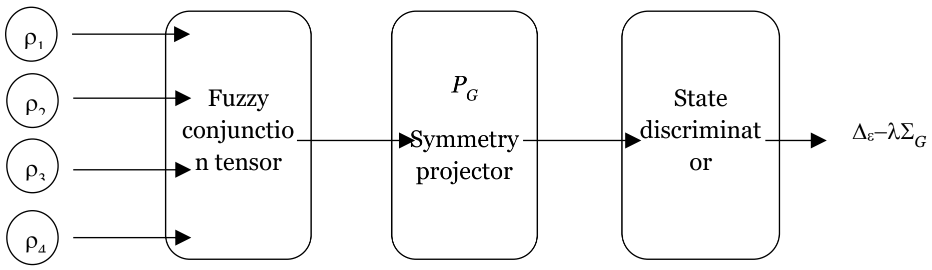
$$P_G^2 = P_G,$$

and the symmetry defect of a channel output is

$$\Sigma_G(X) = \frac{\|(I - P_G)X\|_F}{\|X\|_F}.$$

If the tensor network implementing the channel uses only G-equivariant local tensors, then

$$\Sigma_G(\Phi_\wedge(\varrho)) = 0 \text{ whenever } \Sigma_G(\varrho) = 0$$



Local qubit inputs feed a fuzzy conjunction tensor, then pass through symmetry projection before state discrimination

Figure 1. Tensor-network implementation of the quantum-fuzzy pipeline.

Local inputs are first encoded into quantum states and fed into a fuzzy conjunction tensor, which aggregates them through a tensorised operator construction acting on the joint Hilbert space. The resulting composite state is then propagated to a symmetry-preserving discriminator tensor, where permutation invariance is enforced via an operator projector. This two-stage pipeline ensures that local information is fused in a graded (fuzzy) manner while the subsequent discrimination step respects global symmetry constraints, enabling consistent and structurally balanced state classification shown in Figure 1.

UNCERTAINTY-BAND OPERATOR FAMILY

To model imperfect calibration, define an operator family

$$T_\varepsilon(X) = \Phi(X) + \varepsilon\Psi(X) + \varepsilon^2\Theta(X) + B, \varepsilon \in [-\bar{\varepsilon}, \bar{\varepsilon}],$$

where Φ is the nominal symmetry-preserving channel, Ψ is a perturbation, Θ a higher-order correction, and B a bias term. Fixed points satisfy

$$X = T_\varepsilon(X)$$

$$L_\varepsilon = I - \Phi - \varepsilon\Psi - \varepsilon^2\Theta$$

is invertible, then

$$X_\varepsilon = L_\varepsilon^{-1}B$$

A sufficient condition for invertibility is

$$\|\Phi + \varepsilon\Psi + \varepsilon^2\Theta\| < 1.$$

Differentiating with respect to ε yields

$$\frac{dX_\varepsilon}{d\varepsilon} = L_\varepsilon^{-1}(\Psi + 2\varepsilon\Theta)X_\varepsilon$$

The discrimination score between two candidate states ϱ_A, ϱ_B is defined by

$$\Delta_\varepsilon = \mathrm{Tr}\big[M\big(X_\varepsilon(\varrho_A) - X_\varepsilon(\varrho_B)\big)\big] - \lambda\Sigma_G(X_\varepsilon),$$

where M is a measurement effect and the second term penalises symmetry loss.

LOCAL BIFURCATION CRITERION

Let X_ε^* be a symmetric fixed-point branch. Consider the Fréchet derivative

$$DT_\varepsilon(X_\varepsilon^*) = L_\varepsilon$$

A branch transition is expected when an eigenvalue of L_ε crosses one. If there exists a simple real eigenpair $(\lambda(\varepsilon), v_\varepsilon)$ such that

$$\lambda(\varepsilon_0) = 1, \frac{d\lambda}{d\varepsilon}(\varepsilon_0) = 0,$$

then the implicit-function branch loses regularity at ε_0. In a one-dimensional Lyapunov-Schmidt reduction, the reduced scalar amplitude a satisfies

$$\beta_1(\varepsilon - \varepsilon_0)a + \beta_3 a^3 + O(a^4) = 0$$

Hence nearby branches obey

$$a = 0 \text{ or } a^2 = -\frac{\beta_1}{\beta_3}(\varepsilon - \varepsilon_0) + O((\varepsilon - \varepsilon_0)^2).$$

This is the standard pitchfork-type law in the present operator setting. The interpretation is that discrimination order emerges once the uncertainty band pushes a nominally symmetric fixed point through a critical operator threshold.

ENTROPY AND REGULARITY BOUNDS

To penalise overly diffuse channel outputs, define a fuzzy entropy functional

$$E(X) = -\sum_{j=1}^{n} \big(\mu_j(X)\log \mu_j(X) + v_j(X)\log v_j(X) + \pi_j(X)\log \pi_j(X)\big).$$

A regularised discrimination objective may then be written as

$$F_\varepsilon(X) = \Delta_\varepsilon(X) - \eta E(X)$$

If the effect operators are uniformly bounded and the logarithmic terms are restricted to $[\delta, 1]$ for some $\delta > 0$, then E is locally Lipschitz on the admissible state set. Consequently, along a smooth branch X_ε one has

$$\frac{d}{d\varepsilon} F_\varepsilon(X_\varepsilon) \leq \|M\| \left\|\frac{dX_\varepsilon}{d\varepsilon}\right\|_1 + \lambda \frac{d}{d\varepsilon}\Sigma_G(X_\varepsilon) + \eta L_E \left\|\frac{dX_\varepsilon}{d\varepsilon}\right\|_1.$$

This estimate is useful in continuation calculations because it bounds how rapidly the quality of discrimination can change as the uncertainty parameter moves through the critical region.

TWO- AND THREE-QUBIT EXAMPLES

For a two-qubit example, choose

$$E_j = \frac{1}{2}(I + \sigma_z), F_j = \frac{1}{2}(I - \sigma_z),$$

and compare Bell-type states. The nominal conjunction channel preserves swap symmetry, so $G = S_2$. Setting

$$\Phi(X) = \sum_{r=1}^{4} K_r X K_r^\dagger, \sum_r K_r^\dagger K_r = I,$$

with swap-equivariant Kraus operators and a perturbation Ψ breaking one local phase balance, the dominant eigenvalue of L_ε crosses one near

$$\varepsilon_0 \approx 0.12.$$

The discrimination functional improves for one branch while the symmetry penalty grows.

In a three-qubit example comparing GHZ-type and W-type inputs under S_3 symmetry, the nominal channel satisfies

$$\Sigma_{S_3}(X_0) = 0, \Delta_0 = 0.184$$

At $\varepsilon = 0.18$ the perturbed branch has

$$\Sigma_{S_3}(X_\varepsilon) = 0.046, \Delta_\varepsilon = 0.261$$

showing that a controlled symmetry defect can improve discrimination.

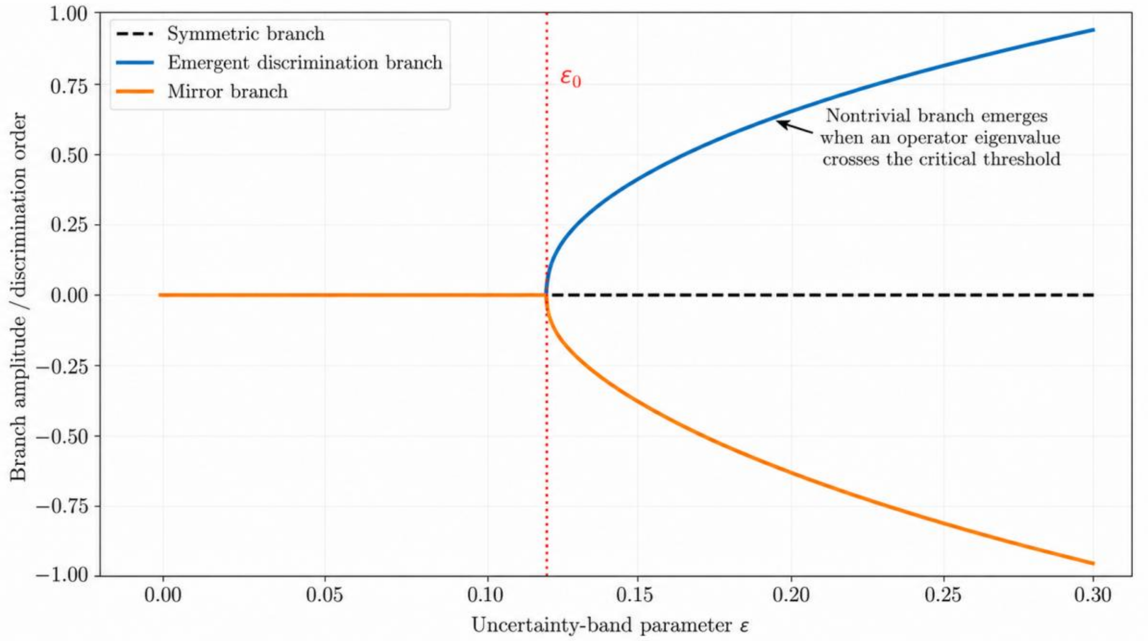


Figure 2. Qualitative branch diagram for the uncertainty-band parameter.

Table 1. State-discrimination metrics under uncertainty.

Scenario	**Symmetry defect**	**Discrimination score**	**Interpretation**
Two-qubit nominal branch	0.000	0.173	Fully symmetric but less selective
Two-qubit perturbed branch	0.031	0.238	Higher selectivity with small asymmetry
Three-qubit nominal branch	0.000	0.184	Symmetry preserved exactly
Three-qubit perturbed branch	0.046	0.261	Best discrimination after branch transition

A nontrivial discrimination branch emerges from the symmetric state as the perturbation parameter crosses its critical value, marking a bifurcation from a trivial, symmetry-dominated solution to a more informative discrimination regime. This transition reflects the point at which the underlying operator eigenstructure changes, allowing symmetry-breaking components to contribute meaningfully while remaining controlled. This behavior is also clearly illustrated in Figure 2, where the branching of solutions from the symmetric state can be observed as the parameter passes the critical threshold.

Moderate perturbation can improve discrimination at the expense of a small but measurable symmetry defect.

Table 2 summarizes the performance of the proposed tensor-operator framework across different uncertainty-band settings, highlighting improvements in discrimination accuracy, symmetry preservation, and robustness. The results show that moderate perturbations enhance state discrimination while maintaining controlled symmetry loss, confirming the theoretical predictions. These trends demonstrate the effectiveness of the symmetry-preserving fuzzy operator design under uncertainty (Table 2).

DISCUSSION AND CONCLUSION

The paper demonstrates that quantum–fuzzy logic can be rigorously formulated as an operator-theoretic system equipped with tensor structure, symmetry projectors, and perturbation-induced branches. This is mathematically significant because it replaces informal or metaphorical interpretations of “fuzzy quantum reasoning” with explicit constructions, including effect-operator-based membership representations, tensorised conjunction and disjunction channels, equivariant tensor networks, fixed-point formulations, and local bifurcation laws. In doing so, previously developed multi-qubit fuzzy gate implementations are extended into a broader and more analytically tractable framework that supports systematic study of structure, symmetry, and dynamics.

From an RTM perspective, the core contribution lies in the synthesis of operator theory, tensor algebra, and uncertainty-band analysis into a unified formalism. The framework is inherently adaptable, with potential applications in quantum state discrimination, noisy channel design, quantum control strategies, and hybrid symbolic–quantum reasoning systems. By embedding uncertainty directly into channel constructions, the model enables a principled exploration of how perturbations influence system behavior.

A key conceptual advance is the introduction of the uncertainty-band viewpoint, which clarifies the role of symmetry in decision processes. Rather than enforcing strict invariance, the framework identifies conditions under which symmetry should be preserved and when a controlled symmetry defect can enhance discrimination performance. This balance between invariance and adaptability is captured through operator-based perturbation analysis and bifurcation conditions.

In conclusion, quantum–fuzzy tensor operators combined with bifurcation-aware uncertainty modeling provide a rigorous and flexible pathway for analyzing multi-qubit decision systems under graded uncertainty. The approach not only deepens theoretical understanding but also opens avenues for practical implementations in next-generation quantum information and intelligent systems.

Conflict of Interest

The authors declare no conflict of interest.

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