

# Analyzing the Electromagnetic Transients and Corona Performance of Long Overhead Lines with Multiple Tower Spans

Mohamed Mostafa Saied\*

## Abstract

*This paper addresses the simulation of the electromagnetic transients developed in an important class of non-uniform high voltage power lines. The resulting information is of paramount importance for the proper planning, design, operation, and protection of electric power networks. It deals particularly with long overhead high voltage transmission lines composed of several tower spans which are connected in cascade. The presented approach considers eventually existing localized corona discharges at some points along the line. The derived mathematical model leads to a system composed of two simultaneous differentials as well as several algebraic equations which can be solved numerically in terms of parametric functions using the software Mathematica using the statement ParametricNDSolve. The location, the Laplace operator, and the loading conditions of the lines will all affect the resulting current and voltage distributions along the line. To obtain the appropriate findings in the time domain using the well-known Hosono algorithm, the paper offers a code for the numerical Laplace inversion of these formulas. This code has already been tested and validated in several studies that are available in literature. By contrasting the outcomes of the case studies that are supplied with those that are accessible elsewhere, the model and computer code are verified. It is believed that this study will be helpful to the high voltage and protection engineers in addition to power engineering students and trainees.*

**Keywords:** Electromagnetic, transients, simulation, high voltage lines, non-uniform, Mathematica, towers, span, corona, differential equations, parametric functions, numerical Laplace inversion, Hosono algorithm, parameter study

## INTRODUCTION

The analyses, mitigation, and the protection against the transient stresses developed in high voltage power networks have attracted the researchers' interest especially in recent years [1–13]. A large

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percentage of the resulting investigations focuses on issues related to the overhead transmission lines and/or underground cables [6–13]. The mostly numerical solution approaches can be broadly classified as time-domain and Laplace  $s$ -domain based techniques [4–8]. Nonlinear situations, such as those including corona discharges and magnetic saturation, require time-domain solutions, [6, 7, 9–13]. In some cases, special consideration is devoted to the location-dependence of the transmission lines' circuit parameters [13]. About uniform lines, they can be approximated as independent, which is the mainstream or traditional situation. Practical and

more realistic nonuniform lines appear in several interesting cases such as the river crossings and high voltage towers. As far as the sources initiating the transients are concerned, they can be of the usual standard time wave forms (DC, sinusoidal, double-exponential surges, etc.) or of non-standard wave shapes such as multi-pulse lightning discharge. To the best of the author's knowledge, no detailed information is available on the combined effect of the simultaneous presence of non-uniform overhead lines overhead lines (e.g., in cases involving cascade-connected multi-tower spans) and the eventual presence of corona discharge. This paper is a contribution to this interesting area of research.

### THE METHOD OF ANALYSIS

This study examines how the wave form of the starting stimuli and the sag of the conductors combine to affect electromagnetic transients in high voltage lines made up of numerous supporting towers. Typically, time-domain models are used for their simulation. To begin, a set of simultaneous partial differential and algebraic equations regulating the voltage and current distributions in terms of time and position are derived. It is important that they consider the pertinent beginning and boundary conditions. They provide a numerical solution with an effective Mathematica code.

It is well-known that the Laplace- or  $s$ -domain techniques can be efficiently used when dealing with the mainstream linear situations, otherwise, time-domain solutions are needed. The location-dependence of the transmission lines' circuit characteristics is given particular attention. Nonuniform lines appear in several interesting cases such as the river crossings, transformer bushings and overhead lines with multiple high voltage towers. As far as the sources initiating the transients are concerned, they are mostly of the usual standard time wave forms (DC, sinusoidal, double-exponential surges, etc.). Nevertheless, sources with non-standard wave forms such as multi-pulse lightning discharges and chopped waves can also be encountered.

To the best of the author's knowledge, no detailed information is currently available in the literature on the combined presence of lines' nonuniformity, such as those including cascaded multi-tower spans, as well as presence of corona.

### METHOD OF ANALYSIS

The surge impedance,  $z_o$ , will fluctuate with  $x$  in the manner roughly depicted in Figure 1 since the line inductance  $l$  and capacitance  $c$  per unit length of the non-uniform line are both assumed functions of location  $x$ . The plot shows  $z_o(x)$  for a sample 380-kV overhead wire across the range of  $x = 0$  to  $x = 3000$  m. The plot covers 10 span lengths of 300-m length, each. By carefully choosing the two constants  $a$  and  $b$ , the following expression can be obtained by trial and error if the number of span lengths taken into consideration is  $N = 10$ :

$$z_o(x) = a \left( 2 + b \cos \left[ \frac{\pi x}{400} \right]^4 \right) \text{ ohms} \quad (1)$$

which has 10 span lengths plotted. It is thought to fluctuate between the lowest value of 290 ohm and the maximum value of 417.6 ohm. The following relationships can be inferred from the established relationships between the speed of light, the surge impedance  $z_o$ , and the line parameters per meter  $l$  and  $c$ :

$$c(x) = \frac{1}{\text{speed} \cdot z_o(x)} \quad (2)$$

$$l(x) = \frac{z_o(x)}{\text{speed}} \quad (3)$$

The investigation starts with the assumption of the following equation for the location-dependence of the line's surge impedance  $z_o(x)$  as a function of the location  $x$ , measured from the line's source terminal for an overhead line of span length  $\text{span}$  between two consecutive towers:

$$z_o(x) = 145 \left( 2 + 0.88 \left[ \cos \left( \frac{\pi x}{span} \right) \right]^4 \right) \quad (4)$$

It is illustrated by the plot in Figure 1 describing a 3-km overhead line having 10 equidistant towers and a span length of 300 m. In other words, the towers are located at distances  $x = 300$  m, 600 m, 900 m, etc. The sag of the surge impedance is assumed to be 25%. Accordingly, the surge impedance  $z_o(x)$  exhibits its highest value (456 ohm) at the tower locations and its lowest value (274 ohm) at the midpoints between each two neighboring towers. Its average value is seen to be close to 365 ohm.

The line's transient response is governed by the following simultaneous differential equations:

$$\frac{\partial v}{\partial x} = -l(x) \frac{\partial i}{\partial t} - i \cdot r \quad (5)$$

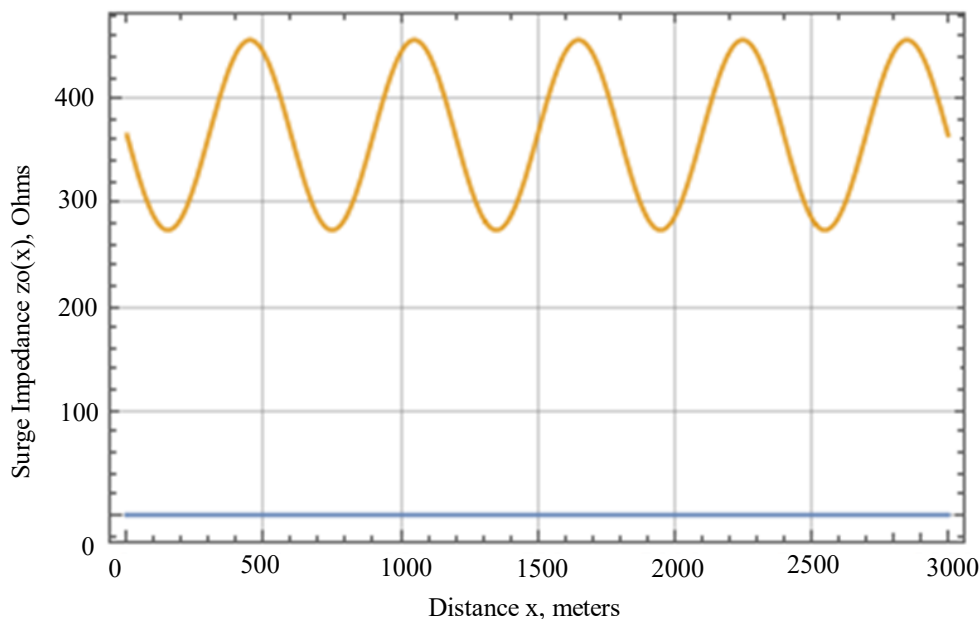
and

$$\frac{\partial i}{\partial x} = -C(x) \frac{\partial v}{\partial t} \quad (6)$$

where  $r$  denotes the line's series resistance per unit length which is assumed 0.1 mohm/m. Their solution should consider the boundary conditions expressed in terms of the assumed source voltage and the line's receiving end termination impedance expressed in the  $s$ -domain. For instance, this impedance can assume the values zero, infinity,  $s \cdot L$ ,

$1/(sC)$  for the short-circuit, open-circuit, inductive termination by the inductance  $L$  or capacitive termination by a capacitance  $C$ .

The above system of equations can be solved for the  $s$ -domain expressions of the voltage and current using the statement ParametricNDSolve of the software Mathematica (Versions 11.3 and 12.2) [14]. Hosono's algorithm, which was previously discussed, can then be used to obtain the temporal response for the numerical Laplace inversion [15].



The source  $x = 0$ , the load  $x = length$

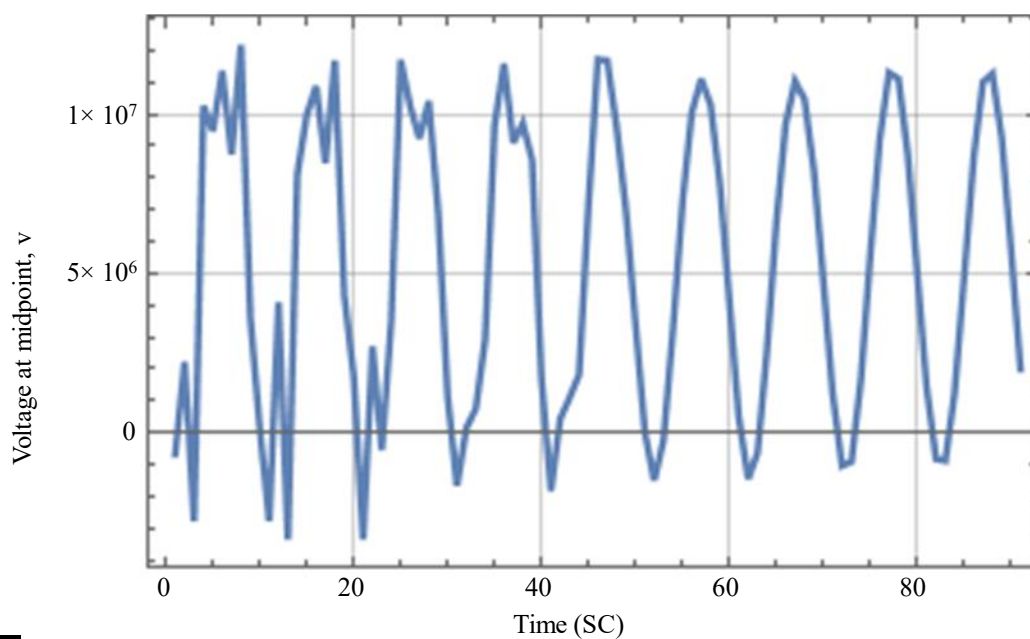
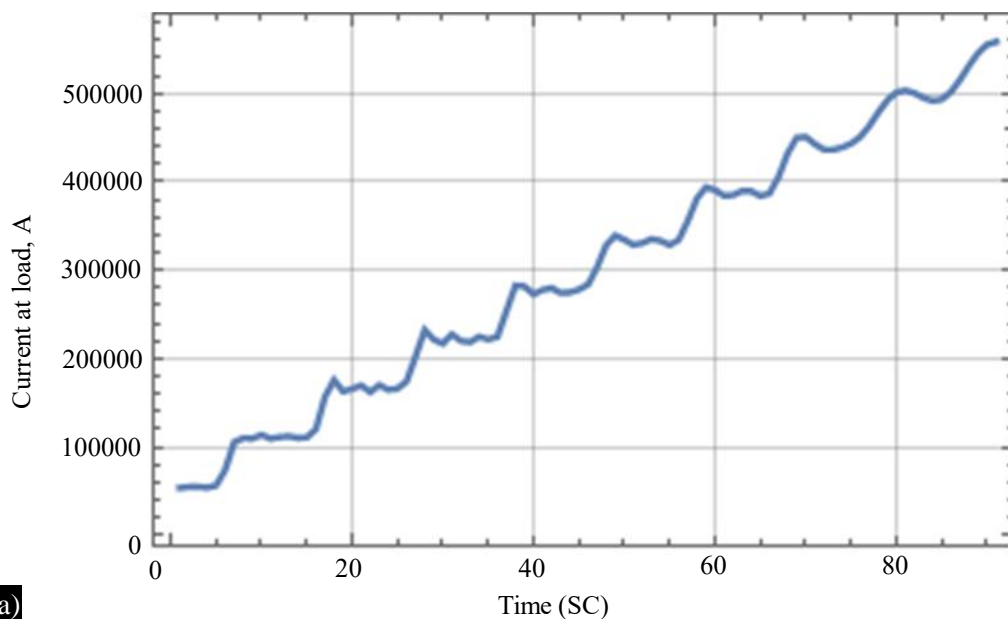
**Figure 1.** The line's surge impedance  $z_o(x)$  as a function of the location  $x$ .

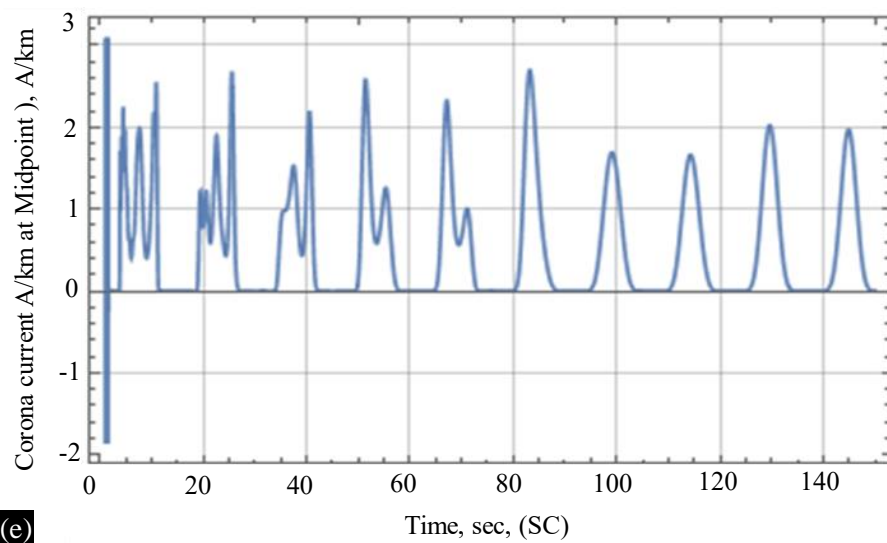
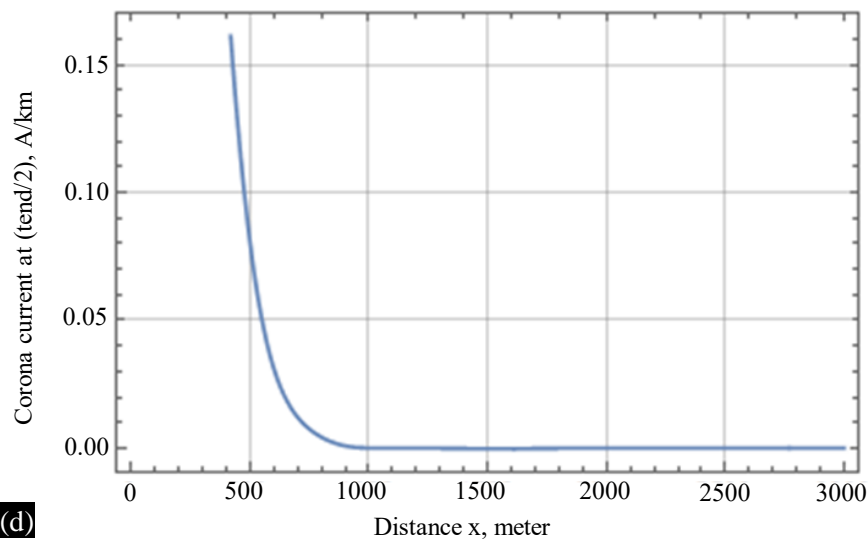
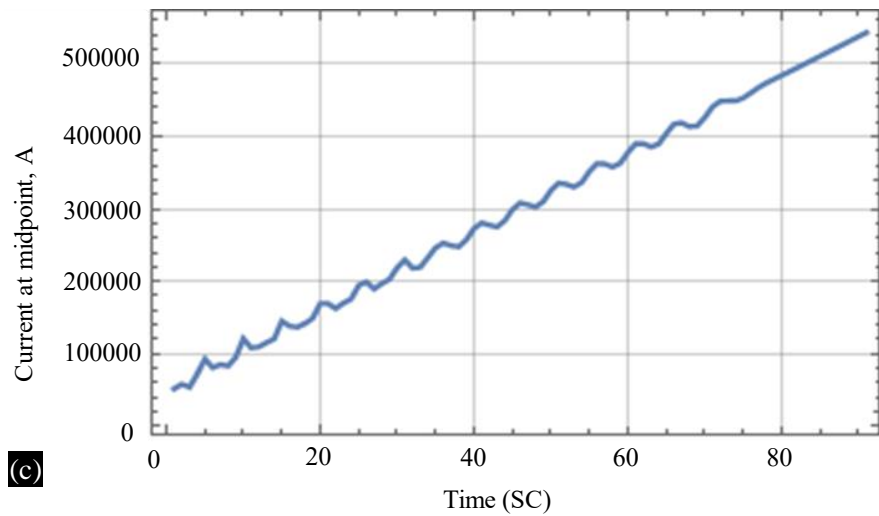
## SAMPLE RESULTS

As an example, this section presents the line's transient response if its receiving end is short-circuited. current at load shown in Figure 2(a). The source voltage is assumed to be a 10 MV step function. As a first approximation, the relatively very small corona currents can be temporarily ignored. The voltage at midpoint shown in Figure 2(b) and the current at midpoint shown in Figure 2(c).

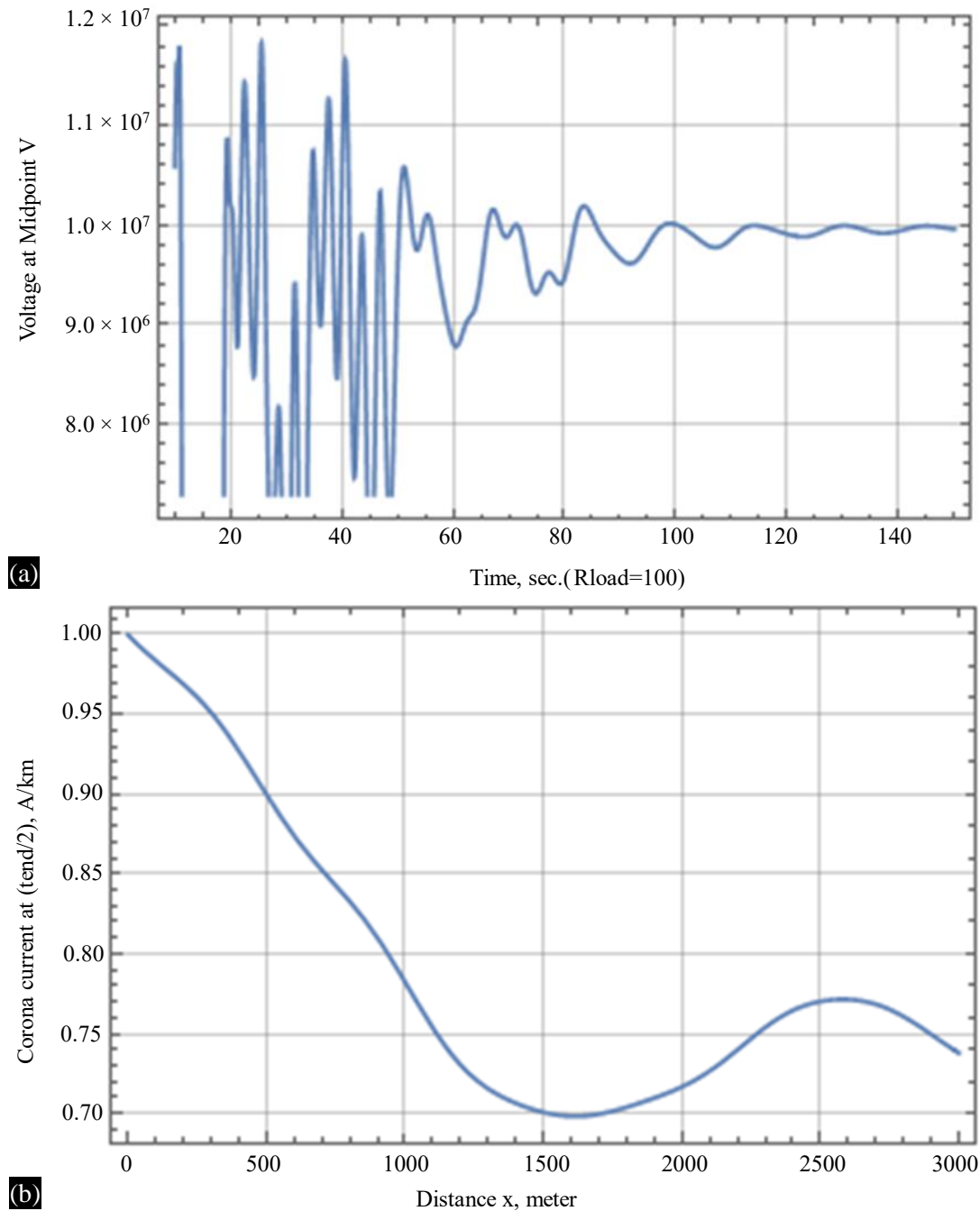
After getting the voltage distribution along the line, the corona current at any point  $x$  can be obtained using the following empirical equation expressing the corona current in terms of the instantaneous local voltage at that point [12]: The corona current/km as a function of the location  $x$  at  $t = 100 \mu\text{s}$  is shown in Figure 2(d) and the corona current/km as a function of the time  $t$  at the location  $x = 1500 \text{ m}$ ,  $1 \text{ div} = 1.333 \mu\text{s}$  is shown in Figure 2(a-e).

$$i_c(t) = (10^{-35}) \cdot v(t)^5 \quad (7)$$





**Figure 2.** The voltage and current transients in the short-circuit case due to a 10 MV step voltage source. The time range is 200  $\mu$ s. (a) Current at load. (b) Voltage at midpoint. (c) Current at midpoint. (d): The corona current/km as a function of the location  $x$  at  $t = 100 \mu$ s. (e) The corona current/km as a function of the time  $t$  at the location  $x = 1500$  m, 1 div = 1.333  $\mu$ s.



**Figure 3.** The result of a 100 ohm line termination. (a) Midpoint voltage versus time. (b) Corona current at  $t = 100$  as a function of location  $x$ .

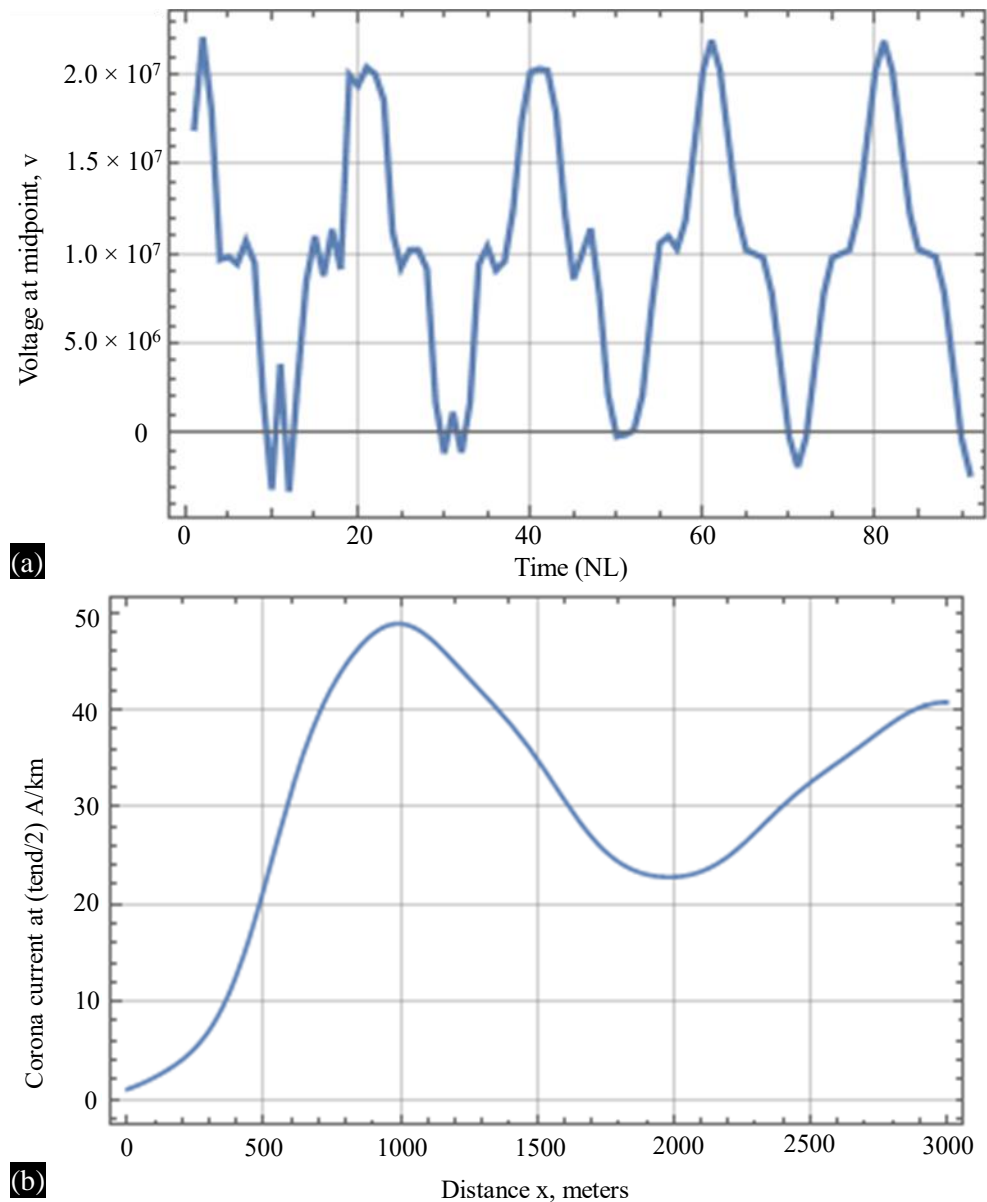
The plot illustrated in Figure 2(a) depicts the wave form of the transient current at the load (i.e.,  $x = 3000$  m).

The initial source current (22 kA) agrees with the value determined by the source step voltage of 10 MV and the line's surge impedance at the first tower (456 ohm). The associated traveling wave needs  $10 \mu\text{s}$  in order to reach the load terminal as described by Figure 2(a). In the steady state, the total loop resistance is 0.3 ohm and the final value of the current is expected to be around 33.3 MA. The traveling wave reaches the line's middle point ( $x = 150$  m) after  $0.5 \mu\text{s}$  as shown in the voltage Figure 2(c). Similar observations can be made for the Figure 2(b) of the voltage at the line's midpoint.

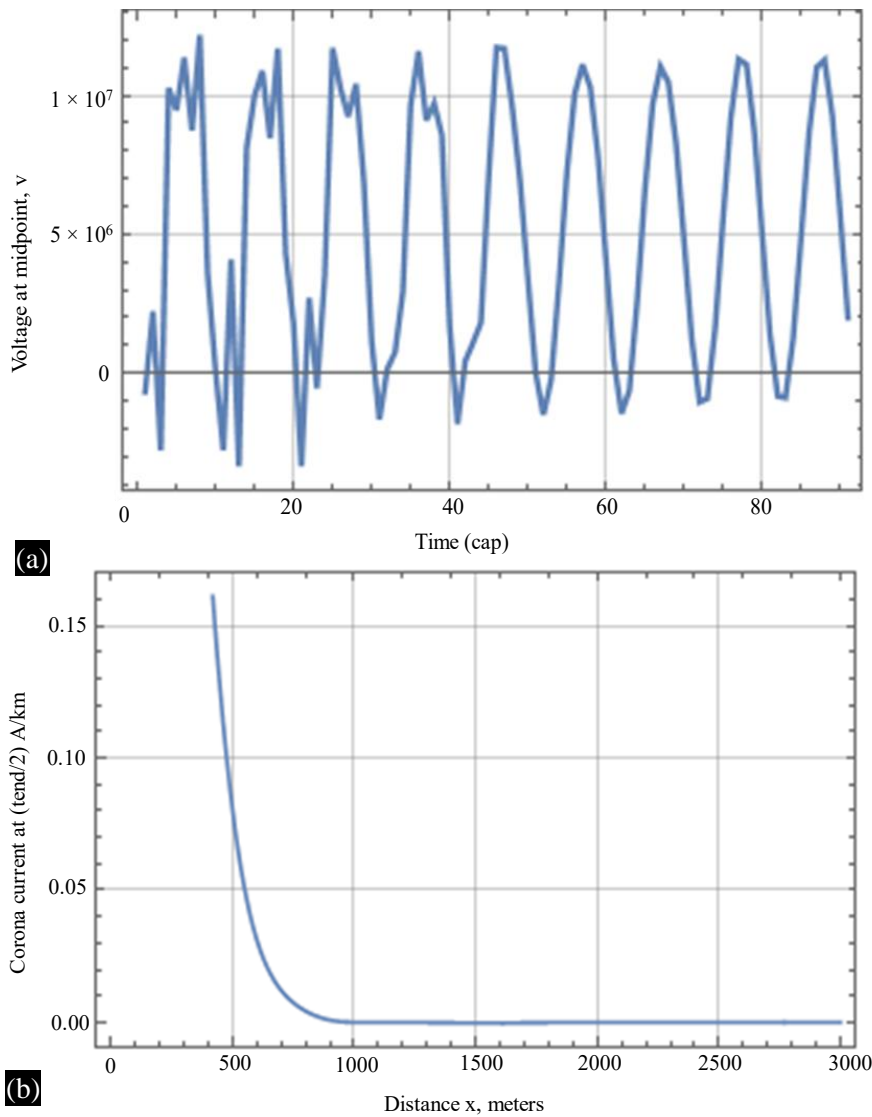
A snapshot of the corona current per kilometer at the time point  $t = 100 \mu\text{s}$ , which can be obtained by applying Equation (6), is depicted in Figure 2(d). For completeness, the plots in Figures 3, 4, and 5 show the results pertinent to the following cases:

Figure 3 depicts 100 ohm pure resistive termination, Figure 4 depicts open-circuited line, and Figure 5 shows 1 farad pure capacitive termination. The plots show that the magnitude of the current corona per unit length lies in the range of several amperes per kilometer.

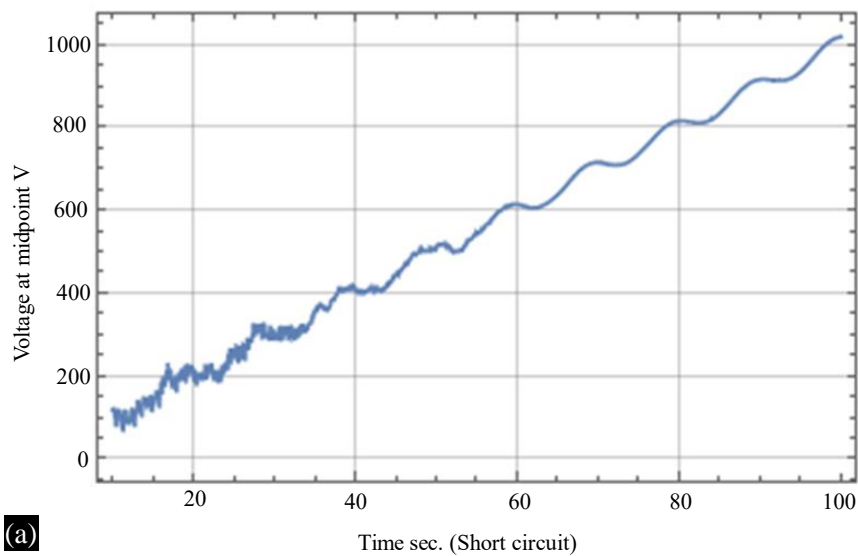
It should be noted that these results are sensitive to changes in the atmospheric conditions (temperature and pressure) as well as to the conductors' surface conditions such as their roughness. The impact of the shape of the source voltage as a function of time is illustrated in Figures 6(a) (b) depicting both the time response of the midpoint voltage and the corresponding shunt corona current as a function of location  $x$ , respectively. The relatively small values of the corona current are due to the fact that the transient conductor voltage is below the corona onset voltage along most of the line length.

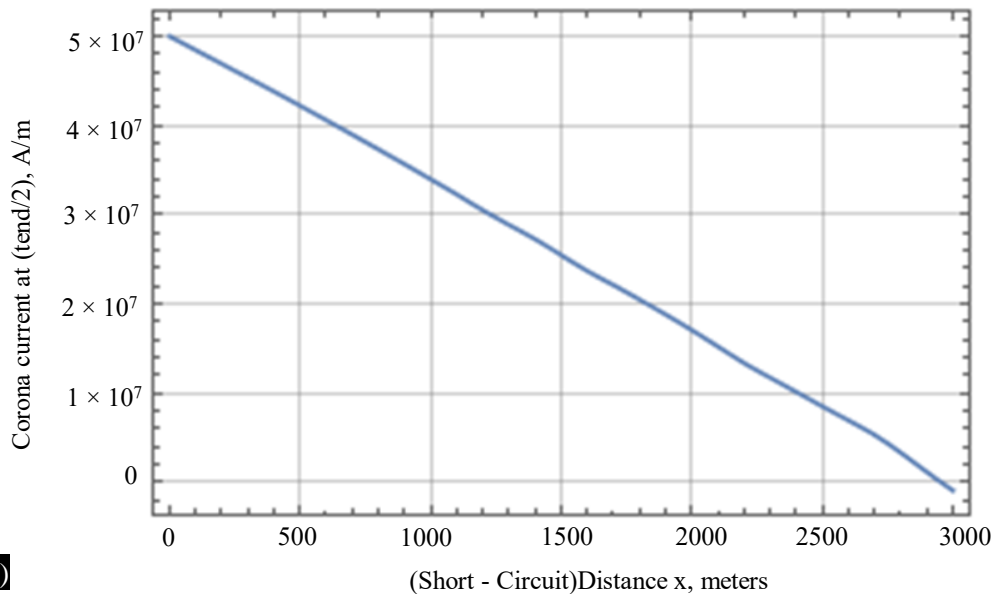


**Figure 4.** The result of an open-circuited line. (a) Midpoint voltage versus time. (b) Corona current at  $t = 100 \mu\text{s}$  as function of location  $x$ .



**Figure 5.** The result of a 1 farad line termination. (a) Midpoint voltage versus time. (b) Corona current at  $t = 100 \mu\text{s}$  as function of location  $x$ .





**(b)** **Figure 6.** Results for 10 MV/s ramp source voltage  $e(t)$ . (a) Voltage at midpoint versus time. (b) Corona current (A/m) versus location.

## CONCLUSIONS

This study deals with the characteristics of long high voltage overhead transmission lines composed of multiple equidistant tower spans. Any number of towers can be dealt with. The presented model can take the time wave forms of the sources initiating the transients, the lines' data and circuit parameters as well as their loading conditions into consideration. The paper presents a detailed analysis and a computer program for investigating the electromagnetic transients and the eventually existing corona current. The derived system of the governing simultaneous differential and algebraic equations is solved numerically using a Mathematica computer code in terms of parametric equations and functions in the Laplace  $s$ -domain. The voltage and current distributions'  $s$ -domain expressions are produced by the program. The corresponding time domain results are obtained through the numerical Laplace inversion using the Hosono algorithm.

The paper discusses the results of several case studies involving sources of step- and ramp-form voltage sources,  $e(t)$ , as well as the pure resistive, pure capacitive, open-circuit as well as short-circuit line terminations. The results indicate, among other facts, that the shunt corona currents, if any, can assume values of several amperes per kilometer.

## REFERENCES

1. Grainger J, Stevenson W. Power System Analysis. International edition. New York, NY, USA: McGraw-Hill; 1994. Chapter 6 and Appendix A.
2. Bewley LV. Travelling waves on transmission systems. New York, NY, USA: Dover; 1963.
3. Greenwood A. Electrical Transients in Power Systems. Hoboken, NJ, USA: John Wiley & Sons; 1991.
4. Dommel HW. Electromagnetic Transients Program Manual (EMTP Theory Handbook). Portland, OR, USA: Bonneville Power Administration; 1986.
5. van der Sluis L. Transients in Power Systems. Hoboken, NJ, USA: John Wiley & Sons; 2001.
6. Begamudre RD. Extra-High Voltage AC Transmission Engineering. 2nd edition. New Delhi, India: Wiley-Eastern Limited; 1986.
7. Ovick NL. Travelling Waves on Transmission Lines Including Corona Effects. PhD Thesis. Pittsburgh, PA, USA: University of Pittsburgh; 1982.
8. Correa J. EMTP Theory Book System Engineering. Portland, OR, USA: Bonneville Power Administration; 1983.

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9. Saied M, Safar Y, Salama M. Line transients with corona. *J Univ Kuwait Sci.* 1987; 14 (1): 77–95.
  10. Saied MM, Safar Y. Electromagnetic transients on compensated lines under corona. *Electric Mach Power Syst.* 1989; 16 (6): 441–462.
  11. Khawaja AH, Huang Q. Estimating sag and wind-induced motion of overhead power lines with current and magnetic-flux density measurements. *IEEE Trans Instrum Meas.* 2017 May;66(5):897–909. DOI: 10.1109/TIM.2017.2676140.
  12. Saied MM. Corona modeling for the transient analysis, steady-state and corona loss performance of transmission lines. *Electric Power Qual Util J.* 2013; XVI (2): 11–20.
  13. Saied MM. A contribution to the frequency analysis and transient response of power transformer windings. *Electr Power Compon Syst.* 2014;42(11):1143–51. DOI: 10.1080/15325008.2014.921952.
  14. Wolfram Research. *Wolfram Mathematica: Tutorial Collection. Advanced Numerical Differential Equation Solving.* Champaign, IL, USA: Wolfram Research; 2008.
  15. Hosono T. Numerical inversion of Laplace transform and some applications to wave optics. *Radio Sci.* 1981; 16 (6): 1015–1019.