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Mathematical Modelling on Unsteady Blood Flow and Unsteady Heat Transfer Through a Stenosed Artery with Hematocrit and Slip Velocity

A. Jimoh[1], E.O. Senewo[2,*], Y.N. Yahaya[3], K. Ibrahim[4], S. Yunisa[5], S.M. Ameh[6], O.O. Rasaq[7], D.I. Ajiola[8]

Abstract

A theoretical investigation concerning variable viscosity dependent on percentage concentration of red blood cell interm of total volume of blood (hematocrit) and slip velocity on unsteady flow of blood and heat transfer has been carried out. The analytical expressions for the flow characteristics and the heat transfer rate, including the velocity profile, volumetric flow rate, flow resistance, wall shear stress, and temperature profile, were obtained using the weighted residual method, specifically the Galerkin method and the fourth-order Runge-Kutta method. The results are shown graphically and it was revealed from the graphs that, hematocrit parameter increases with flow resistance, shear stress and heat transfer rate but reduces the flow velocity and volumetric flow rate. It was also revealed that volumetric flow rate, shear stress and flow velocity are positively influence by higher values of the slip velocity while resistance to flow and heat transfer rate decreases as the slip velocity reduces. Higher values of magnetic field parameter offer more resistance to the flow but reduces the flow velocity, volumetric flow rate and shear stress. Finally, time positively influence both the flow velocity and heat transfer rate. It was interestingly observed that, hematocrit parameter shown more noticeable effects on all the flow characteristics and the heat transfer rate compared to slip velocity and magnetic field parameter.

Keywords: Unsteady blood flow, unsteady heat transfer, slip velocity, hematocrit, magnetic, Third grade fluid models.

INTRODUCTION

***Author for Correspondence**
E.O. Senewo
E-mail: eajoge@oauife.edu.ng

[1]Associate Professor, Department of Mathematics and Statistics, Confluence University of Science and Technology, Osara, Kogi State, Nigeria.
[2–7]Research Scholar, Department of Mathematics and Statistics, Confluence University of Science and Technology, Osara, Kogi State, Nigeria.
[8]Research Scholar, Department of Applied Physical Science (Materials and Coatings Science Option), Georgia Southern University, USA.

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Scientists and researchers have recently received great interest in the study of blood flow and its diseases. The interest is due to the fact that, the important substances, nutrients and oxygen are transported by blood to all parts of the body and this in turns helps to regulate the human body temperature. During the transportation process, the arteries play a very significant role and as such the study of diseases that affect the arteries becomes very imperative and it has received great attention in recent times. Atheroclerosis, known as stenosis, is one of the diseases that affect the arteries and also affect the process of blood flow. When there is fatty substances deposition in the lumen of the arteries, it can in effect lead to significant decrease in the blood flow to the corresponding body organs which can result to disturbance in the blood circulation. These interesting areas of research had captivated the attentions of many researchers and scientists to find solution for the problems of blood flow in arteries that contain stenosis using different methods for different mathematical models of blood flow such as; Akhtar et al [1] considered mechanics of non-Newtonian blood flow in an artery having multiple stenosis and electro-osmotic effect. Hasen and Abdullahi [2] investigated and obtained an analytical solution to the effect of soret and Dufour on the electro-osmotic peristaltic flow of Rabinowitsch fluid. Non-Newtonian blood flow through a stenosed artery in the presence of magnetic field was investigated by Haleh et al [3]. In their investigation they used magnetic field after the stenosis region to destroy the vortex created. Varshney et al [4] numerically studied the incompressible non-Newtonian, fully developed and laminar blood flow in an artery with multiple stenosis under the action of traversed magnetic field. At the same time, the study of blood flow through a stenosed artery having slip velocity at the arterial wall had received the attentions of some researchers (Omamoke and Amos [5], Arun [6], Geeta and Siddique [7], Ghasemi et al [8], Amos et al [9]). Furthermore, the viscosity of blood is not constant in a true physiological system but rather changes with either hamatocrit or temperature and pressure. This has also received the attention of some researchers like Veena and Arundhati [10] used Frobenius method to obtain solutions to effects of externally applied traversed magnetic fields on blood flow in tapered stenosed artery with variable viscosity. Sanjeev and Chandrashekhar [11] studied the hematoicrit effects of the asymmetric blood flow through an artery with stenosis arteries. They solved numerically the unsteady non-linear. Navier-Stokes equations that assume laminar flow conditions and govern flow. The following are some of the researchers that investigated variable viscosity in their investigations; Akram et al [12], Abo-Elkhair [13], Priyadharshini and Ponalgusamy [14], Chitra and Karthikeyan [15].

Heat transfer from an area of higher concentration to one of lower concentration occurs once fluid movement occurs, however the pace of heat transfer is not taken into account by any of the aforementioned researchers in their research. Accordingly, Jimoh et al. [16] looked into the effects of hematocrit and slip velocity on third-grade blood flow and heat transmission via stenosed arteries. According to the findings of their study, slip velocity has a negative impact on heat transfer rate and flow resistance while having a good impact on flow velocity, flow rate, and shear stress. Their findings also show that as the hematocrit parameter rises, flow velocity and flow rate decrease while flow resistance, heat transfer rate, and shear stress increase. Jimoh et al [17] used Galerkin and Newton Raphson methods to obtain solution to a third-grade blood flow and heat transfer models with the consideration of magnetic field and slip velocity. Some of the other researchers that considered heat transfer in their studies include Abubakar and Adeoye et al [18], Saleem et al [19], Zaman et al [20], Misra and Sharma [21], Ramesh [22], Misra and Adhikary [23] to mention but a few. The combined effects of hematocrit, magnetic field, and slip velocity remain outstanding in literature.

This paper therefore is concerned with investigating the effects of hematocrit, magnetic field, and slip velocity on unsteady blood flow and heat transfer through a stenosed artery.

MATHEMATICAL MODELS

By modifying the work of Mohammed [24], the momentum equation for the unsteady fluid flow and the energy equation for the unsteady heat transfer with variable viscosity dependence on red blood cell concentration (Hematocrit) using a third-grade fluid model can be obtained respectively as

$$\frac{\partial w}{\partial t}=\frac{\mu(r)}{\rho}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2}+\frac{2\beta_3}{r\rho}\left(\frac{\partial w}{\partial r}\right)^2+\frac{\alpha_1}{r\rho}\frac{\partial^2 w}{\partial r\partial t}+\frac{\alpha_1}{\rho}\frac{\partial^3 w}{\partial r^2\partial t}-\frac{\partial \hat{P}}{\rho\partial z}-\frac{\sigma\beta_0^2 w}{\rho} \tag{1}$$

and

$$\frac{\partial T}{\partial t}=\frac{\mu(r)}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^2+\frac{\alpha_1}{\rho c_p}\left(\frac{\partial^2 w}{\partial r\partial t}\right)\frac{\partial w}{\partial r}+\frac{2\beta_3}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^4+\frac{K}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{2}$$

According to Einstein formula, the variable viscosity $\mu(r)$ of blood is taken to be

$$\mu(r) = \mu_0\big(1 + \beta h(r)\big) \tag{3}$$

and the hematocrit h(r) is described by Lih [25]

$$h(r) = H\left(1 - \left(\frac{r}{R_0}\right)^m\right),\ \mathrm{m} \geq 2 \tag{4}$$

The first term in the RHS of (1) can be re-written as

$$\frac{\mu(r)}{\rho}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) = \frac{\mu(r)}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) \tag{5}$$

putting (3), (4) and (5) into (1) and simplifying, we obtain

$$\frac{\partial w}{\partial t} = \frac{\mu_0}{\rho}\left[1 + N_1\left(1 - \left(\frac{r}{R_0}\right)^m\right)\right]\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2\left(\frac{\partial^2 w}{\partial r^2}\right) + \frac{2\beta_3}{r\rho}\left(\frac{\partial w}{\partial r}\right)^3 + \frac{\propto_1}{r\rho}\frac{\partial^2 w}{\partial r\partial t} \tag{6}$$

$$+ \frac{\propto_1}{\rho}\frac{\partial^3 w}{\partial r^2\partial t} - \frac{1}{\rho}\frac{\partial \hat{P}}{\partial Z} - \sigma\frac{\beta_0^2 w}{\rho}$$

As a result of the employed velocity slip at the constricted artery as shown in Figure 1 below, the associated slip conditions to (6) are

$$\begin{cases} w = w_s\ at\ r = R(z) \\ \frac{\partial w}{\partial r} = 0\ at\ r = 0 \end{cases} \tag{7}$$

Similarly, the last term in the RHS of (2) can be re-written as:

$$\frac{K}{\rho C_\rho}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) = \frac{K}{r\rho C_\rho}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{8}$$

Substituting (3), (4) and (8) into (2) and after simplifying, we obtain

$$\frac{\partial T}{\partial t} = \frac{\mu_0}{\rho C_\rho}\left[1 + N\left(1 - \left(\frac{r}{\mathrm{R}_0}\right)^m\right)\right].\left(\frac{\partial \omega}{\partial r}\right)^2 + \frac{2\beta_3}{\rho C_\rho}\left(\frac{\partial \omega}{\partial r}\right)^4 + \frac{\alpha_1}{\rho C_\rho}\left(\frac{\partial^2 \omega}{\partial r\partial t}\right)\left(\frac{\partial \omega}{\partial r}\right) \tag{9}$$

$$+ \frac{K}{\rho C_\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

The associated slip conditions to (9) are:

$$\begin{cases} T = T_w \quad at\ r = R(z) \\ \frac{\partial T}{\partial r} = 0\ at\ r = 0 \end{cases} \tag{10}$$

In order to dimensionalize the equations (6), (7), (9) and (10), we introduced the following parameters and variables.

$$\begin{cases} \overline{w} = \frac{w}{d/t_0}, y = r/R_0 \\ \bar{t} = \frac{t}{t_0},\ V_0 = \frac{w_s t_0}{d} \\ \bar{\theta} = \frac{T - T_w}{T_m - T_w} \end{cases} \tag{11}$$

Substituting (11) into (6) and after simplifying, we obtain

$$\frac{\partial \overline{w}}{\partial \bar{\mathrm{t}}} = \frac{1}{RE_{1N}}[1 + N_1(1 - y^m)].\frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\partial \overline{w}}{\partial y}\right) + \Omega_N\left(6\left(\frac{\partial \overline{w}}{\partial y}\right)^2\left(\frac{\partial^2 \overline{w}}{\partial y^2}\right) + \frac{2}{y}\left(\frac{\partial \overline{w}}{\partial y}\right)^3\right) \tag{12}$$

$$+ \Omega_{1N}\left(\frac{1}{y}\frac{\partial^2 \overline{w}}{\partial y\partial \bar{\mathrm{t}}} + \frac{\partial^3 \overline{w}}{\partial y^2\partial \bar{\mathrm{t}}}\right) + G_{1N} - M_{1N}\overline{w}$$

Where

$$\begin{cases} RE_{1N} = \frac{R_0^2}{t_0\mu_0}, \quad \Omega_N = \frac{\beta_3 d^2}{t_0 \rho R_0^4}, \quad \Omega_{1N} = \frac{\propto_1}{\rho R_0^2} \\ G_{1N} = -\frac{t_0^2}{d^2\rho}\frac{\partial \hat{P}}{\partial Z} \text{ and } M_{1N} = \frac{t_0 \sigma \beta_0^2}{\rho} \end{cases} \tag{13}$$

and the corresponding dimensionless slip conditions to (12) can be simplified as

$$\begin{cases} \bar{w} = V_{01N} \text{ at } y = \frac{R(z)}{R_0} = R_b \\ \frac{\partial \bar{w}}{\partial y} = 0 \quad \text{at} \quad y = 0 \end{cases} \tag{14}$$

Similarly, substituting (11) into (9) and after simplifying, we obtain

$$\frac{\partial \bar{\theta}}{\partial \bar{t}} = E_{n1N}[1 + N_1(1 - y^{\,m})]\left(\frac{\partial \bar{\omega}}{\partial y}\right)^2 + \phi_N\left(\frac{\partial \bar{\omega}}{\partial y}\right)^4 + \phi_{1N}\left(\frac{\partial^2 \bar{\omega}}{\partial y \partial \bar{t}}\right)\left(\frac{\partial \bar{\omega}}{\partial y}\right)$$

$$+ \Lambda_{1N}\frac{1}{y}.\frac{\partial}{\partial y}\left(y\frac{\partial \bar{\theta}}{\partial y}\right) \tag{15}$$

where,

$$\begin{cases} E_{n1N} = \frac{Vd^2}{t_0(T_m - T_w)R_0^2 C_\rho}, \quad \Lambda_{1N} = \frac{Kto}{R_0^2 P C_\rho} \\ \phi_N = \frac{2\beta_3 d^4}{t_0^3 R_0^3 (T_m - T_w) R_0^4 \rho C_\rho}, \quad \phi_{1N} = \frac{\alpha_1 d^2}{t_0^3 (T_m - T_w) R_0^2 \rho C_\rho} \end{cases} \tag{16}$$

and the associated slip conditions to (15) can be simplified as

$$\begin{cases} \bar{\theta} = 0 \quad \text{at} \quad y = R_b \\ \frac{\partial \bar{\theta}}{\partial y} = 0 \quad \text{at} \quad y = 0 \end{cases} \tag{17}$$

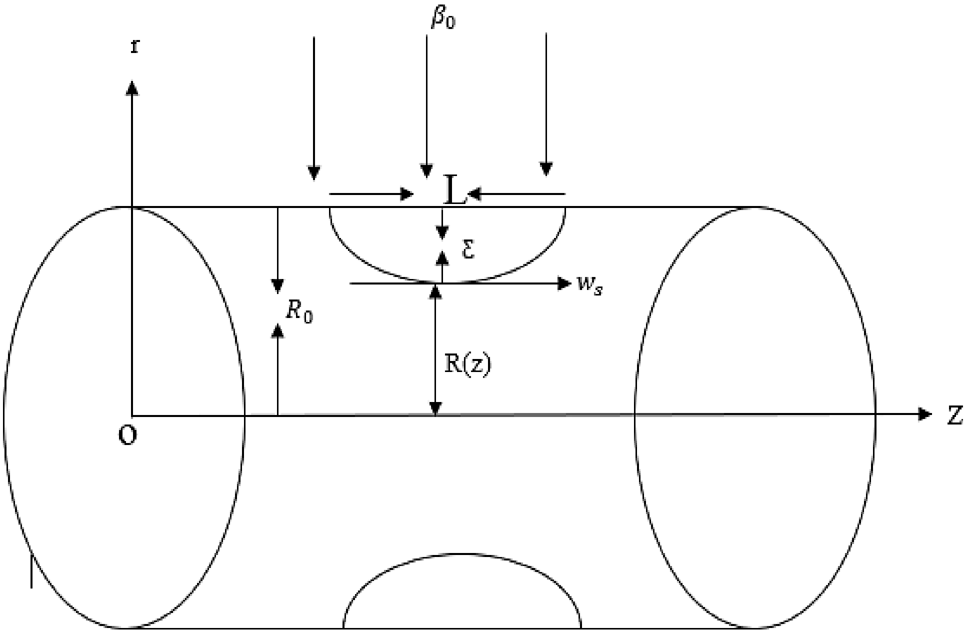

Figure 1. Geometry of the stenosis.

The stenosis has been described by Young [26] and Biswas [27]

$$\begin{cases} \frac{R(z)}{R_0} = 1 - \frac{Ɛ}{2R_0}\left[1 + \frac{cos\pi z}{L}\right], \text{for} |z| < \text{L} \\ R_0, \quad \text{for } |z| > \text{L} \end{cases} \tag{18}$$

SOLUTION

In order to obtain the analytical expression for the velocity profile to (12) using Galerkin's Weighted Residual method, we assume a solution of the form

$$\overline{w}(y,t) = a_0(t) + a_1(t)y + a_2(t)y^2 \tag{19}$$

Subjecting (19) to the slip conditions (14) and after simplifying we obtain

$$\overline{w}(y,\ t) = \frac{V_{01N}y^2}{Rb^2} + a_0(t)\left(1-\frac{y^2}{Rb^2}\right) + a_2(t)y^2\left(1-\frac{y^2}{Rb^2}\right) \tag{20}$$

$$\text{Let } \bar{r} = \frac{y}{R_b} \tag{21}$$

Invoking (21) in (20) and simplifying, we obtain

$$\overline{w}(\bar{r},t\,) = V_{01N}\bar{r}^2 + a_0(t)(1-\bar{r}^2) + a_2(t)\bar{r}^2(1-\bar{r}^2) \tag{22}$$

By dropping the bar for convenience sake, (22) can be written as

$$w(r,\ t) = V_{01N}r^2 + a_0(t)(1-r^2) + a_2(t)r^2(1-r^2) \tag{23}$$

From (23), we have the followings

$$\frac{\partial w}{\partial t} = \dot{a_0}(t)(1-r^2) + \dot{a_2}(t)r^2(1-r^2) \tag{24}$$

$$\frac{1}{r}\frac{\partial^2 w}{\partial r\partial t} = -2\dot{a_0}(t) + 2\dot{a_2}(t) - 4r^2\dot{a_2}(t) \tag{25}$$

$$\frac{\partial^3 w}{\partial r^2\partial t} = -2\dot{a_0}(t) + 2\dot{a_2}(t) - 12r^2\dot{a_2}(t) \tag{26}$$

$$\frac{\partial w}{\partial r} = 2V_{01N}r - 2a_0(t)r + 2a_2(t)r - 2a_2(t)r^3 - 2a_2r^3 \tag{27}$$

$$\frac{\partial^2 w}{\partial r^2} = 2V_{01N} - 2a_0(t) + 2a_2(t) - 12a_2(t)r^2 \tag{28}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = 4V_{01N} + 4a_2(t) - 4a_0(t) - 16a_2(t)r^2 \tag{29}$$

$$\left(\frac{\partial w}{\partial r}\right)^2 = 16a_2^2(t)r^6 - 16V_{01N}a_2(t)r^4 + 16a_0(t)a_2(t)r^4 - 16a_2^2(t)r^4 + 4V_{01N}^2r^2 - 8V_{01N}a_0(t)r^2 + 8V_{01N}a_2(t)r^2 + 4a_0^2(t)r^2 - 8a_0(t)a_2(t)r^2 + 4a_2^2(t)r^2 \tag{30}$$

$$\frac{2}{r}\left(\frac{\partial w}{\partial r}\right)^3 = -128a_2^3(t)r^8 + 192V_{01N}a_2^2(t)r^6 - 192a_0(t)a_2^2(t)r^6 + 192a_2^3(t)r^6 - 96V_{01N}^2a_2(t)r^4 + 192V_{01N}a_0(t)a_2(t)r^4 - 192V_{01N}a_2^2(t)r^4 - 96a_0^2(t)a_2(t)r^4 + 192a_0(t)a_2^2(t)r^4 - 96a_2^3V_{01N}^4 + 16V_{01N}^3r^2 - 48V_{01N}^2a_0(t)r^2 + 48V_{01N}^2a^2(t)r^4 + 48V_{01N}a_0^2(t)r^4 - 96V_{01N}a_0(t)a_2(t)r^2 + 4V_{01N}a_2^2(t)r^2 - 16a_0^3(t)r^2 + 48a_0^2(t)a_2(t)r^2 - 48a_0(t)a_2^2(t)r^2 + 16a_2^3(t)r^2 \tag{31}$$

$$6\left(\frac{\partial^2 w}{\partial r^2}\right)\left(\frac{\partial w}{\partial r}\right)^2 = 48V_{01N}^2r^2 - 144V_{01N}^2a_0(t)r^2 + 144V_{01N}^2a_2(t)r^2 - 480V_{01N}^2a_2(t)r_2 + 144V_{01N}a_0^2(t)r^2 - 40V_{01N}a_0(t)a_2(t)r^2 + 960V_{01N}a_0(t)a_2(t)r^4 + 144V_{01N}a_2^2(t)r^2 - 144V_{01N}a_2^2(t)r^4 + 1344V_{01N}a_2^2(t)r^6 + 144a_0^2(t)a_2(t)r^2 - 480a_0^2(t)a_2(t)r^4 - 144a_0(t)a_2^2(t)r^2 +$$

$960a_0(t)a_2^2(t)r^4 - \quad 1344a_0(t)a_2^2(t)r^6 - 480a_2^3(t)r^4 + 1344a_2^3(t)r^6 - 48a_2^3(t)r^2 + 1152a_2^3(t)r^8$ (32)

where**.** is the differentiation with respect to time t.
The residual for equation (12) can be written as

$$RR(a_0(t), a_2(t), r) = \quad \frac{\partial w}{\partial \mathrm{t}} - G_{1N} - \frac{1}{RE_{1N}}[1 + N_1(1 - r^m)].\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - \Omega_N\left(6\left(\frac{\partial w}{\partial r}\right)^2\left(\frac{\partial^2 w}{\partial r^2}\right) - \frac{2}{r}\left(\frac{\partial w}{\partial r}\right)^3\right) - \Omega_{1N}\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \mathrm{t}} + \frac{\partial^3 w}{\partial r^2 \partial \mathrm{t}}\right) + \ M_{1N}w \quad (33)$$

Taking the shape of the profile (m=2), and substituting (23), (24), (25), (26), (29), (31) and (32) into (33) and after simplifying, we obtain

$RR(a_0(t), a_2(t), r) = -64\Omega_N r^2 V_{01N}^3 + M_{1N} r^2 V_{01N}^3 + 1280a_2^3(t)\Omega_N r^8 - 1536a_2^3(t)\Omega_N r^6 + 576a_2^3(t)\Omega_N r^4 - M_{1N}a_2(t)r^4 + 64a_0^3(t)\Omega_N r^2 - 64a_2^3(t)\Omega_N r^2 - M_{1N}a_0(t)r^2 + M_{1N}a_2(t)r^2 + 16\dot{a_2}(t)\Omega_{1N}r^2 + 4a_0(t) - 4a_2(t) + 16a_2(t)r^2 - 4V_{01N} - 1152a_0(t)a_2(t)\Omega_N r^4 V_{01N} + 384a_0(t)a_2(t)\Omega_N r^2 V_{01N} + 4\ N_1 r^2 V_{01N} - 16N_1 a_2(t)r^4 - \dot{a_2}(t)r^4 - 4N_1 a_0(t)r^2 + 20N_1 a_2(t)r^2 - \dot{a_0}(t)r^2 + \dot{a_2}(t)r^2 + M_{1N}a_0(t) + 4\dot{a_0}(t)\Omega_{1N} - 4\dot{a_2}(t)\Omega_{1N} - G_{1N} - 4N_1 V_{01N} + 4N_1 a_0(t) - 4Na_2(t) + \dot{a_0}(t) + 1536a_0(t)a_2^2(t)\Omega_N r^6 - 1536a_2^2(t)\Omega_N r^6 V_{01N} + 576a_0^2(t)a_2(t)\Omega_N r^4 - 1152a_0(t)a_2^2(t)\Omega_N r^4 + 1152a_2^2(t)\Omega_N r^4 V_{01N} + 576a_2(t)\Omega_N r^4 V_{01N}^2 - 192a_0^2(t)a_2(t)\Omega_N r^2 - 192a_0^2(t)\Omega_N r^2 V_{01N} + 192a_0(t)a_2^2(t)\Omega_N r^2 + 192a_0(t)\Omega_N r^2 V_{01N}^2 - 192a_2^2(t)\Omega_N r^2 V_{01N}(-192a_2(t)\Omega_N r^2 V_{01N}^2$ (34)

By differentiating (23) with respect to $a_0(t)\ and\ a_2(t)$, we obtain the weight functions respectively as

$$w_1(r,t) = (1 - r^2) \quad (35)$$

and

$$w_2(r,t) = r^2(1 - r^2) \quad (36)$$

the orthogonality of the residual $RR(r, a_0(t), a_2(t))$ with respect to the weight functions in equations (35) and (36) lead the following systems in equations (37) and (38).

$$\int_0^1 w_1(r,t)\ RR(a_0(t), a_2(t), r)dr = o \quad (37)$$

$$\int_0^1 w_2(r,t)\ RR(a_0(t), a_2(t), r)dr = 0 \quad (38)$$

Equation (39) were obtained upon the substitution (35) and (36) into (37) and fully simplified.

$14784a_0^3(t)\Omega_N RE_{1N} + 12672a_0^2(t)a_2(t)\Omega_N RE_{1N} - 44352a_0^2(t)\Omega_N RE_{1N}V_{01N} + \quad 14784a_0(t)a_2^2(t)\Omega_N RE_{1N} - 25344a_0(t)a_2(t)\Omega_N RE_{1N}V_{01N} + \quad 44353a_0(t)\Omega_N RE_{1N}V_{01N}^2 + 2560a_2^3(t)\Omega_N RE_{1N} - 14784a_2^2(t)\Omega_N RE_{1N}V_{01N} + \quad 12672a_2(t)\Omega_N RE_{1N}V_{01N}^2 - 14784\Omega_N RE_{1N}V_{01N}^3 + 924M_{1N}a_0(t)RE_{1N} + \quad 132M_{1N}a_2(t)RE_{1N} + 231M_{1N}RE_{1N}V_{01N} + 4620\dot{a}_0(t)\Omega_{1N}RE_{1N} - \quad 924\dot{a}_2(t)\Omega_{1N}RE_{1N} - 1155G_{1N}RE_{1N} + 3696N_1a_0(t) - 1584N_1a_2(t) - 3696N_1V_{01N} + $ (39) $\quad 924\dot{a}_0(t)RE_{1N} + 132\dot{a}_2(t)RE_{1N} + 4620a_0(t) - 924a_2(t) - 4620V_{01N} = 0$ (39)

Similarly, we obtained (40) upon the substitution of equations (35) and (36) into (38) and fully simplified.

$$-82368a_0^3(t)\Omega_N RE_{1N} - 164736a_0^2(t)a_2(t)\Omega_N RE_{1N} + 257104a_0^2(t)\Omega_N RE_{1N}V_{01N} - 122304a_0(t)a_2^2(t)\Omega_N RE_{1N} + 329472a_0(t)a_2(t)\Omega_N RE_{1N}V_{01N} - 247104a_0(t)\Omega_N RE_{1N}V_{01N}^2 - 33792a_2^3(t)\Omega_N RE_{1N} + 122304a_2^2(t)\Omega_N RE_{1N}V_{01N} - 164736a_2(t)\Omega_N RE_{1N}V_{01N}^2 + 82368\Omega_N RE_{1N}V_{01N}^3 - 1716M_{1N}a_0(t)RE_{1N} - 572M_{1N}a_2(t)RE_{1N} - 1287M_{1N}RE_{1N}V_{01N} - 12012\dot{a}_0(t)\Omega_{1N}RE_{1N} - 8580\dot{a}_2(t)\Omega_{1N}RE_{1N} + 3003G_{1N}RE_{1N} - 6864N_1a_0(t) - 2288N_1a_2(t) + 6864N_1V_{01N} - 1716\dot{a}_0(t)RE_{1N} - 572\dot{a}_2(t)RE_{1N} - 12012a_0(t) - 8580a_2(t) + 12012V_{01N} = 0 \quad (40)$$

By simulating equations (39) and (40), we respectively obtained (41) and (42),

$$3.20000000\dot{a}_0(t) - 0.4571428571\dot{a}_2(t) + 85.33333333a_0^3(t) + 14.77633478a_2^3(t) - 64.00000000a_0^2(t) - 21.33333333a_2^2(t) + 73.14285714a_0^2(t)a_2(t) + 85.33333333a_2^2(t)a_0(t) - 36.57142858a_0(t)a_2(t) + (41) \quad 23.89037037a_0(t) + 1.973756614a_2(t) = 4.247592593 \quad (41)$$

and

$$0.6095238095\dot{a}_0(t) - 0.4063492063\dot{a}_2(t) + 36.57142857a_0^3(t) + 15.00366300a_2^3(t) - 27.42857142a_0^2(t) - 13.57575758a_2^2(t) + 73.14285714a_0^2(t)a_2(t) + 54.30303030a_2^2(t)a_0(t) - 36.57142858a_0(t)a_2(t) + (42) \quad 8.153650793a_0(t) + 5.229347442a_2(t) = 1.083888889 \quad (42)$$

Numerical techniques were used to solve the systems of non-linear first order ordinary differential equations (41) and (42) with initial conditions $a_0(0) = 0$ and

$a_2(0) = 0$, we obtain the values of $a_0(t)$ and $a_2(t)$ and when substituted into (23) after simplifying, we obtain

$$w(r,t) = 0.225004442 - 0.025004443r^2 + 0.011932224r^2(1 - r^2) \quad (43)$$

as the velocity profile of the unsteady blood flow with hematocrit, magnetic field and slip effects.

Similarly, a trial solution of the form (44) is assumed in order to obtain the temperature profile for the unsteady heat transfer model.

$$\overline{\theta}(y,t) = C_0(t) + C_1(t)y + C_2(t)y^2 \quad (44)$$

Subjecting (44) to the slip conditions (17) and after simplifying, we obtain

$$\overline{\theta}(y,t) = a_3(t)\left(1 - \frac{y^2}{R_b^2}\right) + a_4(t)\frac{y^2}{R_b^2}\left(1 - \frac{y^2}{R_b^2}\right) \quad (45)$$

By using the transformation (21) and dropping bar, equation (45) can be written as

$$\theta(r,t) = a_3(t)(1 - r^2) + a_4(t)r^2(1 - r^2) \quad (46)$$

From (3.5) and (3.28) we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = -4a_3 + 4a_4 - 16a_4r^2 \quad (47)$$

$$\left(\frac{\partial^2 w}{\partial r \partial t}\right)\left(\frac{\partial w}{\partial r}\right) = -4V_{01N}r\dot{a}_0(t) + 4V_{01N}r\dot{a}_2(t) - 8r^4V_{01N}\dot{a}_2(t) +$$

$$4r^2a_0(t)\dot{a_0}(t) - 4r^2a_2(t)\dot{a_2}(t) - \quad (48)$$

$$4r^2a_0(t)\dot{a_2}(t) + 8r^4a_0(t)\dot{a_2}(t) - 4r^2a_2(t)\dot{a_0}(t) + 8r^4a_2(t)\dot{a_2}(t) + 8r^4a_2(t)\dot{a_0}(t) - 8r^4a_2(t)\dot{a_2}(t) + 16r^6a_2(t)\dot{a_2}(t) \quad (48)$$

$$\left(\frac{\partial w}{\partial r}\right)^4 = 64V_{01N}^2a_2r^2 - 64V_{01N}^3a_0r^4 - 64V_{01N}^3a_2r^4 - 128V_{01N}^3a_2r^6 + 96V_{01N}^2a_2^2r^4 + 96V_{01N}^2a_0^2r^4 - 384V_{01N}^2a_2^2r^6 + 384V_{01N}^2a_2^2r^8 - 512V_{01N}a_2^3r^{10} - 384V_{01N}a_2^3r^6 - 64V_{01N}a_0^3r^4 + 768V_{01N}a_2^3r^8 + 64V_{01N}a_2^3r^4 - 64a_0^3a_2r^4 + 128a_0^3a_2r^6 + 96a_0^2a_2^2r^4 - 384a_0^2a_2^2r^6 + 384a_0^2a_2^2r^8 + 512a_0a_2^3r^{10} + 384a_0a_2^3r^6 - 768a_0a_2^3r^8 - 64a_0a_2^3r^4 + 16a_2^3r^4 - 128a_2^3r^6 + 16a_0^4r^4 - 512a_2^4r^{10} + 384a_2^4r^8 + 16V_{0N}^4r^4 + 256a_2^4r^{12} + 192V_{01N}a_0^2a_2r^4 - 384V_{01N}a_0^2a_2r^6 - 192V_{01N}a_0a_2^2r^4 + 768V_{01N}a_0a_2^2r^6 - 768V_{01N}a_0a_2^2r^8 - 192V_{01N}^2a_0a_2r^4 + 384V_{01N}^2a_0a_2r^6 \quad (49)$$

The residual for equation (15) can be written as

$$RR_1(r, a_3(t), a_4(t)) = \frac{\partial\theta}{\partial r} - E_{n1N}\big(1 + N_1(1 - r^m)\big)\left(\frac{\partial w}{\partial r}\right)^2 + \phi_N\left(\frac{\partial w}{\partial r}\right)^4 + \phi_{1N}\left(\frac{\partial^2 w}{\partial r\partial t}\right)\left(\frac{\partial w}{\partial r}\right) + \Lambda_{1N}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) \quad (50)$$

Substituting (30), (47),(48) and (49) into (50) and after simplifying, we obtain

$$RR_1(r, a_3(t), a_4(t)) = 16\Lambda_{1N}a_4(t)r^2 - 32\phi_NV_{01N}^4r^4 - 32\phi_Na_0^4(t)r^4 - 512\phi_Na_2^4(t)r^{12} + 1024\phi_Na_2^4(t)r^{10} - 768\phi_Na_2^4(t)r^8 + 256\phi_Na_2^4(t)r^6 - 32\phi_Na_2^4(t)r^4 - 4E_{n1N}V_{01N}^2r^2 - 4E_{n1N}a_0^4(t)r^2 - 16E_{n1N}a_2^2(t)r^6 + 16E_{n1N}a_2^2(t)r^4 - 4E_{n1N}a_2^2(t)r^2 + \dot{a}_4(t)r^2 + 4\Lambda_{1N}a_3(t) - 4\Lambda_{1N}4(t) - \dot{a}_3(t)r^2 - \dot{a}_4(t)r^4 - 8E_{n1N}V_{01N}r^4N_1 + 8E_{n1N}V_{01N}r^2N_1 - 16E_{n1N}V_{01N}a_2(t)N_1r^6 + 24E_{n1N}V_{01N}a_2(t)r^4 - 8E_{n1N}V_{01N}a_2(t)N_1r^2 + 16E_{n1N}a_0(t)a_2(t)N_1r^6 - 24E_{n1N}a_0(t)a_2(t)N_1r^4 + 8E_{n1N}a_0(t)a_2(t)N_1r^2 + 384\phi_NV_{01N}a_0(t)a_2^2(t)r^4 + 1536\phi_NV_{01N}a_0(t)a_2^2(t)r^8 + 768\phi_NV_{01N}a_0^2(t)a_2(t)r^6 - 384\phi_NV_{01N}a_0^2(t)a_2(t)r^4 - 1536\phi_NV_{01N}a_0(t)a_2^2(t)r^6 - 768\phi_NV_{01N}^2a_0(t)a_2(t)r^6 + 384\phi_NV_{01N}^2a_0(t)a_2(t)r^4 + \dot{a}_3(t) - 4E_{n1N}V_{01N}^2N_1r^2 - 4E_{n1N}a_0^2(t)N_1r^2 + 4E_{n1N}a_0^2(t)N_1r^4 + 4E_{n1N}V_{01N}^2Nr^4 + 16E_{n1N}a_2^2(t)Nr^8 - 32E_{n1N}a_2^2(t)Nr^2 + 20E_{n1N}a_2^2(t)Nr^4 - 4E_{n1N}a_2^2(t)Nr^2 + 16E_{n1N}V_{01N}a_2(t)r^4 - 8E_{n1N}V_{01N}a_2(t)r^2 + 8E_{n1N}V_{01N}a_0(t)r^2 + 8E_{n1N}a_0(t)a_2(t)r^2 - 16E_{n1N}a_0(t)a_2(t)r^4 - 256\phi_Na_0^3(t)a_2(t)r^6 + 128\phi_Na_0^3(t)a_2(t)r^4 + 768\phi_Na_0^2(t)a_2^2(t)r^6 - 1024\phi_Na_0(t)a_2^3(t)r^{10} + 1536\phi_Na_0(t)a_2^3(t)r^8 - 768\phi_Na_0(t)a_2^3(t)r^6 + 128\phi_Na_0(t)a_2^3(t)r^4 + 128\phi_NV_{01N}^3a_0(t)r^4 - 768\phi_NV_{01N}^2a_2^2(t)r^8 - 192\phi_NV_{01N}^2a_0^2(t)r^4 + 256\phi_NV_{01N}^3a_2(t)r^6 + 768\phi_NV_{01N}^2a_2^2(t)r^6 - 128\phi_NV_{01N}^3a_2(t)r^4 - 192\phi_NV_{01N}^2a_2^2(t)r^4 + 128\phi_NV_{01N}a_0^3(t)r^4 + 1024\phi_NV_{01N}a_2^3(t)r^{10} - 1536\phi_NV_{01N}a_2^3(t)r^8 + 768\phi_NV_{01N}a_2^3(t)r^6 - 128\phi_NV_{01N}a_2^3(t)r^4 - 192\phi_Na_0^2(t)a_2^2(t)r^4 - 768\phi_Na_0^2(t)a_2^2(t)r^8 + 4\dot{a}_0(t)\phi_{1N}V_{01N}r^2 - 4\phi_{1N}\dot{a}_0(t)a_0(t)r^2 - 8\phi_{1N}\dot{a}_0(t)a_2(t)r^4 + 4\phi_{1N}\dot{a}_0(t)a_0(t)r^2 + 8\phi_{1N}\dot{a}_2(t)V_{01N}r^4 - 8\phi_{1N}\dot{a}_2(t)a_0(t)r^4 - 16\phi_{1N}\dot{a}_2(t)a_2(t)r^6 + 16\phi_{1N}\dot{a}_2(t)a_2(t)r^4 - 4\phi_{1N}\dot{a}_2(t)V_{01N}r^2 + 4\phi_{1N}\dot{a}_2(t)a_0(t)r^2 - 4\phi_{1N}\dot{a}_2(t)a_2(t)r^2 \quad (51)$$

By taking the derivative of (46) with respect to $a_3(t)$ and $a_4(t)$we obtained the weight functions as obtained in (32) and (33).

We obtained equations (52) and (53) by taking the orthogonality of the residual $RR_1(r, a_3(t), a_4(t))$ with respect to the weight functions in equations (32) and (33)

$$\int_0^1 RR_1(r, a_3(t), a_4(t))\, w_1(r) dr = 0 \tag{52}$$

$$\int_0^1 RR_1(r, a_3(t), a_4(t))\, w_2(r) dr = 0 \tag{53}$$

We obtained equation (54) by putting equations (32) and (47) into (52) and simplified fully

$$-\frac{652}{45045}\phi_N a_2^4(t) - \frac{8}{63}E_{n1N}a_2^2(t) - \frac{64}{35}\phi_N a_0^4(t) - \frac{8}{15}E_{n1N}a_0^2(t) - \frac{8}{15}E_{n1N}V_{01N}^2 - \frac{64}{35}\phi_N V_{01N}^4 - \frac{256}{715}\phi_N a_0(t)a_2^3(t) - \frac{256}{715}\phi_N V_{01N}a_2^3(t) - \frac{2432}{1155}\phi_N a_0^2(t)a_2^2(t) - \frac{2432}{1155}\phi_N V_{01N}^2 a_2^2(t) - \frac{32}{385}E_{n1N}a_2^2(t)N - \frac{256}{315}\phi_N a_0^3(t)a_2(t) + \frac{256}{315}\phi_N V_{01N}^3 a_2(t) - \frac{8}{63}\phi_N \dot{a}_2(t)a_2(t) + \frac{256}{35}\phi_N V_{01N}a_0^3(t) - \frac{384}{35}\phi_N V_{01N}^2 a_0^2(t) + \frac{256}{35}\phi_N V_{01N}^3 a_0(t) - \frac{32}{105}E_{n1N}a_0^2(t)N_1 + \frac{16}{105}E_{n1N}a_0(t)a_2(t) - \frac{16}{105}E_{n1N}V_{01N}a_2(t) + \frac{8}{105}\phi_{1N}\dot{a}_0(t)a_2(t) + \frac{8}{105}\phi_{1N}\dot{a}_2(t)a_0(t) - \frac{8}{105}\phi_{1N}\dot{a}_2(t)V_{01N} + \frac{16}{15}E_{n1N}V_{01N}a_0(t) - \frac{8}{15}\phi_{1N}\dot{a}_0(t)a_0(t) + \frac{8}{15}\phi_{1N}\dot{a}_0(t)V_{01N} - \frac{32}{105}E_{n1N}V_{01N}^2 N_1 + \frac{8}{3}\Lambda_{1N}a_3(t) - \frac{8}{15}\Lambda_{1N}a_4(t) + \frac{8}{15}\dot{a}_3(t) + \frac{8}{105}\dot{a}_4(t) + \frac{4864}{1155}\phi_N V_{01N}a_0(t)a_2^2(t) + \frac{256}{105}\phi_N V_{01N}a_0^2(t)a_2(t) - \frac{256}{105}\phi_N V_{01N}a_0(t)a_2(t) + \frac{64}{315}E_{n1N}a_0(t)a_2(t)N_1 - \frac{64}{315}E_{n1N}V_{01N}a_2(t)N_1 + \frac{64}{105}E_{n1N}V_{01N}a_0(t)N_1 = 0 \tag{54}$$

Also, putting (33) and (47) into (53), we integrate and simplified to obtained

$$-\frac{8384}{85085}\phi_N a_2^4(t) - \frac{152}{3465}E_{n1N}a_2^2(t) - \frac{64}{63}\phi_N a_0^4(t) - \frac{8}{35}E_{n1N}a_0^2(t) - \frac{8}{35}E_{n1N}V_{01N}^2 - \frac{64}{63}\phi_N V_{01N}^4 - \frac{21248}{45045}\phi_N a_0(t)a_2^3(t) + \frac{21248}{45045}\phi_N V_{01N}a_2^3(t) - \frac{3968}{3003}\phi_N a_0^2(t)a_2^2(t) + \frac{3968}{3003}\phi_N V_{01N}^2 a_2^2(t) - \frac{736}{45045}E_{n1N}a_2^2(t)N_1 - \frac{256}{231}\phi_N a_0^3(t)a_2(t) + \frac{256}{231}\phi_N V_{01N}^3 a_2(t) - \frac{152}{3465}\phi_N \dot{a}_2(t)a_2(t) + \frac{256}{63}\phi_N V_{01N}a_0^3(t) - \frac{128}{21}\phi_N V_{01N}^2 a_0^2(t) + \frac{256}{63}\phi_N V_{01N}^3 a_0(t) - \frac{32}{315}E_{n1N}a_0^2(t)N_1 - \frac{16}{315}E_{n1N}a_0(t)a_2(t) + \frac{16}{315}E_{n1N}V_{01N}a_2(t) - \frac{8}{315}\phi_{1N}\dot{a}_0(t)a_2(t) - \frac{8}{315}\phi_{1N}\dot{a}_2(t)a_0(t) + \frac{8}{315}\phi_{1N}\dot{a}_2(t)V_{01N} + \frac{16}{35}E_{n1N}V_{01N}a_0(t) - \frac{8}{35}\phi_{1N}\dot{a}_0(t)a_0(t) + \frac{8}{35}\phi_{1N}\dot{a}_0(t)V_{01N} - \frac{32}{315}E_{n1N}V_{01N}^2 N_1 + \frac{8}{15}\Lambda_{1N}a_3(t) + \frac{8}{21}\Lambda_{1N}a_4(t) + \frac{8}{105}\dot{a}_3(t) + \frac{8}{315}\dot{a}_4(t) + \frac{7936}{3003}\phi_N V_{01N}a_0(t)a_2^2(t) + \frac{256}{77}\phi_N V_{01N}a_0^2(t)a_2(t) - \frac{256}{77}\phi_N V_{01N}^2 a_0(t)a_2(t) + \frac{64}{3465}E_{n1N}a_0(t)a_2(t)N_1 - \frac{64}{3465}E_{n1N}V_{01N}a_2(t)N_1 + \frac{64}{315}E_{n1N}V_{01N}a_0(t)N_1 = 0 \tag{55}$$

Substituting the appropriate values of the parameters ϕ_N, E_{n1N}, N_1, ϕ_{1N}, Λ_{1N} and V_{01N} into (54) and (55), we obtain

$$\frac{8}{105}\dot{a}_0(t)a_2(t) - \frac{8}{63}\dot{a}_2(t)a_2(t) + \frac{8}{105}\dot{a}_2(t)a_0(t) - \frac{8}{15}\dot{a}_0(t)a_0(t) - 1.219047619a_0^3(t)a_2(t)$$
$$+ 0.06095238096a_0(t)a_2(t) + 1.579220779a_0(t)a_2^2(t)$$
$$+ 0.9142857142a_0^2(t)a_2(t) - 0.5370629371a_0(t)a_2^3(t)$$
$$- 3.158441558a_0^2(t)a_2^2(t) + 2.742857142a_0^3(t) + 0.1342657343a_2^3(t)$$
$$- 2.742857143a_0^4(t) + 0.2195138195a_2^4(t) - 2.742857143a_0^2(t)$$
$$- 0.6372294373a_2^2(t) + \frac{8}{15}\dot{a}_3(t) + 0.133333333\dot{a}_0(t) - 0.01904761905\dot{a}_2(t)$$
$$- 0.7200000000a_4(t) + 3.600000000a_3(t) - 0.1904761906a_2(t)$$
$$+ 1.02871429a_0(t)$$
$$= 0.1178571428 \tag{56}$$

and

$$\frac{8}{315}\dot{a}_0(t)a_2(t) - \frac{152}{3465}\dot{a}_2(t)a_2(t) - \frac{8}{315}\dot{a}_2(t)a_0(t) - \frac{8}{15}\dot{a}_0(t)a_0(t) - 1.662337662a_0^3(t)a_2(t)$$
$$- 0.3324675325a_0(t)a_2(t) + 0.9910089910a_0(t)a_2^2(t)$$
$$+ 1.246753247a_0^2(t)a_2(t) - 0.7075591076a_0(t)a_2^3(t)$$
$$- 1.982017982a_0^2(t)a_2^2(t) + 1.523809524a_0^3(t) + 0.1768897769a_2^3(t)$$
$$- 1.523809524a_0^4(t) + 0.1478051360a_2^4(t) - 1.219047619a_0^2(t)$$
$$- 0.2386946387a_2^2(t) + \frac{8}{105}\dot{a}_3(t) + \frac{8}{315}\dot{a}_4(t) + 0.05714285714\dot{a}_0(t)$$
$$+ 0.0063492063\dot{a}_2(t) + 0.5142857143a_4(t) + 0.7200000000a_3(t)$$
$$- 0.03116883117a_2(t) + 0.4190476190a_0(t)$$
$$= 0.04642857143 \tag{57}$$

Solving the system of non-linear first order ordinary differential equations (56) and (57) with initial conditions $a_0(t) = 0$, $a_2(t) = 0$, $a_3(t) = 0$, $a_4(t) = 0$ and $t = 0.5$ using fourth order Runge Kutta method, we obtain the values of $a_3(t)$ and $a_4(t)$ and when substituted into (46), we obtain

$$\theta(r,t) = 0.00176686582597 - 0.0017668758r^2 + 0.002739491r^2(1 - r^2) \tag{58}$$

as the temperature profile of the unsteady blood flow with hematocrit, magnetic field and slip effects.
By substituting the appropriate values of the parameters ϕ_N, E_{n1N}, N, ϕ_{1N}, Λ_{1N} and V_{01N}, and the constants $a_0(t)$ and $a_2(t)$ in (54) and (55) and following the same procedures above, we obtain the corresponding values of a_3(t), a_4(t) and $\theta(r,t)$.

Volume Flow Rate

The volume flow rate denoted by Q is given by

$$Q = 2\pi \int_0^{R(z)} rw(r)dr \tag{59}$$

Putting (23) into (59) and evaluate, we obtain

$$Q = 12\left[3V_0\big(R(z)\big)^4 + a_0(t)\left(6\big(R(z)\big)^2 - 3\big(R(z)\big)^4\right) + a_2(t)\left(3\big(R(z)\big)^4 - 2\big(R(z)\big)^6\right)\right] \tag{60}$$

Shear Stress

The shear stress denoted by $\tau_S$is given as

$$\tau_s = \mu \frac{\partial w}{\partial r}\Big|_{r=R(z)} + 2\beta_3\left(\frac{\partial w}{\partial r}\right)^3\Big|_{r=R(z)} \tag{61}$$

Simplified (61) to obtain

(62) $$\tau_s = 2\mu R(Z)\left(V_0 - a_0(t) + a_2(t) - 2R\left((Z)\right)^2 a_2\right) + 16R(Z)\beta_3\left(V_0 - a_0(t) + a_2(t) - 2\left(R(Z)\right)^2 a_2(t)\right) \quad (62)$$

Resistance to Flow

The resistance to flow can be denoted as ψ and is given by

$$\psi = \frac{-\frac{\partial \hat{P}}{\partial z}}{12\left[3V_0\left(R(z)\right)^4 + a_0(t)\left(6\left(R(z)\right)^2 - 3\left(R(z)\right)^4\right) + a_2(t)\left(3\left(R(z)\right)^4 - 2\left(R(z)\right)^6\right)\right]} \quad (63)$$

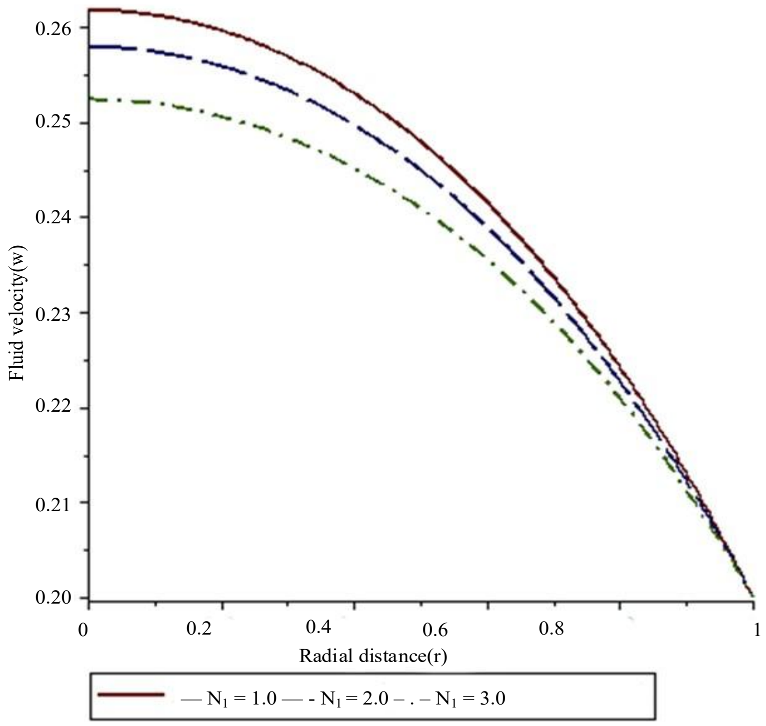

Figure 2. Variation of Velocity Profile of Unsteady Blood Flow along the radial distance for different values of Hematocrit Parameter.

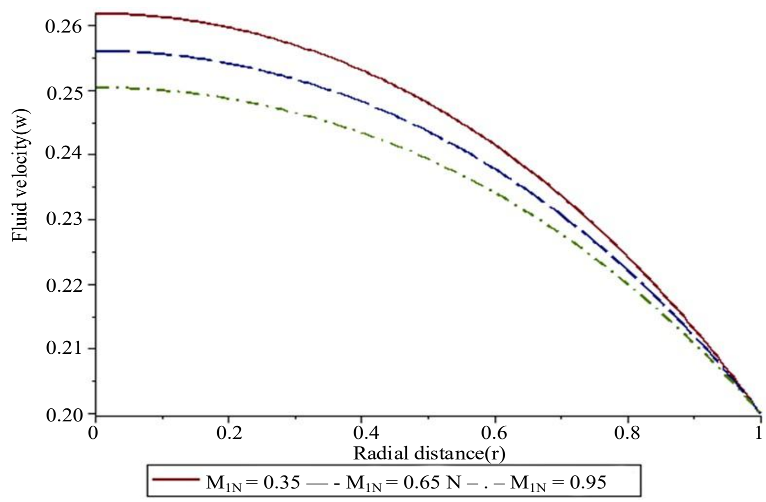

Figure 3. Variation of Velocity Profile of Unsteady Blood Flow along the radial distance for different values of the Magnetic Field Parameter.

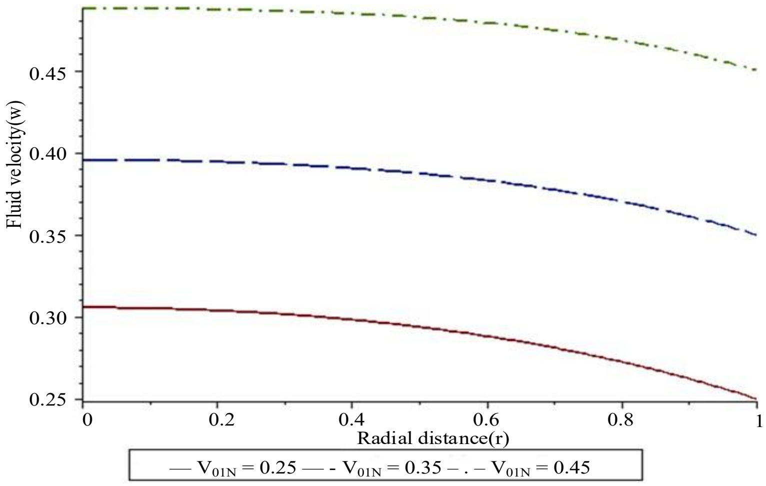

Figure 4. Variation of Velocity Profile of Unsteady Blood Flow along the radial distance for different values of the Slip Velocity

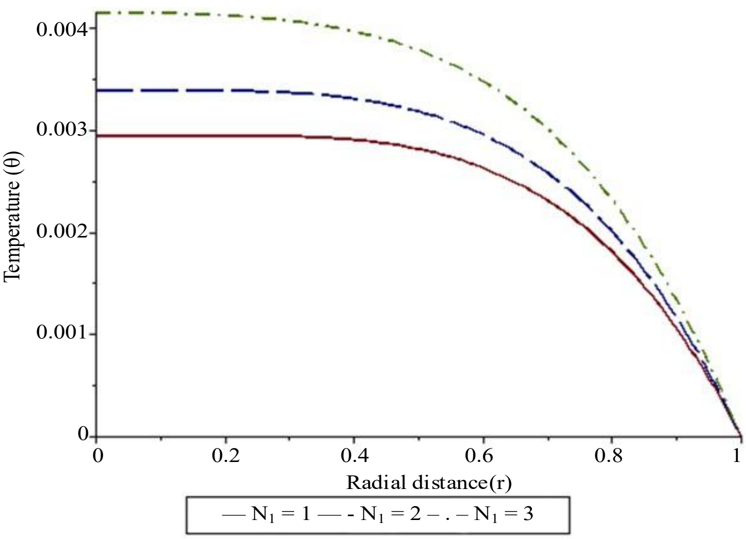

Figure 5. Variation of Temperature Profile of Unsteady Heat Transfer along the radial distance for different values of Hematocrit Parameter.

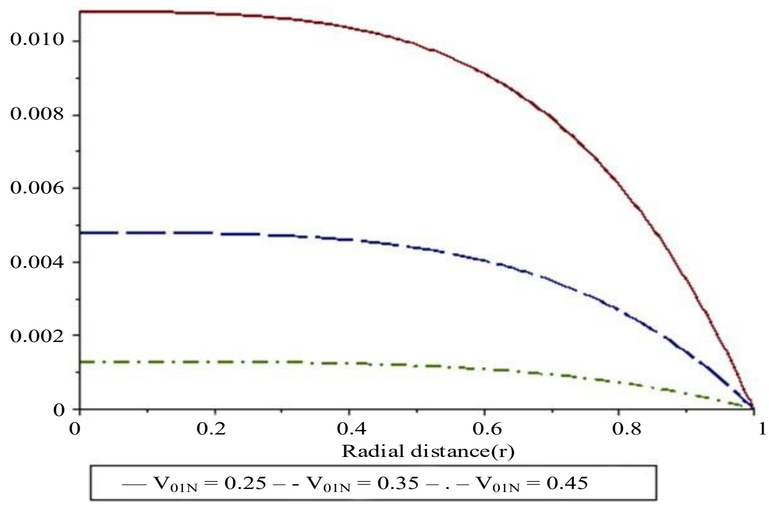

Figure 6. Variation of Temperature Profile of Unsteady Heat Transfer along the radial distance for different values of Slip Velocity.

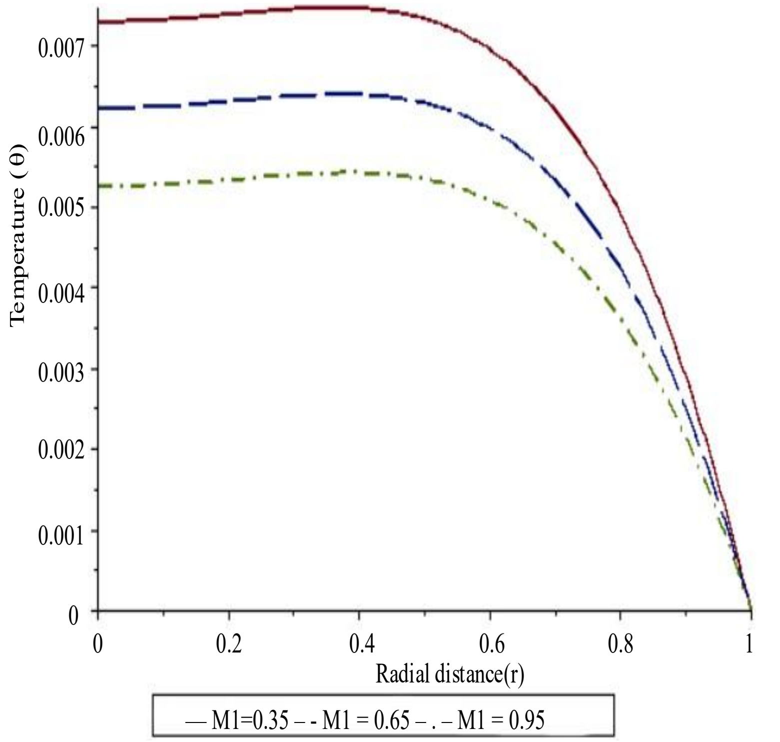

Figure 7. Variation of Temperature Profile of Unsteady Heat Transfer along the radial distance for different values of magnetic field parameter.

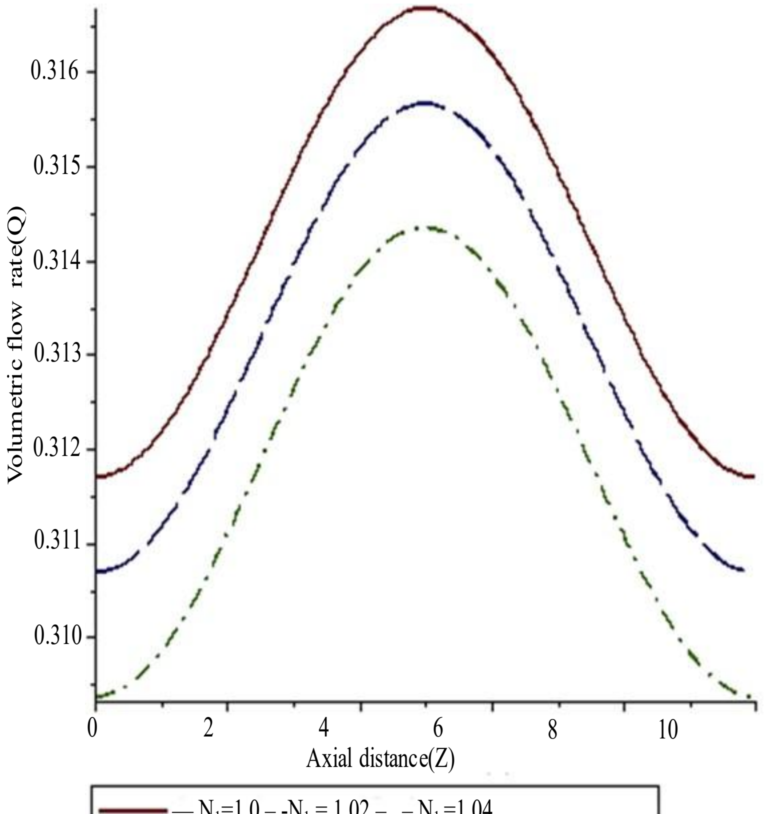

Figure 8. Variation of Volumetric Flow Rate of Unsteady Blood Flow with increasing values of the Hematocrit Parameter in the entire arterial region along the axial direction.

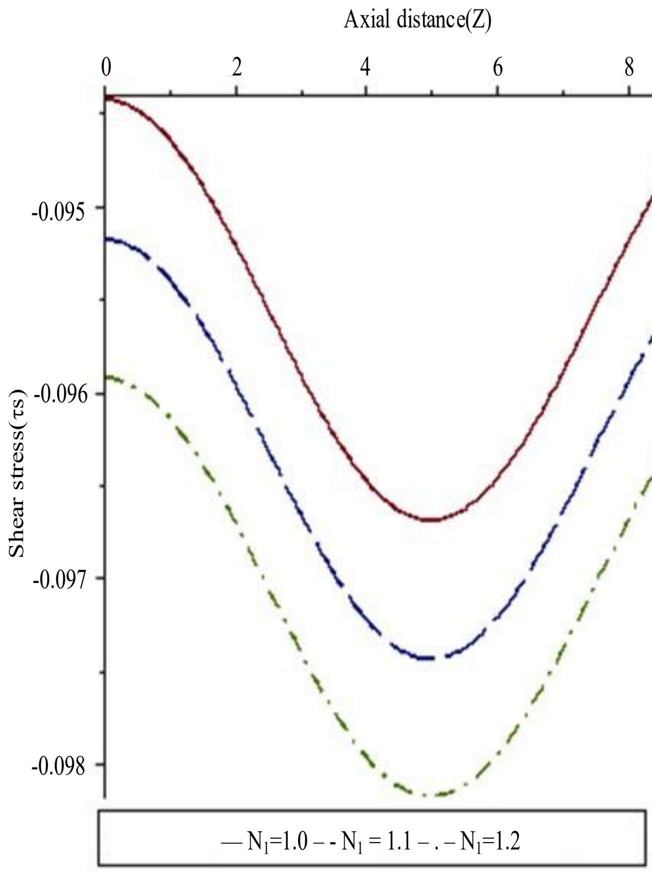

Figure 9. Variation of Wall Shear Stress of Unsteady Blood Flow with increasing values of the Hematocrit Parameter in the entire arterial region along the axial direction.

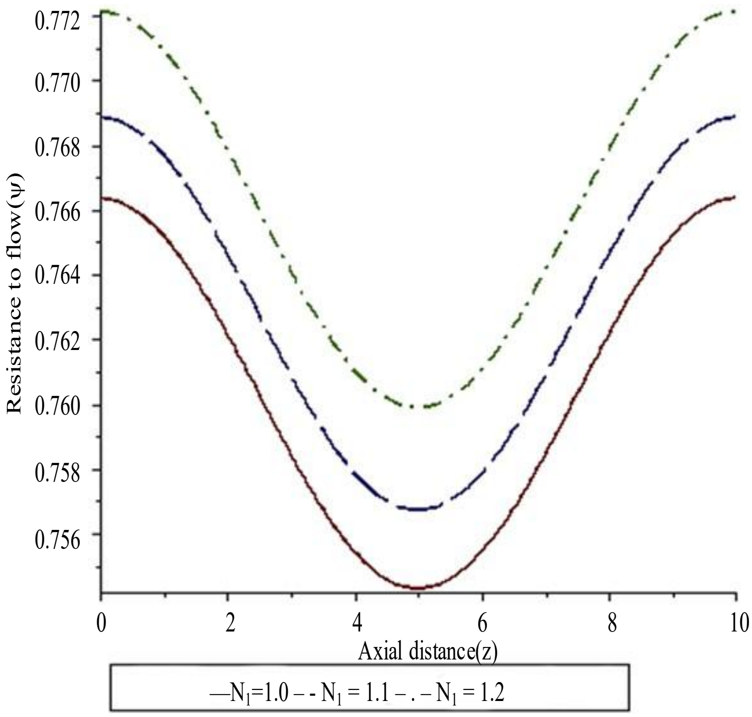

Figure 10. Variation of Resistance to Unsteady Blood Flow with increasing values of the Hematocrit Parameter in the entire arterial region along the axial direction.

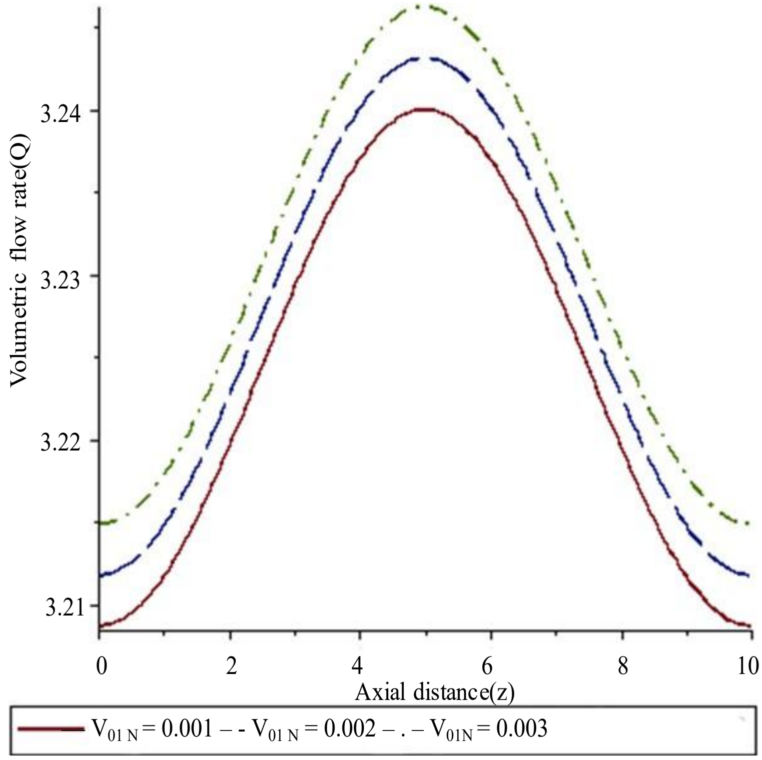

Figure 11. Variation of Volumetric Flow Rate of Unsteady Blood Flow with increasing values of Slip Velocity in the entire arterial region along the axial direction.

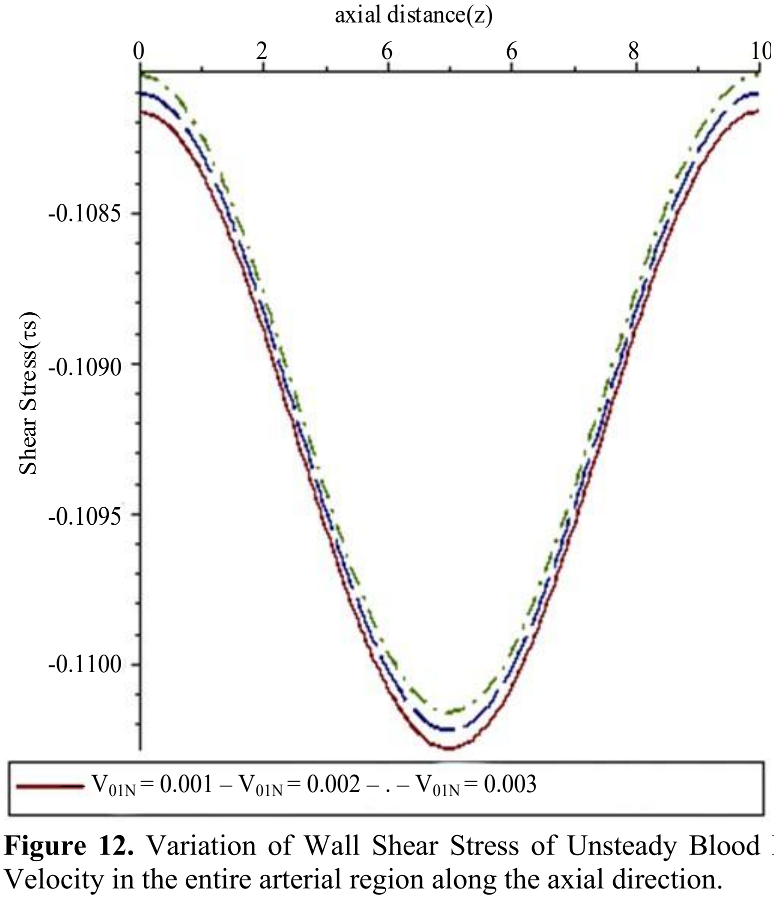

Figure 12. Variation of Wall Shear Stress of Unsteady Blood Flow with increasing values of Slip Velocity in the entire arterial region along the axial direction.

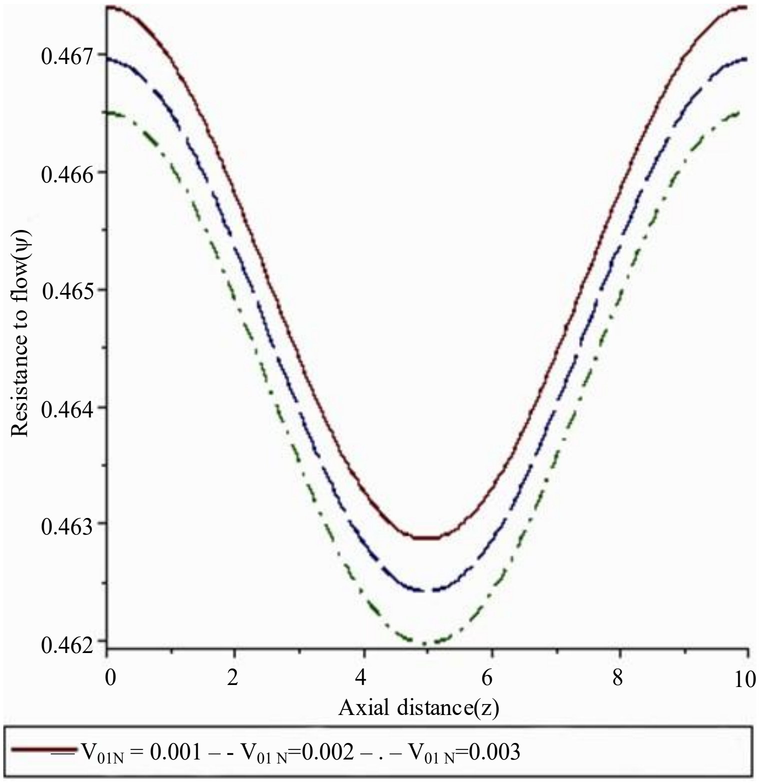

Figure 13. Variation of Resistance to Unsteady Blood Flow with increasing values of the Slip Velocity in the entire arterial region along the axial direction.

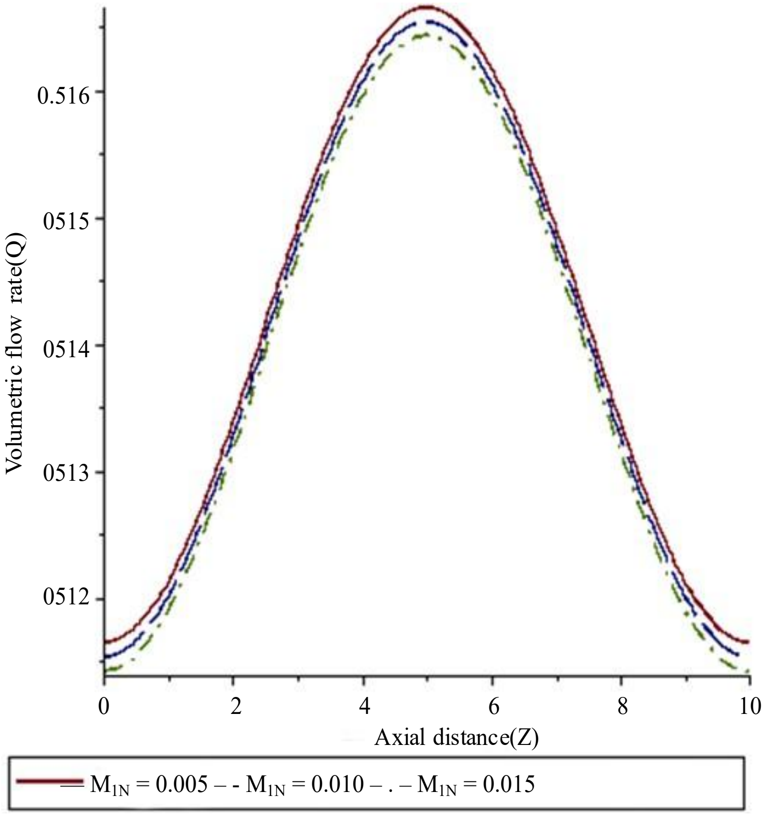

Figure 14. Variation of Volumetric Flow Rate of Unsteady Blood Flow with increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.

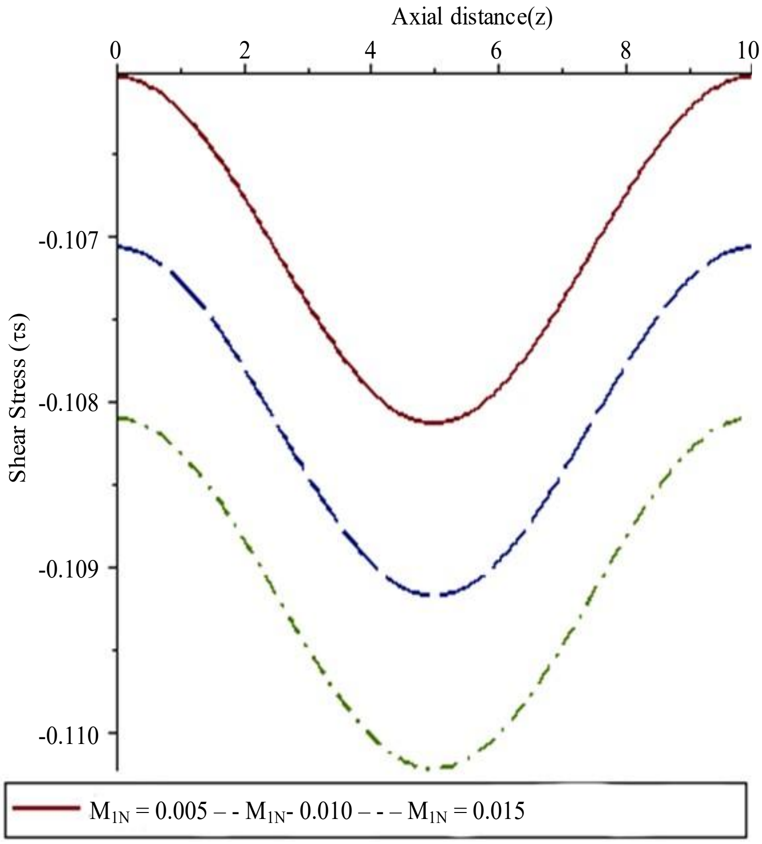

Figure 15. Variation of Wall Shear Stress of Unsteady Blood Flow with increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.

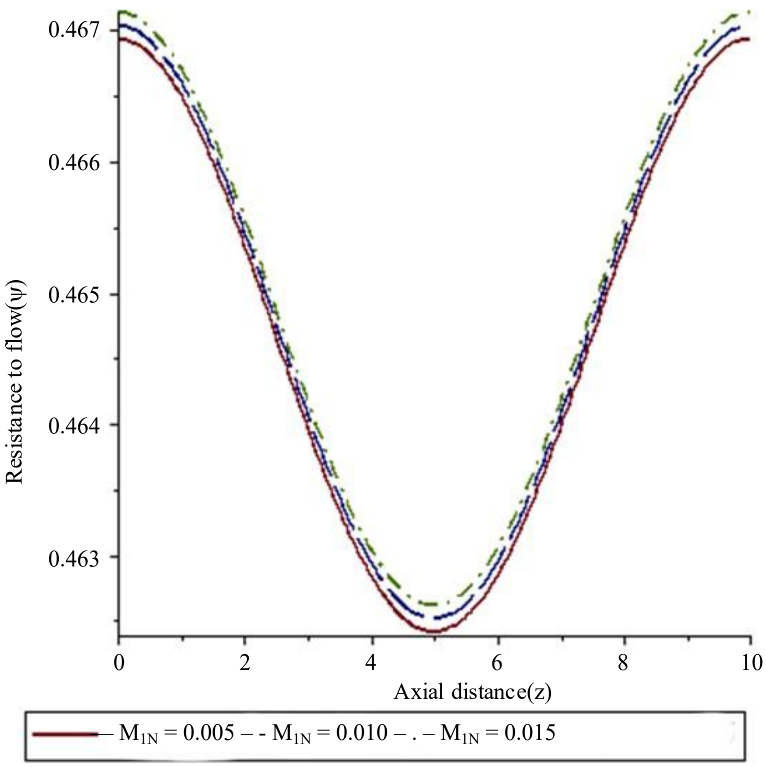

Figure 16. Variation of Resistance to Unsteady Blood Flow with increasing values of the Magnetic Field Parameter in the entire arterial region along the axial direction.

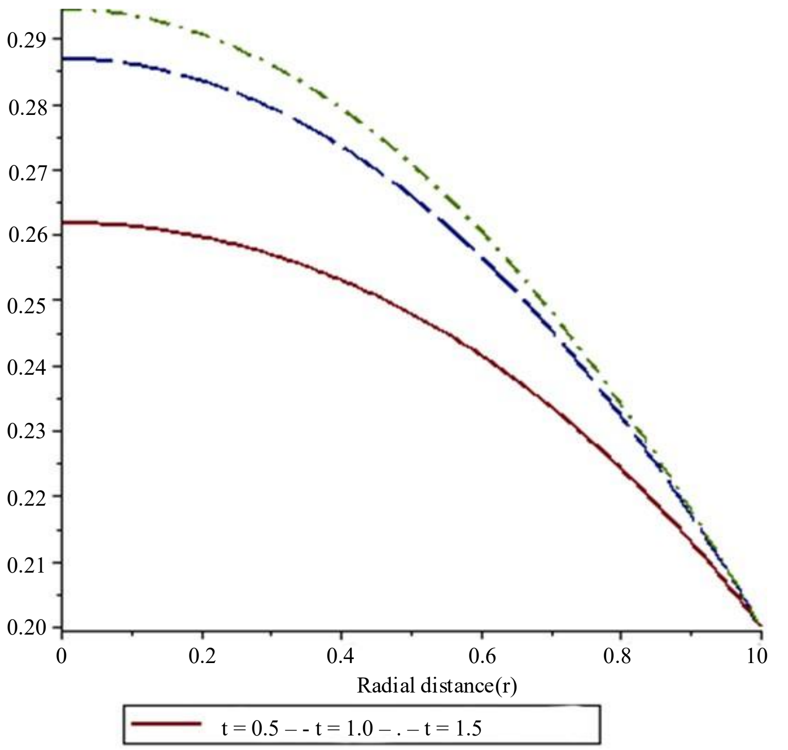

Figure 17. Variation of Velocity Profile of Unsteady Blood Flow along radial distance at various time.

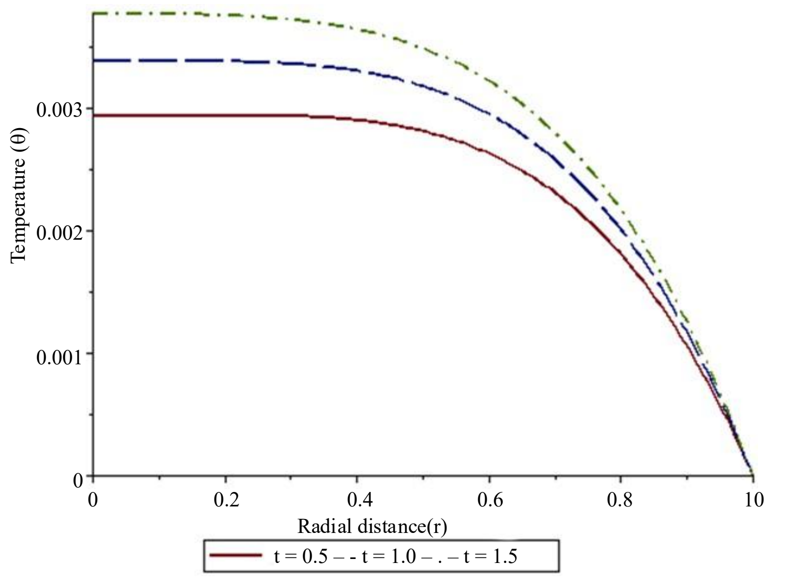

Figure 18. Variation of Temperature Profile of Unsteady Heat Transfer along radial distance at various times.

DISCUSSION OF RESULTS

In this work, we have investigated the effects of hematocrit, slip velocity, and magnetic field parameter on the unsteady flow and unsteady heat transfer models by considering variable viscosity of blood dependent on red blood cell consideration, velocity slip at the boundary of the artery and externally applied magnetic field and the results based on the mathematical analysis carried out indicate that increase in hematocrit parameter leads to increase in temperature profile and also offer more resistance to the flow as indicated in Figures 5 and 10 respectively but reduces significantly the flow velocity, volumetric flow rate and wall shear stress as shown in Figures 2, 8 and 9 respectively. Also, increase in magnetic field offer more resistance to the flow as shown in Figure 14 but reduces the velocity profile, the temperature profile, volumetric flow rate and wall shear stress as indicated in Figures 3, 7, 15, and 16 respectively. In the same vein, it is seen from Figures 4, 11, and 12 respectively that increases in slip velocity leads to increases in velocity profile, volumetric flow rate, and wall shear stress but reduces the temperature profiles as shown in Figures 6 and 13 respectively. Finally, it is revealed from Figures 17 and 18 that, time positively influence the flow velocity and heat transfer rate.

CONCLUSION

In this investigation, effects of hematocrit and slip velocity on unsteady blood flow and unsteady heat transfer under the influence of applied magnetic field have been analyzed. Governing nonlinear differential equation have been solved using Galerkin's weighted residual and Fourth order Runge-Kutta methods. Effects of some parameters such as hematocrit parameter (N_1), slip velocity V_{01N}, and magnetic field parameter (M_{1N}) on flow velocity, temperature profile, flow rate, shear stress and flow resistance have been sketch graphically as shown below. Some of the findings are summarized below:

i. Flow velocity and flow rate increases significantly as we increase the slip velocity while it decreases when the values of hematocrit and magnetic field parameters increase. This controlled behavior of the velocity profile and flow rate under the influence of hematocrit, magnetic field and slip velocity can respectively control blood viscosity, helps medical staff in their medical surgical procedures and restoring blood as well as reducing the damage caused to the vessel wall.
ii. Wall shear stress increases as we increase hematocrit parameter and slip velocity while it decreases when the values of magnetic field parameter increase.
iii. Flow resistance increases as we increases hematocrit and magnetic field parameters while it decreases when the values of the slip velocity increase.
iv. Temperature profile increases as the values of hematocrit parameter increase while it decreases when the values of the slip velocity decrease.
v. Time positively influence both the flow velocity and temperature profile.

Nomenclatures

w - Fluid velocity
$\bar{w}$ - Dimensionless fluid velocity
t - Time component
$\bar{t}$ - Dimensionless time component
r - Radial distance
y - Dimensionless radial distance
z - Axial distance
w_s - Slip velocity
V_{01N} - Dimensionless Slip velocity for the unsteady blood flow with hematocrit
T - Temperature profile
T_w - Pipe temperature
$\bar{\theta}$ - Dimensionless temperature profile
T_m - Fluid temperature
R_0 - Radius of the normal artery
β_0 - Magnetic Field Strength
R(z) - Radius of the artery in a stenotic region
σ - Electrical Conductivity
ψ - Resistance to flow
K - Thermal conductivity
Q - Volumetric flow rate
τ_s - Wall Shear Stress
Ƹ - Maximum height of the stenosis
L - Length of the stenosis
N_1 = Hβ = Hematocrit parameterfor the unsteady blood flow
h(r) = Hematocrit at a distance r
μ_0 = Viscosity coefficient for plasma
β = A constant whose value for blood equal 2.5
μ(r) = Coefficient of viscosity of blood at radial distance
G_{1N} = Pressure gradient for the unsteady blood flow model with hematocrit
V_{01N} = Slip velocity for the unsteady blood flow with hematocrit
M_{1N} = Magnetic field parameter for the unsteady blood flow model with hematocrit
Ω_N = Shear thinning for the unsteadyblood flow model with hematocrit
E_{n1N} = Eckert number for the unsteadyheat transfer model with hematocrit
ϕ_{1N} = Shear thinning for the unsteady heat transfer model with hematocrit
Λ_{1N} =Third grade parameter for the unsteadyheat transfer model with hematocrit

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