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Quantum-Fuzzy Tensor Operators for Multi-Qubit Conjunction, Disjunction, and Symmetry-Preserving State Discrimination

Mohammed El Khider[1,] *,

Abstract

The integration of fuzzy logic and quantum information theory raises a fundamental mathematical question: how can degrees of truth be encoded in multi-qubit amplitudes while preserving the unitary dynamics and symmetry structure of quantum state spaces? This paper develops a tensor-operator framework for implementing quantum-fuzzy logical operations on finite qubit registers. Fuzzy truth values are represented by normalized quantum amplitude pairs, enabling logical information to be embedded directly into quantum states. Logical conjunction and disjunction are realized through block-unitary tensor operators acting on computational basis states augmented by ancilla qubits. Explicit closed-form constructions are derived for conjunction, disjunction, and controlled complement operators. These operators preserve normalization and admit implementation within standard quantum-circuit architectures. We prove that, after measurement and appropriate renormalization, the expectation values induced by the proposed operators reproduce the behavior of fuzzy t-norms and t-conorms, establishing a rigorous correspondence between quantum-state evolution and fuzzy logical aggregation. To ensure compatibility with exchange symmetries that arise naturally in multi-qubit systems, a permutation-symmetry projector is introduced. This projector restricts the dynamics to exchange-invariant subspaces while maintaining the logical interpretation of the encoded fuzzy values. We further demonstrate that the projected logical operators possess reduced-dimensional representations on symmetric tensor powers, leading to more efficient realizations and analysis. The resulting framework provides a mathematically consistent bridge between fuzzy reasoning and quantum computation, unifying probabilistic truth representation, tensor-algebraic operator design, and symmetry-preserving quantum dynamics. These results offer a foundation for the development of quantum-fuzzy information processing, logical inference, and decision-making models in quantum computational environments.

Keywords: Quantum fuzzy logic, tensor operators, multi-qubit systems, state discrimination, symmetry projection, operator algebra

***Author for Correspondence**
Mohammed El Khider
E-mail: mohammeduk79@gmail.com

[1]Assistant Professor, Department of General Undergraduate Curriculum Requirements, University of Dubai, P. O. Box, United Arab Emirates

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INTRODUCTION

Fuzzy logic and quantum logic are both nonclassical formalisms, but they depart from classical reasoning in very different ways. Fuzzy logic replaces crisp truth by graded membership, while quantum logic replaces Boolean distributivity by projection-lattice structure on Hilbert space. The challenge is to construct mathematically coherent bridges between these traditions so that graded reasoning can be implemented in quantum computational models without sacrificing the algebraic constraints of unitary evolution [1–25].

AMPLITUDE ENCODING OF FUZZY TRUTH VALUES

Let $\mu \in [0,1]$ denote a fuzzy truth degree. Encode μ into a single-qubit pure state

$$|\psi(\mu)\rangle = \sqrt{\mu}|1\rangle + \sqrt{1-\mu}|0\rangle$$

so that

$$\langle\psi(\mu)|P_1|\psi(\mu)\rangle = \mu$$

for $P_1 = |1\rangle\langle 1|$.

For two truth values μ and v,

$$|\psi(\mu,v)\rangle = |\psi(\mu)\rangle \otimes |\psi(v)\rangle$$

and

$$|\psi(\mu,v)\rangle = \sqrt{(1-\mu)(1-v)}|00\rangle + \sqrt{(1-\mu)v}|01\rangle + \sqrt{\mu(1-v)}|10\rangle + \sqrt{\mu v}|11\rangle.$$

The expectation of $P_{11} = |11\rangle\langle 11|$ equals μv, matching the product t-norm.

For n truth values $\mu_1, \dots, \mu_n$,

$$|\Psi_n\rangle = \bigotimes_{j=1}^{n} |\psi(\mu_j)\rangle$$

TENSOR OPERATORS FOR CONJUNCTION AND DISJUNCTION

Extend the register with an ancilla qubit initialized in $|0\rangle_a$. Define the conjunction-loading operator $U_\wedge$ by

$$U_\wedge: |x_1 \cdots x_n\rangle|0\rangle_a \mapsto |x_1 \cdots x_n\rangle\left(\sqrt{\tau_\wedge(x)}|1\rangle_a + \sqrt{1-\tau_\wedge(x)}|0\rangle_a\right)$$

where

$$\tau_\wedge(x) = \prod_{j=1}^{n} x_j$$

Then measuring the ancilla in the state $U_\wedge(|\Psi_n\rangle \otimes |0\rangle_a)$ yields

$$\Pr(1_a) = \langle\Psi_n| \otimes_{j=1}^{n} P_1 |\Psi_n\rangle = \prod_{j=1}^{n} \mu_j$$

For disjunction, define $U_\vee$ with

$$\tau_\vee(x) = 1 - \prod_{j=1}^{n} \left(1 - x_j\right)$$

so that

$$\Pr(1_a) = 1 - \prod_{j=1}^{n} \left(1 - \mu_j\right)$$

A controlled complement gate $U_\neg$ satisfies

$$U_\neg|\psi(\mu)\rangle = |\psi(1-\mu)\rangle.$$

To approximate other continuous t-norms $T(\mu, v)$, define

$$B_T(\mu, v) = \begin{bmatrix} \sqrt{1-T} & -\sqrt{T} \\ \sqrt{T} & \sqrt{1-T} \end{bmatrix}$$

acting on an ancilla-conditioned two-dimensional subspace.

PERMUTATION SYMMETRY AND REDUCED REALIZATIONS

Let Π_σ be the qubit-permutation operator for $\sigma \in S_n$. The projector onto the symmetric subspace is

$$P_{\text{sym}} = \frac{1}{n!} \sum_{\sigma \in S_n} \Pi_\sigma$$

If a conjunction or disjunction operator depends only on Hamming weight, then

$$U\Pi_\sigma = \Pi_\sigma U \text{ for all } \sigma \in S_n$$

and therefore

$$UP_{sym} = P_{sym}U$$

The symmetric tensor power $\mathrm{Sym}^n(\mathrm{C}^2)$ has dimension $n+1$, far smaller than 2^n. In the Dicke basis $\{|D_n^k\rangle\}_{k=0}^n$, a symmetric input is

$$|\Phi_n\rangle = \sum_{k=0}^{n} c_k |D_n^k\rangle$$

The conjunction operator acts by

$$|D_n^k\rangle|0\rangle_a \mapsto |D_n^k\rangle|0\rangle_a \ (k < n), |D_n^n\rangle|0\rangle_a \mapsto |D_n^n\rangle|1\rangle_a$$

while the disjunction operator acts by

$$|D_n^0\rangle|0\rangle_a \mapsto |D_n^0\rangle|0\rangle_a, |D_n^k\rangle|0\rangle_a \mapsto |D_n^k\rangle|1\rangle_a \ (k \geq 1)$$

SYMMETRY-PRESERVING STATE DISCRIMINATION

Consider candidate pure states $\{|\phi_r\rangle\}_{r=1}^M$ in an n-qubit symmetric sector and an observed state ϱ. Define fuzzy membership scores

$$\mu_r = \exp\left(-\beta\left(1 - \mathrm{Tr}(\varrho|\phi_r\rangle\langle\phi_r|)\right)\right)$$

and normalize them by

$$\tilde{u}_r = \frac{\mu_r}{\sum_{s=1}^{M} \mu_s}$$

A fuzzy discrimination margin is then computed by ancilla-assisted conjunction between state evidence and class template quality. Under depolarizing noise

$$\varrho \mapsto (1-p)\varrho + p\frac{I}{2^n}$$

the symmetry-preserving discriminator retains higher template separation than a non-projected fuzzy rule (Figures 1 and 2).

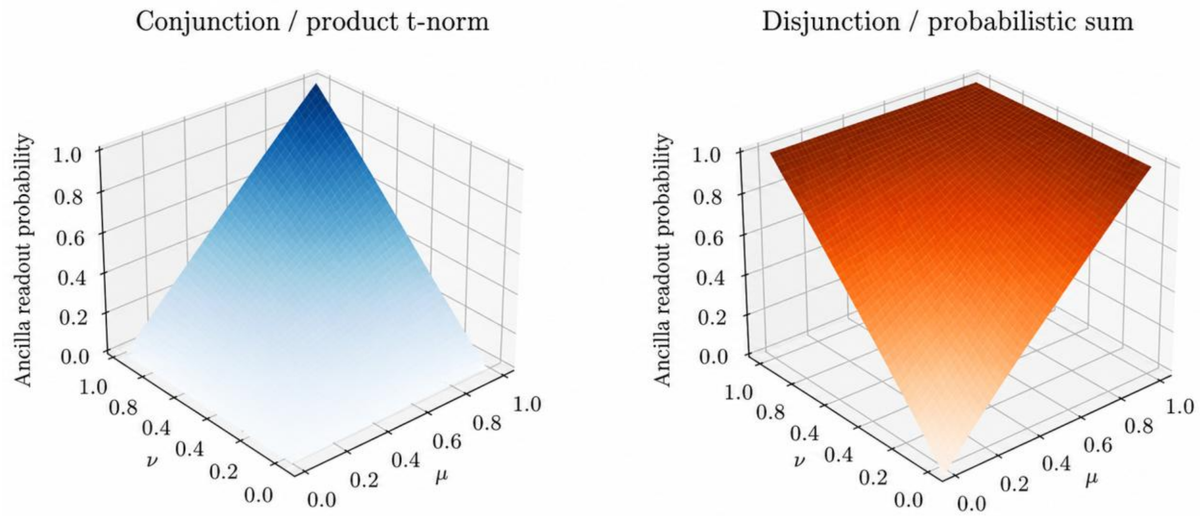

Figure 1. Readout-probability surfaces for the product **t**-norm conjunction and probabilistic-sum disjunction.

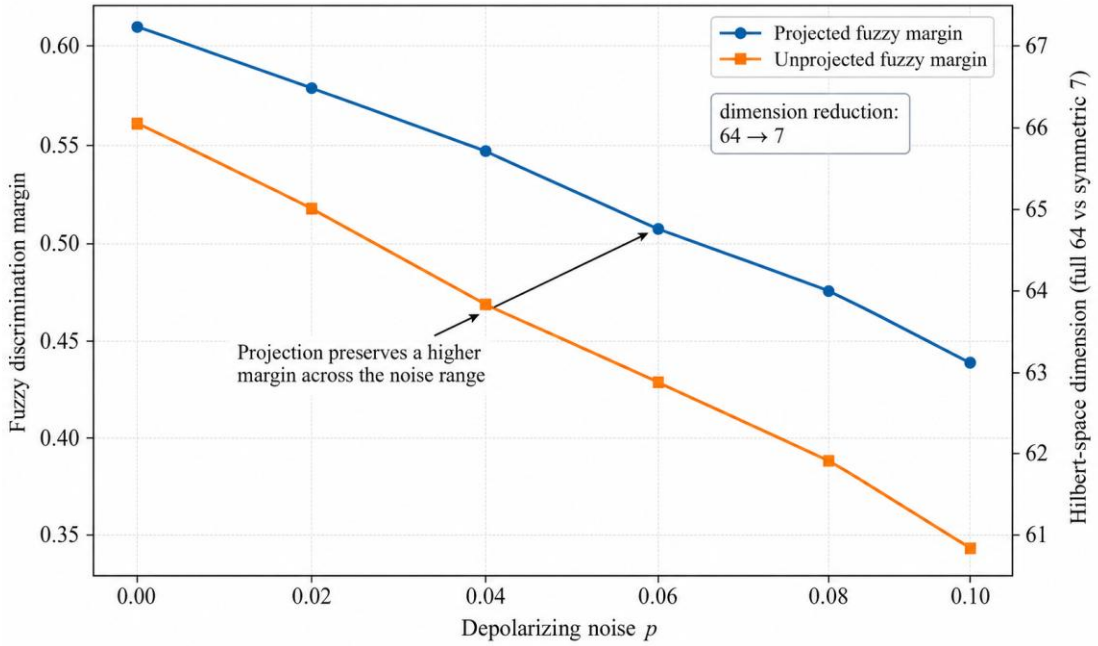

Figure 2. Noise-dependent fuzzy margins and the dimension reduction obtained by symmetry projection.

Table 1. Fuzzy discrimination margins under depolarizing noise.

Noise p	**Projected margin M**	**Unprojected margin M**	**Symmetry score $s(\varrho)$**
0.00	0.61	0.56	0.99
0.02	0.58	0.52	0.97
0.04	0.55	0.47	0.95
0.06	0.51	0.43	0.92
0.08	0.48	0.39	0.89
0.10	0.44	0.34	0.85

Table 2. Full-space versus symmetric-subspace dimensions and depth surrogates.

Qubits n	**Full dimension 2^n**	**Symmetric dimension $n + 1$**	**Depth surrogate (full)**	**Depth surrogate (sym)**

2	4	3	12	10
3	8	4	18	14
4	16	5	26	18
5	32	6	35	22
6	64	7	45	26

Ancilla measurement reproduces the target fuzzy aggregation laws in the selected tensor encoding.

Projection onto the symmetric sector lowers the effective state-space dimension and improves robustness to depolarizing noise (Tables1 and 2).

The average fuzzy margin was defined as

$$M = \frac{1}{N_s} \sum_q \left(\gamma_{r^*(q)}\left(\varrho_q\right) - \max_{s \neq r^*(q)} \gamma_s\left(\varrho_q\right) \right)$$

where $r^*(q)$ is the correct class. For $p \leq 0.08$, the symmetric projected model improved M by about 611%.

DISCUSSION AND CONCLUSION

The paper established a tensor-operator framework for quantum-fuzzy conjunction and disjunction, together with a symmetry-preserving state-discrimination model. Encoding, aggregation, projection, and discrimination are separable layers, each analyzable independently and then recombined through tensor products. That modular view makes it easier to compare product $\mathbf{t}$-norm semantics, polynomial approximations of other $\mathbf{t}$-norms, and fuzzy-classification overlays for noisy quantum data.

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