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Comparative Study of Clamped-Clamped and Clamped-Free Elastic Beams Resting on Bi-Parametric Subgrades and Subjected to Concentrated Moving Loads

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Abstract

In this paper, a comparative study of clamped-clamped and clamped-free elastic beams resting on bi-parametric subgrades subjected to concentrated moving load has been investigated. This work takes into account the viscous effects of the moving load. The findings are shown graphically. The methods of solution incorporate integral transformation in combination with convolution theory, a version of Struble's asymptotic approaches, and Fourier integral translate with the series representation of the Dirac-delta symbol. Special cases of moving force and moving mass problems traversed by concentrated moving load and the influence of axial force, shear modulus, and foundation modulus are taken into consideration. For both clamped-clamped and clamped-free elastic beams, it was found that when the beam is subjected to concentrated moving load, the response amplitude of the beam decreases as the sums of the axial force N, shear modulus G, and foundation modulus K increase. However, for all cases taken into consideration, the deflection profiles of the clamped-clamped beam are higher compared with that of the clamped-free beam for a range of values of axial force, shear modulus, and foundation modulus. Nevertheless, greater shear modulus G values will be required for a more pronounced impact than foundation modulus K values. Because the critical velocity for the system traversed by mobile component is likewise determined to be less than that under the consequences of replacing mass, resonance is obtained now in the moving mass problem as opposed to the moving force problem. As a result, the moving force problem cannot be used as a safe approximation to the moving mass problem.

Keywords: Bi-parametric subgrades, concentrated loads, resonance, moving force, moving mass, clamped-clamped elastic beam, clamped-free elastic beam

INTRODUCTION

The investigation of forced vibration of elastic bodies such as beam resting on elastic foundation subjected to moving load has great importance for the engineering structures such as railways, concrete pavements, etc. In general, and based on open literature, the foundation model can be classified as Winkler elastic foundation or one-parameter foundation and bi-parametric foundations. The Winkler or one-parameter model is the most common basic foundation representation based on distributed unconnected linear elastic strings that behave independently.

Some of the researchers who have worked on one-parameter foundation model are Jimoh et al. [1], Omolofe and Ogunbamike [2], Mukherjee et al. [3], Seref et al. [4], Oluwatoyin [5], and Olotu et al. [6] to mention but a few.

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One of the great shortcomings of the one-parameter foundation model is the discontinuous behavior of the surface displacement beyond the load region and this contravene the observation made in practice. Since the bi-parametric foundation model enables surface displacement continuity outside of the load zone, it is seen to be more realistic.

Some of the researchers who considered bi-parametric foundation model in their studies are Jimoh and Ajoge [7], Saurabh [8], Baran [9], Jimoh [10], Jimoh and Ajoge [11], Anague et al. [12], Rajib et al. [13], Davood and Mohammed [14], and Jimoh and Ajoge [15].

All the above-mentioned researchers considered simply supported boundary conditions. Some researchers [16–19] who considered other boundary conditions such as clamped-clamped and clamped-free in their studies are very scanty in the literature and none of those researchers considered comparative studies between the other boundary conditions.

This paper therefore focused on the comparative studies of clamped-clamped and clamped-free uniform elastic beam resting on bi-parametric subgrades and subjected to concentrated moving load.

MATHEMATICAL MODEL

The governing partial differential equation for a uniform elastic beam of length L on two parameters foundation and traversed by a concentrated load $P(x, t)$ of mass M moving with velocity c is given by [20].

$$\frac{\mathrm{EI}}{\mu}\frac{\partial^4 W(x,t)}{\partial x^4}+\frac{\partial^2 W(x,t)}{\partial t^2}-\frac{N}{\mu}\frac{\partial^2 W(x,t)}{\partial x^2}-G\frac{\partial^2 W(x,t)}{\partial x^2}+\frac{k}{\mu}\psi(x,t)$$
$$\frac{-M}{\mu}\delta(x-\mathrm{ct})\left[\frac{\partial^2 W(x,t)}{\partial t^2}+\frac{2v\partial^2 W(x,t)}{\partial x\partial t}+\frac{c^2\partial^2 W(x,t)}{\partial x^2}\right]=\frac{\mathrm{Mg}}{\mu}\delta(x-\mathrm{ct}) \tag{1}$$

The beam's flexural rigidity is expressed as by E, its Young's modulus by I, its constant moment of inertia by EI, its foundation modulus by K, its shear modulus by G, its transverse deflection by $\psi(x, t)$, its steady mass per unit length by μ, its constant axial force by N, its spatial coordinate along the beam's axis by x, and its time variable by t.

If the inertia effect of the moving load is taken into consideration, the transverse load $P(x, t)$ is expressed as [20]

$$P(x,t)=P_f(x,t)\left[1-\frac{1}{g}\frac{d^2 W(x,t)}{\mathrm{dt}^2}\right] \tag{2}$$

where the continuous moving force $P_f(x, t)$ acting on the beam model is given as

$$P_f(x,t)=\mathrm{Mg}\delta(x-\mathrm{ct}) \tag{3}$$

where M and c are the mass and the speed of the moving load, respectively, g is the acceleration due to gravity, and $\frac{d^2}{\mathrm{dt}^2}$ is a convective acceleration operation defined as

$$\frac{d^2 W}{\mathrm{dt}^2}=\frac{\partial^2 W}{\partial t^2}+\frac{2c\partial^2 W}{\partial x\partial t}+\frac{c^2\partial^2 W}{\partial x^2} \tag{4}$$

It is assumed that the beam in question is uniform, meaning that its characteristics, including its mass per unit length (μ) and moment of inertia (I), do not change along its span (L). Equation (1) may be rearranged to produce what follows by using Equations (2), (3), and (4).

$$\frac{\mathrm{EI}}{\mu}\frac{\partial^4 W(x,t)}{\partial x^4}+\frac{\partial^2 W(x,t)}{\partial t^2}-\frac{N}{\mu}\frac{\partial^2 W(x,t)}{\partial x^2}-G\frac{\partial^2 W(x,t)}{\partial x^2}+\frac{k}{\mu}W(x,t)$$
$$\frac{-M}{\mu}\delta(x-\mathrm{ct})\left[\frac{\partial^2 W(x,t)}{\partial t^2}+\frac{2c\partial^2 W(x,t)}{\partial x\partial t}+\frac{c^2\partial^2 W(x,t)}{\partial x^2}\right]=\frac{\mathrm{Mg}}{\mu}\delta(x-\mathrm{ct}) \tag{5}$$

Clamed-clamped and clamed-free are the bordering conditions of the structures in question, and the beginning conditions are taken below without sacrificing generality:

$$W(x,t) = 0 = \frac{\partial W(x,t)}{\partial t} \tag{6}$$

SOLUTION PROCEDURES

To tackle the initial boundary value problem, a generic method has been provided in this section. The generalized finite integral transform (1) is used to reduce the modified version of the fourth order partial differential equation above after the Dirac-delta signal has been expressed as a Fourier cosine series. The modified Struble's asymptotic method is then used to simplify the resultant pair changed differential equation.

The generalized integral transform is defined by
with inverse

$$W(x,t) = \sum_{m=1}^{\infty} \frac{\mu}{Q_m} W(m,t) V_m(x) \tag{7}$$

where

$$Q_m = \int \mu V_m^2(x)\text{dx} \tag{8}$$

Q_m (x) is any function specified so that the clamped-clamped boundary conditions are met, and it implements the generalized integral transform. Beam mode shapes are a suitable choice of functions for beam difficulties.

$$V_m(x) = \text{Sin}\frac{\alpha_m x}{L} + A_m \text{Cos}\frac{\alpha_m x}{L} + B_m \text{Sinh}\frac{\alpha_m x}{L} + C_m \text{Cosh}\frac{\alpha_m x}{L} \tag{9}$$

OPERATIONAL SIMPLIFICATION

Equation (10) emerged by utilizing the generalized finite integral reduce in (1).

$$I_l R_l(0,L,t) + I_l R_l(t) + W_{tt}(m,t) + I_2 R_2(t) + I_3 W(m,t)$$
$$+ R_3(t) + R_4(t) + R_5(t) = \frac{\text{Mg}}{\mu} R_6(t) \tag{10}$$

where

$$I_1 = \frac{\text{EI}}{\mu}, I_2 = \frac{N+G}{\mu}, I_3 = \frac{K}{\mu} \tag{11}$$

$$\text{R}_1(0,\text{L},\text{t}) = \left[\frac{\partial^2 W(x,t)}{\partial x^2} V_m(x) + \frac{\partial^2 W(x,t)}{\partial x^2} V_m^1(x) + \frac{\partial W(x,t)}{\partial x} V_m^{11}(x) - V(x,t) V_m^{111}(x)\right]_0^L \tag{12}$$

$$R1(t) = \int 0L(Wxx(x,t)Vm(x))dx \tag{13}$$

$$R2(t) = \int 0LW(x,t)Vm(x)dx \tag{14}$$

$$R3(t) = CdtdW \tag{15}$$

$$R4(t) = Fext(t) \tag{16}$$

$$R5(t) = \int 0LP(x,t)dx \tag{17}$$

and

$$R6(t) = MWtt(t) \tag{18}$$

In order to evaluate the integrals (13) to (18), use is made of the property of the Dirac-delta function as an even function to express it in Fourier cosine series, namely:

$$\delta(x - \text{ct}) = \frac{1}{L} + \sum_{n=1}^{\infty} \text{Cos}\frac{n\pi\text{ct}}{L} \text{Cos}\frac{n\pi x}{L} \tag{19}$$

In view of (7), using Equation (19) in Equation (10), after some simplification and rearrangement one obtains

$+ \Gamma$k,t)

$$+2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\mathrm{Cos}\frac{n\pi\mathrm{ct}}{L}W_{\mathrm{tt}}(k,t)R_c(n,k,m)+2v\sum_{k=1}^{\infty}W_t(k,t)R_d(k,m)$$

$$+4c\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\mathrm{Cos}\frac{n\pi\mathrm{ct}}{L}W_t(k,t)R_e(n,k,m)+c^2\sum_{k=1}^{\infty}W(k,t)R_a(k,m)$$

$$+2c^2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\mathrm{Cos}\frac{n\pi\mathrm{ct}}{L}W(k,t)R_f(n,k,m)=\frac{p}{\mu}$$

$$+B_m\mathrm{Sinh}\frac{\alpha_m\mathrm{ct}}{L}+C_m\mathrm{Cosh}\frac{\alpha_m\mathrm{ct}}{L} \tag{20}$$

where

$$\eta=\frac{1}{\mu}(N+G), \Gamma=\frac{M}{L\mu}, p=\mathrm{Mg} \tag{21}$$

Equation (15) is the transformed equation governing the problem of a uniform elastic beam resting on bi-parametric foundation.

In what follows, two special cases of Equation (22) are considered.

SOLUTION OF THE TRANSFORMED GOVERNING EQUATION

Case 1: Uniform Elastic Beam Traversed by a Moving Force

By setting Γ = 0, Equation (15) may be used to generate the differential equation that describes the response of a uniform clamped-clamped elastic beam sitting on a foundation with two parameters subject to a moving force traveling at constant velocity. In this particular situation, one gets

$$W_{\mathrm{tt}}(m,t)+\left(\omega_m^2+\frac{k}{\mu}\right)W(m,t)-\eta\sum_{k=0}^{\infty}W(k,t)R_a(k,m)$$

$$\frac{p}{\mu}\left[\mathrm{Sin}\frac{\alpha_m\mathrm{ct}}{L}+A_m\mathrm{Cos}\frac{\alpha_m\mathrm{ct}}{L}+B_m\mathrm{Sinh}\frac{\alpha_m\mathrm{ct}}{L}+C_m\mathrm{Cosh}\frac{\alpha_m\mathrm{ct}}{L}\right] \tag{23}$$

It is clear that Equation (23) cannot be solved theoretically. An approximation analytical approach is used even if the equation lends relatively easily to numerical approaches since the solutions so obtained frequently reveal important details about the vibrating mechanism. To do this, Equation (24) may be rewritten to assume the following form by applying a variation of the asymptotic approach due to Struble.

$$k \neq m$$
$$\infty$$

$$\frac{p}{\mu}\left[\mathrm{Sin}\frac{\alpha_m\mathrm{ct}}{L}+A_m\mathrm{Cos}\frac{\alpha_m\mathrm{ct}}{L}+B_m\mathrm{Sinh}\frac{\alpha_m\mathrm{ct}}{L}+C_m\mathrm{Cosh}\frac{\alpha_m\mathrm{ct}}{L}\right] \tag{24}$$

where

$$\omega_{\mathrm{mf}}^2=\omega_m^2+\frac{k}{\mu}, \eta=\frac{1}{\mu}(N+G), R_a(m,m)=R_a(k,m) \quad {}_{k=m} \tag{25}$$

Equation (24) is then substituted with an analogous free system amplifier defined by adjusted frequency. Therefore, we set (24)'s right-hand side to zero and take into account a parameter α < 1 for any arbitrary ratio that has been defined as

$$\alpha=\frac{\eta}{1+\eta}$$

So that

$$\eta=\alpha+0(\alpha^2) \tag{26}$$

Substituting Equation (26) into the homogeneous part of Equation (24), one obtains

$$k \neq m \tag{27}$$
$$\infty$$

where

$R_a(m,m) = R_a(k,m)/_k = m$ (28)

The solution to Equation (27) may be formulated as follows when we set α = 0, which corresponds to the scenario whenever the mass of the system's inertia effect is ignored:

$$W(m,t) = C_0 \text{Cos}(\omega_{\text{mf}} t - D_0) \quad (29)$$

where, C_0 and D_0 are constants. Furthermore, as Struble's technique requires that the asymptotic solution of the homogeneous part of Equation (30) be of the form

$$W(m,t) = \Lambda(m,t)\cos[\omega_{\text{mf}} t - \beta(m,t)] + \alpha\beta_1 + 0(\alpha^2) \quad (30)$$

where $\Lambda(m, t)$ and $\beta(m,t)$ are slowly varying functions of time.

To obtain the modified frequency, Equation (30) and its derivatives are substituted into Equation (31) and one obtains

$$2(m,t)\beta(m,t)\omega_{\text{mf}} - 2(m,t)\omega_{\text{mf}}\text{Sin}[\omega_{\text{mf}} t - \beta(m,t)] - \sum_{k=1}^{\infty} \lambda T_a(m,t) \wedge (m,t)\text{Cos}[\omega_{\text{mf}} t - \beta(m,t)] = 0 \quad (31)$$

retaining term to (α) only.

By setting the corresponding coefficients of the "Sin" [ω_{mf} t – β(m,t)] and Cos [ω_mf t – β(m,t)] terms on both sides of Equation (32) to zero, the variational equations are created.
Thus

$$-2\,\Lambda\,(\text{m, t})\,\omega_{\text{mf}} = 0 \quad (32)$$

and

$$-\alpha R_a(m,m)\Lambda(m,t) + 2\Lambda(m,t)\beta(m,t)\omega_{\text{mf}} = 0 \quad (33)$$

Solving Equations (34) and (35), respectively, gives

$$\Lambda(m,t) = C^a \quad (34)$$

and

$$\beta(m,t) = \frac{\alpha R_a(m,m)}{2\omega_{\text{mf}}^2} t + C^b \quad (35)$$

Where C^a and C^b are constants.

Therefore, when the effect of the force of the particle is considered, the first approximation to the homogeneous system is

$$W(m,t) = C^a \text{Cos}[\gamma_{\text{mf}} t - \beta_m] \quad (36)$$

where

$$\gamma_{\text{mf}} = \omega_{\text{mf}} \left[1 - \frac{\alpha R_a(m,m)}{2\omega_{\text{mf}}^2}\right] \quad (37)$$

represents the frequency of the free system as the outcome of the moving force's presence and is known as the modified natural frequency. Therefore, the analogous free system operator defined by the changed frequency γ_"mf" is used to solve the non-homogeneous formula (37) in place of the differential operator acting on $W(m, t)$ and $W(k, t)$.

Using Equation (38), the homogeneous part of Equation (37) can be written as

$$\frac{d^2 W(m,t)}{\text{dt}^2} + \gamma_{\text{mf}}^2 W(m,t) = 0 \quad (38)$$

Hence, the entire Equation (4.2) takes the form

$$\frac{d^2W(m,t)}{\mathrm{dt}^2} + \gamma_{\mathrm{mf}}^2 W(m,t)$$
$$\frac{P}{\mu}\left[\mathrm{Sin}\frac{\alpha_m \mathrm{vt}}{L} + A_m \mathrm{Cos}\frac{\alpha_m \mathrm{vt}}{L} + B_m \mathrm{Sinh}\frac{\alpha_m \mathrm{vt}}{L} + C_m \mathrm{Cosh}\frac{\alpha_m \mathrm{vt}}{L}\right] \tag{39}$$

Solving Equation (39) in conjunction with the initial Condition, one obtains

$$W(m,t) = \frac{P}{\mu\gamma_{\mathrm{mf}}}\left\{\frac{\gamma_{\mathrm{mf}}\mathrm{Sin}\varepsilon_c t - \varepsilon_c \mathrm{Sin}\gamma_{\mathrm{mf}} t}{\gamma_{\mathrm{mf}}^2 - \varepsilon_c^2} + \frac{A_m \gamma_{\mathrm{mf}}(\mathrm{Cosh}\varepsilon_c t + \mathrm{Cos}\gamma_{\mathrm{mf}} t)}{\gamma_{\mathrm{mf}}^2 - \varepsilon_c^2}\right.$$
$$\left.\frac{+B_m(\gamma_{\mathrm{mf}}\mathrm{Sinh}\varepsilon_c t - \varepsilon_c \mathrm{Sin}\gamma_{\mathrm{mf}} t)}{\gamma_{\mathrm{mf}}^2 + \varepsilon_c^2} + \frac{C_m \gamma_{\mathrm{mf}}(\mathrm{Cosh}\varepsilon_c t + \mathrm{Cos}\gamma_{\mathrm{mf}} t)}{\gamma_{\mathrm{mf}}^2 + \varepsilon_c^2}\right\} \tag{40}$$

By taking inversion of (4.19), one obtains

$$W(x,t) = \frac{1}{\alpha_m(x)}\sum_{m=1}^{\infty}\frac{P}{\mu\gamma_{\mathrm{mf}}}\left\{\frac{\gamma_{\mathrm{mf}}\mathrm{Sin}\varepsilon_c t - \varepsilon_c \mathrm{Sin}\gamma_{\mathrm{mf}} t}{\gamma_{\mathrm{mf}}^2 - \varepsilon_c^2} + \frac{A_m \gamma_{\mathrm{mf}}(\mathrm{Cos}\varepsilon_c t - \mathrm{Cos}\gamma_{\mathrm{mf}} t)}{\gamma_{\mathrm{mf}}^2 - \Sigma}\right.$$
$$\left.\frac{+B_m(\gamma_{\mathrm{mf}}\mathrm{Sinh}\varepsilon_c t - \varepsilon_c \mathrm{Sin}\gamma_{\mathrm{mf}} t)}{\gamma_{\mathrm{mf}}^2 + \varepsilon_c^2} + \frac{\gamma_{\mathrm{mf}}\mathrm{Sin}\varepsilon_c t - \varepsilon_c \mathrm{Sin}\gamma_{\mathrm{mf}} t}{\gamma_{\mathrm{mf}}^2 + \varepsilon_c^2}\right\}$$
$$\left\{Sin\frac{\alpha_m x}{L} + A_m Cos\frac{\alpha_m x}{L} + B_m Sinh\frac{\alpha_m x}{L} + C_m Cosh\frac{\alpha_m x}{L}\right\} \tag{41}$$

where

$$\varepsilon_c = \frac{\alpha_m c}{L}\,\mathrm{and}\,\alpha_m(x) = \int V_m^2(x)\lambda x \tag{42}$$

Equation (41) depicts the transverse displacement action of a uniform elastic beam sitting on a bi-parameter foundation to a moving focused force moving at a constant frequency.

Case 2: Uniform Elastic Beam Traversed by a Moving Mass

When the mass of the moving load and the structure are the same, the moving load's inertia impact is not insignificant. Therefore, in this instance, Γ≠0, is needed. This phenomenon is referred to as the issue of moving masses. In order to achieve a hypothetical analytical solution, the modified Struble's asymptotic approach is utilized; we ignored the terms that indicate the inertia effect of the moving mass, and the result is Equation (44).

$$W_{\mathrm{tt}}(m,t) + \gamma_{\mathrm{mf}}^2 W(m,t) + \Gamma + 2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty} \mathrm{Cos}\frac{n\pi \mathrm{ct}}{L} W_{\mathrm{tt}}(k,t) R_c(n,k,m)$$
$$+2c\sum_{k=1}^{\infty} W_t(k,t) R_d(k,m) + 4c\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\mathrm{Cos}\frac{n\pi \mathrm{ct}}{L} W_t(k,t) R_e(n,k,m)$$
$$+2c^2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\mathrm{Cos}\frac{n\pi \mathrm{ct}}{L} W(k,t) R_f(n,k,m)$$
$$\frac{P}{\mu}\left\{\mathrm{Sin}\frac{\alpha_m \mathrm{ct}}{L} + A_m \mathrm{Cos}\frac{\alpha_m \mathrm{ct}}{L} + B_m \mathrm{Sinh}\frac{\alpha_m \mathrm{ct}}{L} + C_m \mathrm{Cosh}\frac{\alpha_m \mathrm{ct}}{L}\right\} \tag{43}$$

$$\chi_m = \frac{2\gamma_{\mathrm{mf}}^2 - \Gamma_1\left(\gamma_{\mathrm{mf}}^2 R_b(m,m) - c^2 R_a(m,m)\right)}{2\gamma_{\mathrm{mf}}} \tag{44}$$

where

$\Gamma_1 = \frac{\Gamma}{\Gamma+1}$, $R_a(m,m) = R_a(k,m)/_k = m$ and $R_b(m,m) = R_b(k,m)/_k = m$

retaining to $0(\Gamma_1)$ only.

Thus, Equation (45) takes the form

$$+B_m \mathrm{Sinh}\frac{\alpha_m \mathrm{ct}}{L} + C_m \mathrm{Cosh}\frac{\alpha_m \mathrm{ct}}{L} \tag{45}$$

It is noticed that Equation (45) is analogous to Equation (39). We thus use argument as in case 1 to obtain solution to (45) as

$$W(m,t) = \frac{\Gamma_1 \mathrm{Lg}}{\chi_m} + \frac{A_m\chi_m(\mathrm{Cos}\varepsilon_c t - \mathrm{Cos}\chi_m t)}{\chi_m^2 - \varepsilon_c^2} + \frac{B_m(\chi_m \mathrm{Sinh}\varepsilon_c t - \varepsilon_c \mathrm{Sin}\chi_m t)}{\chi_m^2 + \varepsilon_c^2} + \frac{C_m\chi_m(\mathrm{Cosh}\varepsilon_c t + \mathrm{Cos}\chi_m t)}{\chi_m^2 + \varepsilon_c^2} \quad (46)$$

which on inversion becomes

$$\psi(x,t) = \sum_{m=1}^{\infty} \frac{\Gamma_1 \mathrm{Lg}}{\alpha_m(x)\chi_m}$$
$$+\frac{A_m\chi_m(\mathrm{Cos}\varepsilon_c t - \mathrm{Cos}\chi_m t)}{\chi_m^2 - \varepsilon_c^2} + \frac{B_m(\chi_m \mathrm{Sin}\varepsilon_c t - \varepsilon \mathrm{ctSin}\chi_m t)}{\chi_m^2 - \varepsilon_c^2}$$
$$+\frac{C_m\chi_m(\mathrm{Cosh}\varepsilon_c t - \mathrm{Cos}\chi_m t)}{\chi_m^2 - \varepsilon_c^2}$$
$$+B_m \mathrm{Sinh}\frac{\alpha_m x}{L} + C_m \mathrm{Cosh}\frac{\alpha_m x}{L} \quad (47)$$

Equation (47) represents the transerse response to a moving mass at constant speed of a uniform elastic beam resting on bi-parameters foundation.

APPLICATIONS

In this section, the forgoing analyses are illustrated by clamped-clamped boundary conditions.

Clamped-Clamped End Conditions

At a clamped-clamped ends, both deflection and slope vanish. Thus, when the beam is clamped at $x = 0$ and $x = L$, the conditions are expressed as

$$\psi(0,t) = 0 = \psi(L,t), \frac{\partial\psi(0,t)}{\partial x} = 0 = \frac{\partial\psi(L,t)}{\partial x} \quad (48)$$

and for normal modes

$$V_m(0) = 0 = V_m(L), \frac{\partial V_m(0)}{\partial x} = 0 = \frac{\partial V_m(L)}{\partial x} \quad (49)$$

which implies that

$$V_k(0) = 0 = V_k(L), \frac{\partial V_k(0)}{\partial x} = 0 = \frac{\partial V_k(L)}{\partial x} \quad (50)$$

Thus, it can be shown that

$$A_m = \frac{\mathrm{Sin}\lambda_m - \mathrm{Sinh}\lambda_m}{\mathrm{Cos}\lambda_m - \mathrm{Cosh}\lambda_m} = -C_m \text{ and } B_m = -1 \quad (51)$$

$$A_k = \frac{\mathrm{Sinh}\alpha_k - \mathrm{Sin}\alpha_k}{\mathrm{Cos}\alpha_k + \mathrm{Cosh}\alpha_k} = -C_k \text{ and } B_k = -1 \quad (52)$$

The frequency equation is given by;

$$\mathrm{Cos}\alpha_m \mathrm{Cosh}\alpha_m = 1 \quad (53)$$

Such that

$$\alpha_1 = 4073004, \alpha_2 = 7.85320, \alpha_3 = 10.99561 \quad (54)$$

Clamped-Free End Conditions

At end $x = 0$, the beam is taking to be clamped and at end $x = \mathrm{L}$, the beam is free. Thus, the boundary conditions of the Bernoulli-Euler beam can be written as

$$\psi(0,t) = 0 = \frac{\partial\psi(0,t)}{\partial x}, \frac{\partial^2\psi(L,t)}{\partial x^2} = 0 = \frac{\partial^3\psi(L,t)}{\partial x^3} \quad (55)$$

and for normal modes

$$V_m(0) = 0 = \frac{\partial V_m(0)}{\partial x}, \frac{\partial^2 V_m(L)}{\partial x^2} = 0 = \frac{\partial^3 V_m(L)}{\partial x^3} \quad (56)$$

which implies that

$$V_k(0) = 0 = \frac{\partial V_k(0)}{\partial x}, \frac{\partial^2 V_K(L)}{\partial x^2} = 0 = \frac{\partial^3 V_k(L)}{\partial x^3} \quad (57)$$

Thus, it can be shown that

$$A_m = \frac{\text{Sin}\lambda_m - \text{Sinh}\lambda_m}{\text{Cosh}\lambda_m - \text{Cos}\lambda_m} = -C_m \text{ and } B_m = -1 \quad (58)$$

$$A_k = \frac{\text{Sin}\lambda_k - \text{Sinh}\lambda_k}{\text{Cosh}\lambda_k - \text{Cos}\lambda_k} = -C_k \text{ and } B_k = -1 \quad (59)$$

and the frequency equation for both end conditions is

$$\text{Cosh}\lambda_m \text{Cos}\lambda_m = -1 \quad (60)$$

Such that

$$\lambda_1 = 1.875, \lambda_2 = 4.694, \lambda_3 = 7.855 \quad (61)$$

Using (58), (59), and (61) in Equations (41) and (47), respectively, one obtains the displacement response to a moving force and a moving mass of a uniform clamped-free ends of elastic beam resting on bi-parametric sub-grades.

Comments on Closed Form Solution

In dynamical systems like this, the response intensity can increase without a bond. Resonance conditions are the circumstances in which this occurs. Establishing the conditions under which resonance exists is crucial at this point. Researchers and design engineers are quite concerned about this problem in structural roadway engineering. The clamped-clamped elastic beam supported by a bi-parametric foundation and driven by a moving force obviously exhibits a condition of resonance whenever

$$\gamma_{\text{mf}} = \frac{k\pi c}{L} \quad (62)$$

According to Equation (47), however, an equivalent elastic beam that a moving mass passes through produces a resonance effect at

$$\chi_m = \frac{k\pi c}{L} \quad (63)$$

$$\gamma_{\text{mf}} = \frac{2\gamma_{\text{mf}}^2 - \Gamma_1\left(\gamma_{\text{mf}}^2 R_b(m,m) - c^2 R_a(m,m)\right)}{2\frac{k\pi c}{L}} \quad (64)$$

NUMERICAL CALCULATIONS AND DISCUSSIONS OF RESULTS

The value of axial force *N* is varied between 0 N and 2.0×10^8 N, the values of the shear modulus (*G*) varied between 0 N/m^3 and 900 000 N/m^3. Figures 1(a) and (b), respectively, display the transverse displacement response to a moving force of clamped-clamped and camped-free uniform elastic beams for various values of axial force and for fixed value of shear modulus *G* and foundation modulus *K*. The graphs demonstrate that, given fixed values of ground stiffness *K* and shear modulus *G*, the response amplitudes grow as the axial force magnitude decreases. For fixed levels of shear moduli *G*, axial force *N*, and varying amounts of foundation moduli *K*, Figures 2(a) and (b), respectively, also display the deflection profiles resulting from the moving force of clamped-clamped and camped-free uniform elastic beams. The plots suggest that as the foundation moduli *K* are increased, the beam's response amplitudes drop. The deflection profiles of clamped-clamped and camped-free uniform elastic beams are displayed in Figures 3(a) and (b), respectively, for fixed values for the axial force *N* and foundation modulus *K* and for a range of shear modulus *G* values.

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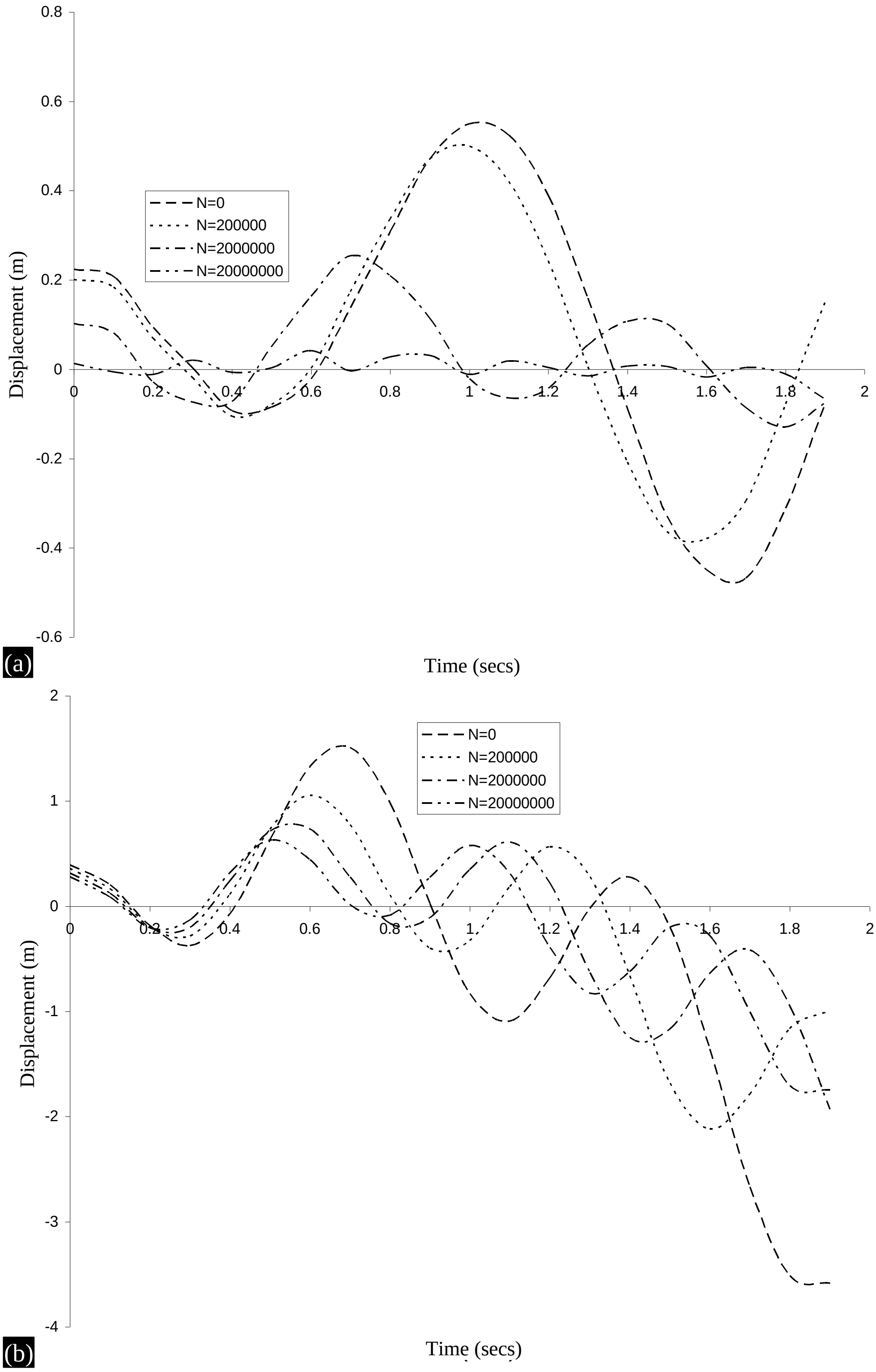
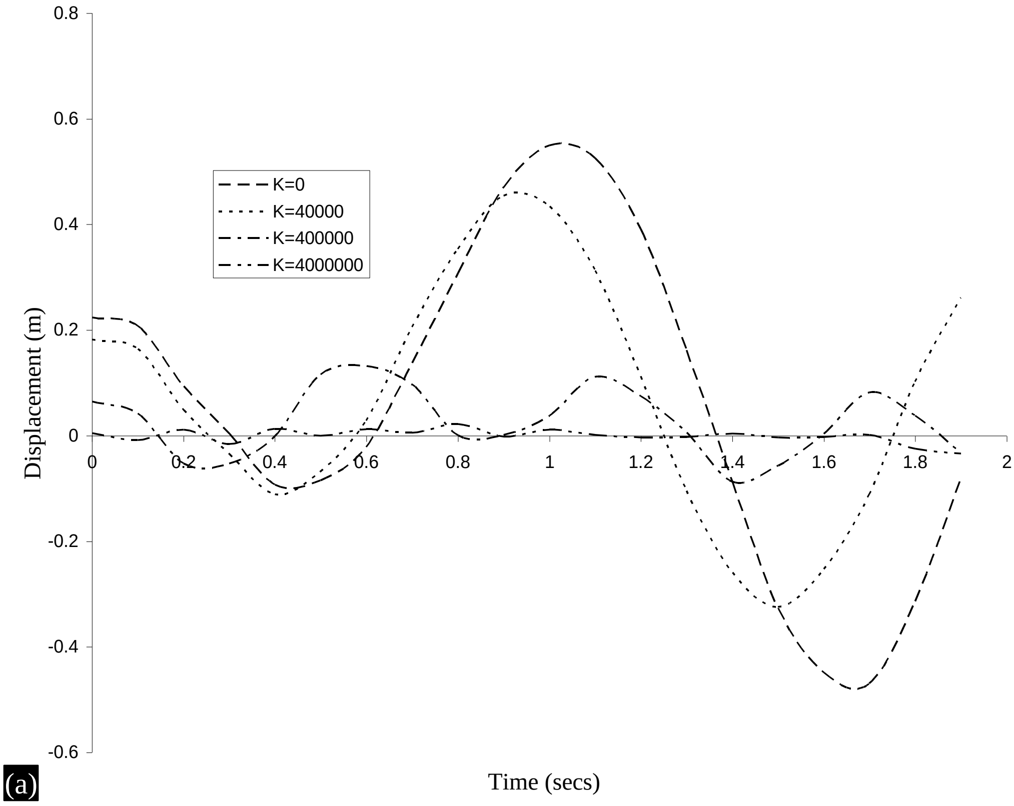
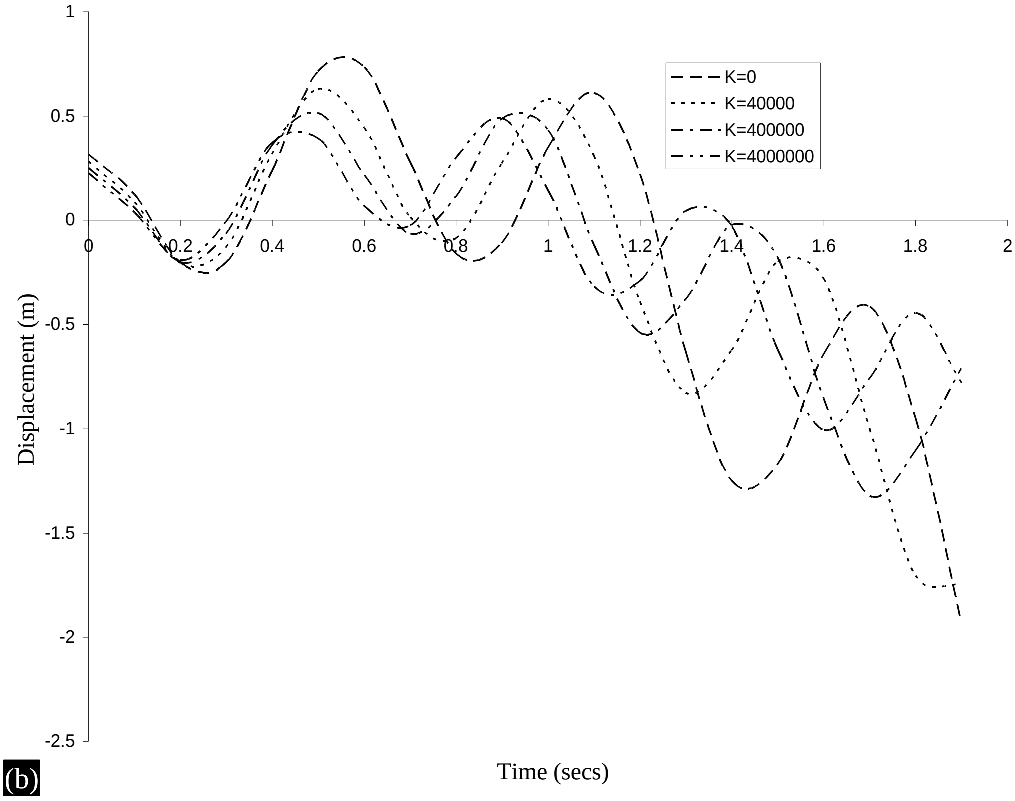


Figure 1. (a) Deflection profile of a clamped-clamped uniform elastic beam under moving force for fixed values of shear modulus (G = 90000), foundation modulus (K = 40000), and various values of axial force (N). (b) Deflection profile of a clamped-free uniform elastic beam under moving force for fixed values of shear modulus (G = 90000), foundation modulus (K = 40000), and various values of axial force (N).

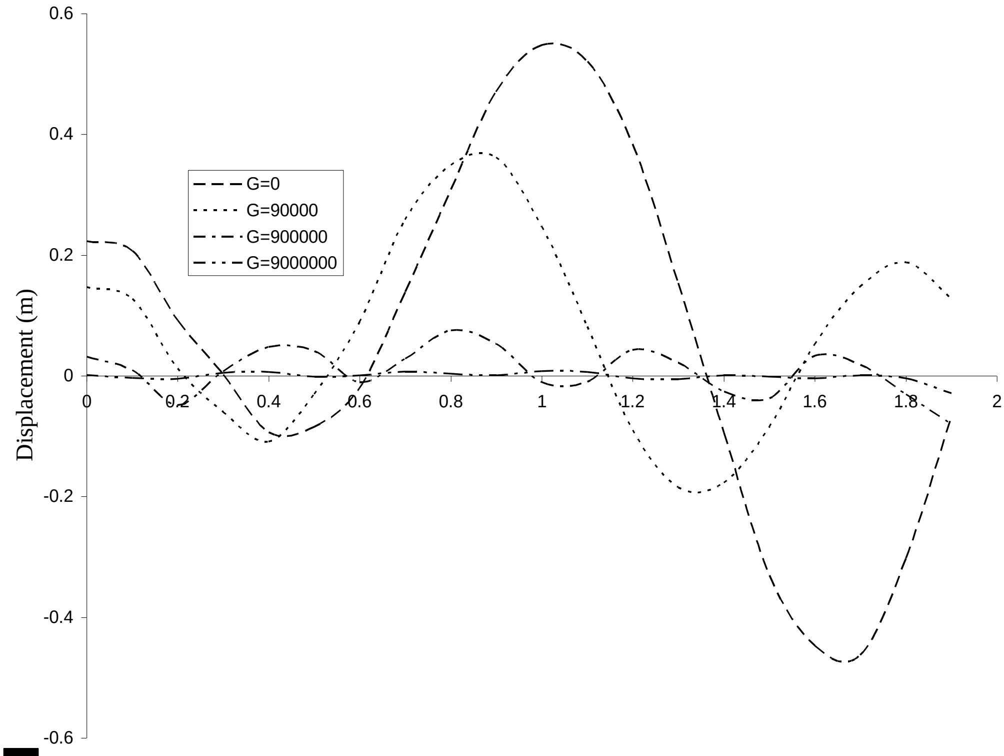


(a)

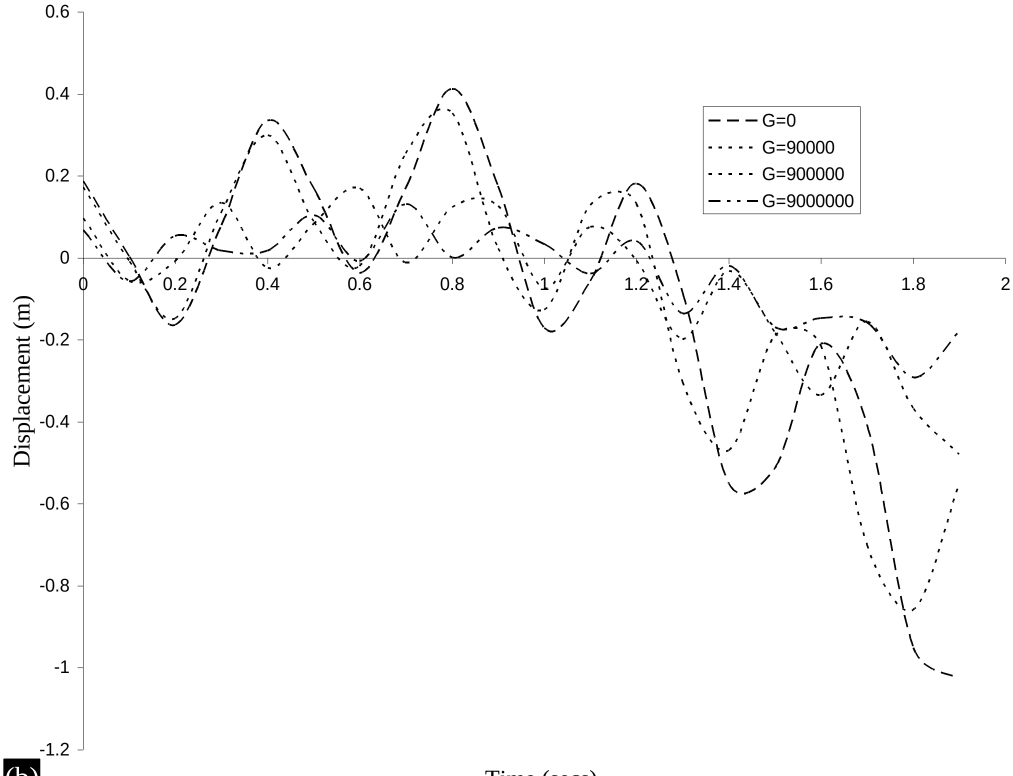


(b)

Figure 2. (a) Deflection profile of a clamped-clamped uniform elastic beam under moving force for fixed values of shear modulus (G = 90000), axial force (N = 40000), and various values of foundation modulus (K). (b) Deflection profile of a clamped-free uniform elastic beam under moving force for fixed values of shear modulus (G = 90000), axial force (N = 40000), and various values of foundation modulus (K).



(a)



(b)

Figure 3. (a) Deflection profile of the clamped-clamped uniform elastic beam under a moving force for various values of shear modulus *G*, and fixed values of axial force *N* (20000), and foundation moduli *K* (400000). (b) Deflection profile of the clamped-free uniform elastic beam under a moving force for various values of shear modulus *G*, and fixed values of axial force *N* (20000), and foundation moduli *K* (400000).

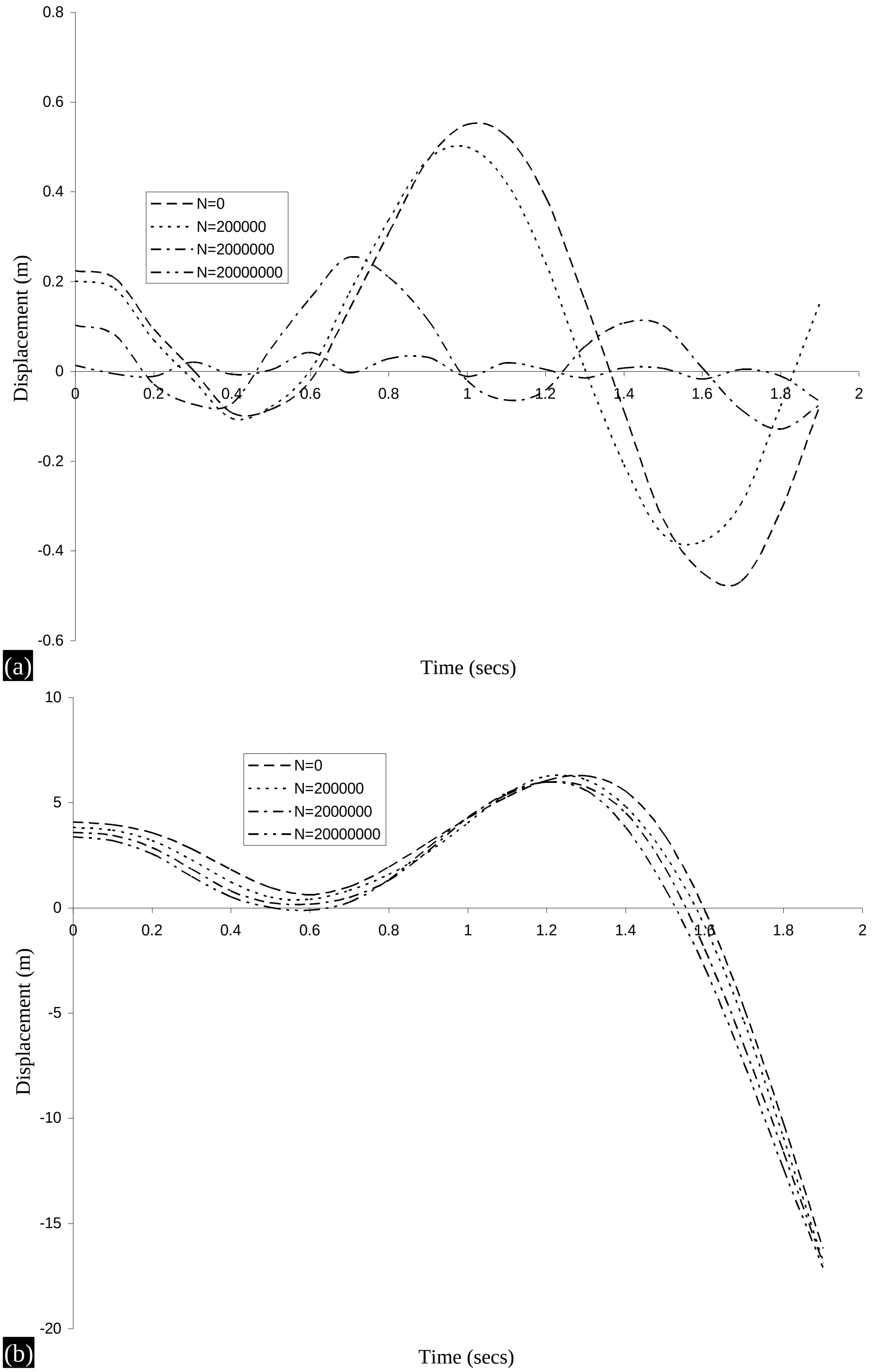


Figure 4. (a) Deflection profile of a clamped-clamped uniform elastic beam under moving mass for fixed values of shear modulus (G = 90000), foundation modulus (K = 40000), and various values of axial force (N). (b) Deflection profile of a clamped-free uniform elastic beam under moving mass for fixed values of shear modulus (G = 90000), foundation modulus (K = 40000), and various values of axial force (N).

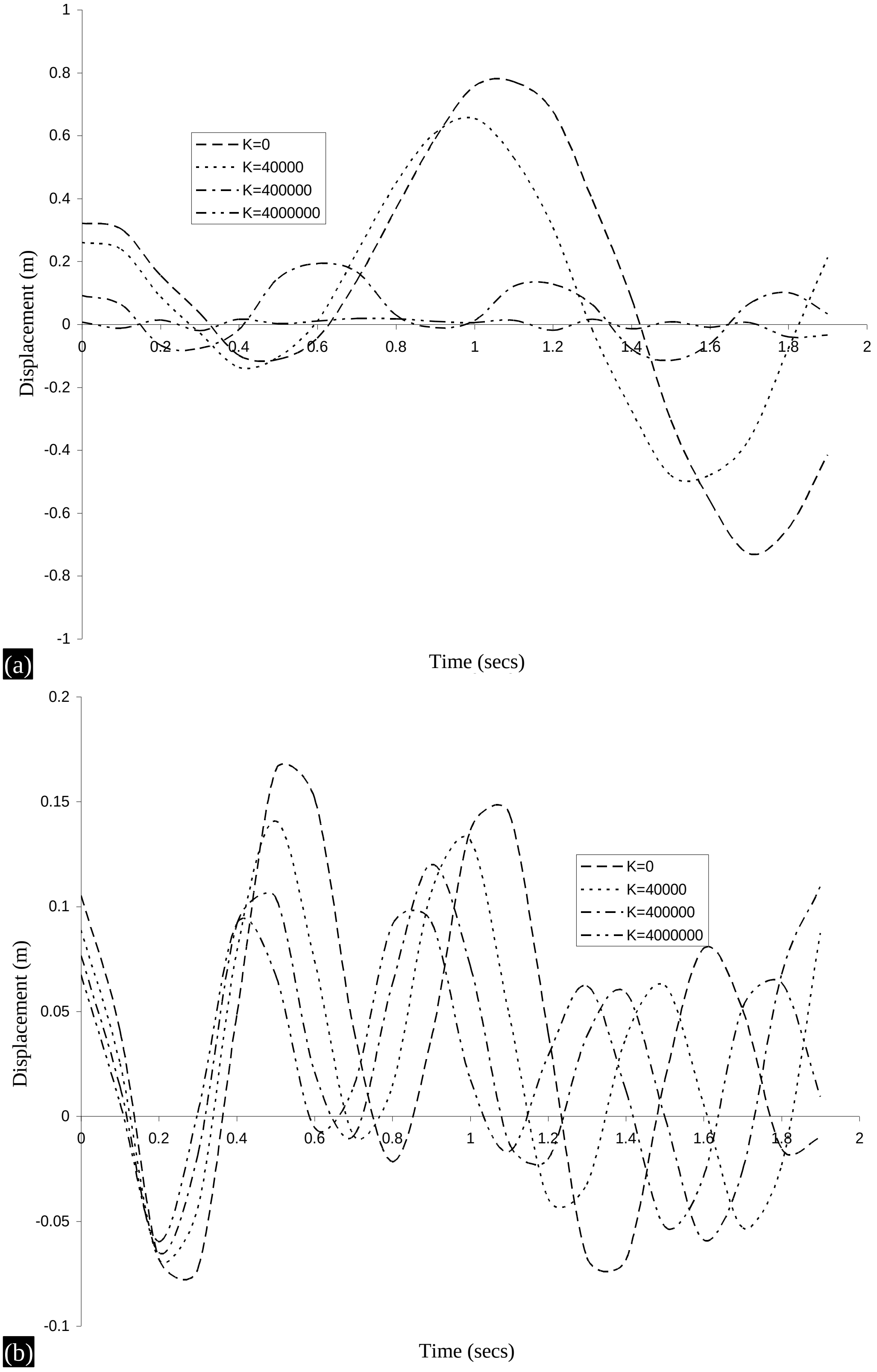


Figure 5. (a) Deflection profile of a clamped-clamped uniform elastic beam under moving mass for fixed values of shear modulus (*G* = 90000), axial force (*N* = 40000), and various values of foundation modulus (*K*). (b) Deflection profile of a clamped-free uniform elastic beam under moving mass for fixed values of shear modulus (*G* = 90000), axial force (*N* = 40000), and various values of foundation modulus (*K*).

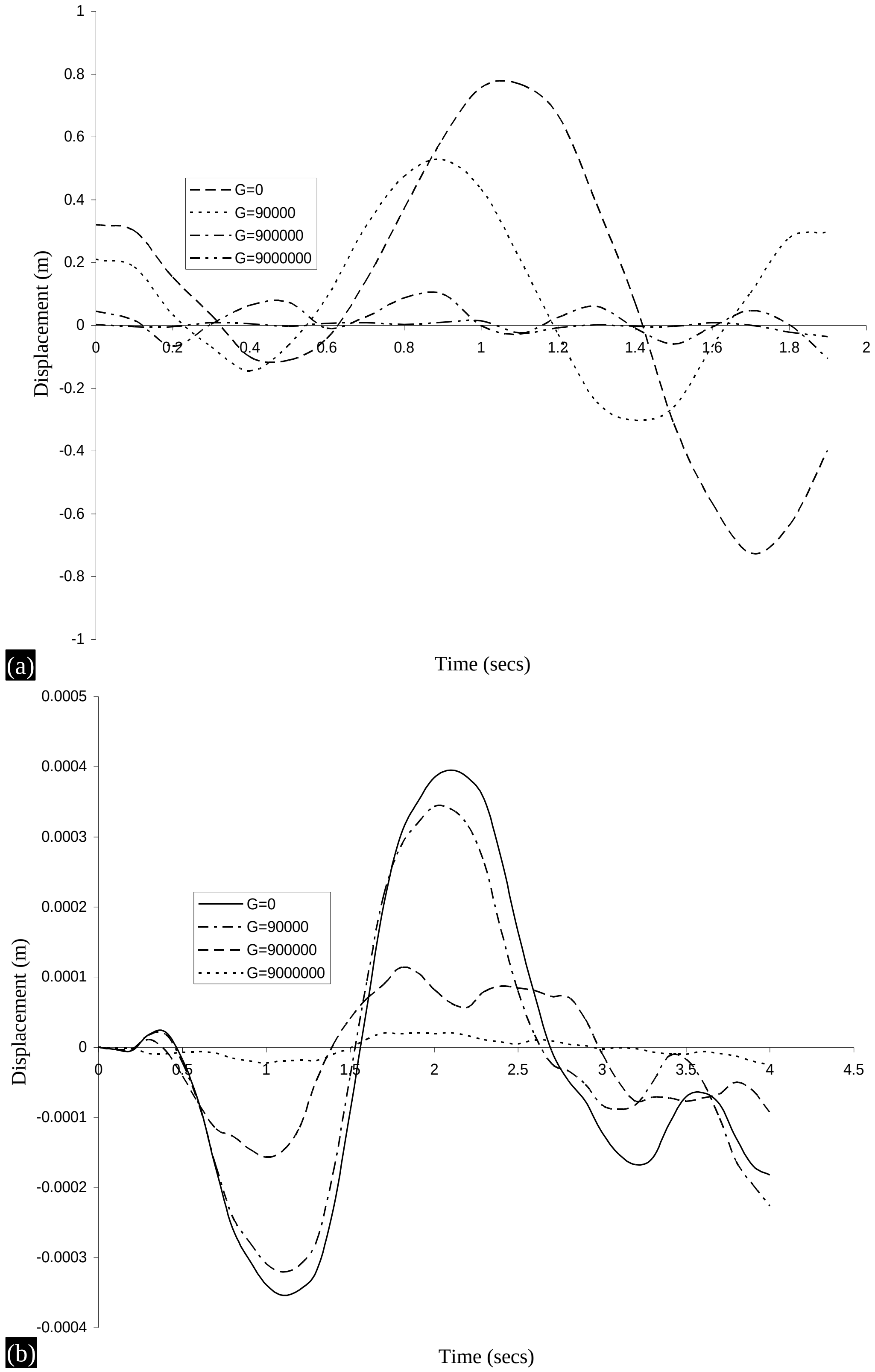
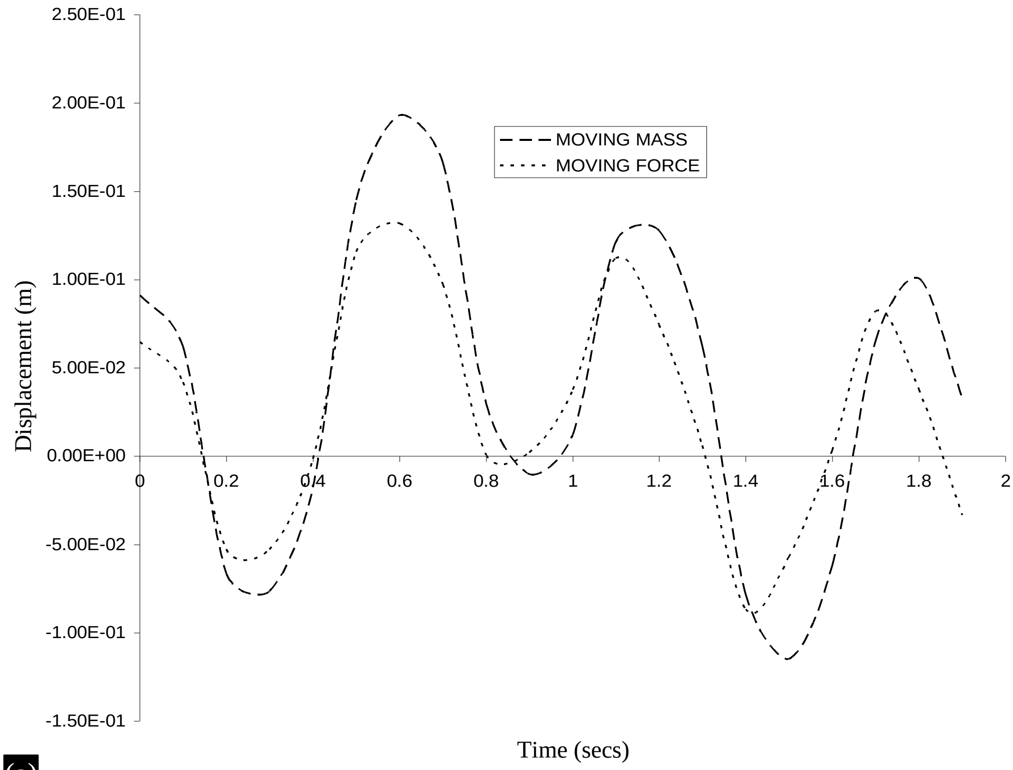
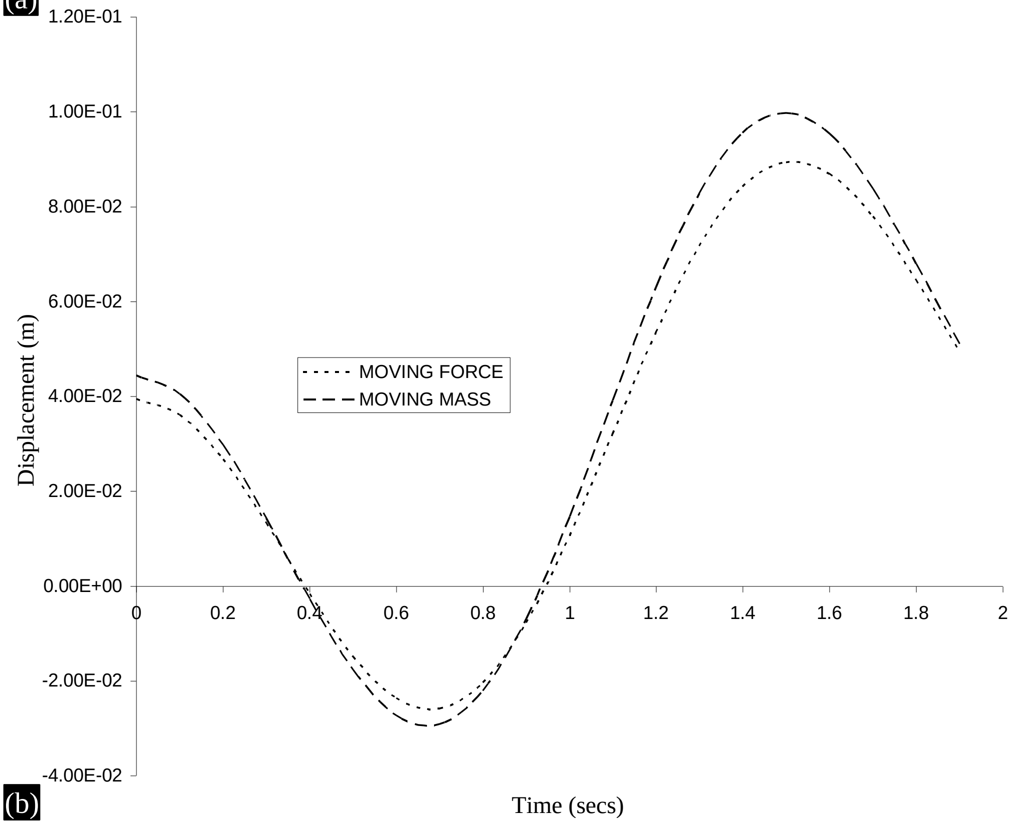


Figure 6. (a) Deflection profile of the clamped-clamped uniform elastic beam under a moving mass for various values of shear modulus *G*, and fixed values of axial force *N* (20000) and foundation modulus *K* (400000). (b) Deflection profile of the clamped-free uniform elastic beam under a moving mass for various values of shear modulus *G*, and fixed values of axial force *N* (20000) and foundation modulus *K* (400000).



(a)



(b)

Figure 7. (a) Comparison of the deflection profile of moving force and moving mass cases of clamped-clamped uniform elastic beam with foundation modulus (K = 400000), shear modulus (G = 90000), and axial force (N = 20000). (b) Comparison of the deflection profile of moving force and moving mass cases of clamped-free uniform elastic beam with foundation modulus (K = 400000), shear modulus (G = 90000), and axial force (N = 20000).

The plots demonstrate that when the shearing modulus increases, the beam's response amplitudes decrease. The deflection profiles caused by moving mass of the uniform clamped-clamped and camped-free elastic beam appear in Figures 4(a) and (b), respectively, for fixed values of foundation modulus *K* and shear modulus *G* as well as for a range of axial force *N* values. The graph indicates that the response intensity of the beam decreases with increasing axial force levels.

Also, Figures 5(a) and (b) are the corresponding curves due to moving masses of the uniform clamped-clamped and camped-free beams. Figure 5(a) shows the transverse displacement response due to concentrated masses of the clamped-clamped uniform elastic beam.

A clamped-clamped uniform beam's transverse displacement in response to moving concentrated masses is illustrated in Figure 6(a) for a range of shear modulus *G* values, fixed axial *N* force values, and foundation modulus *K* values.

In contrast, Figure 6(b) shows that deflecting profile of the clamped-free beam caused by moving concentrated masses diminishes for rising levels of shear modulus *G* and for constant values of core modulus and axial force.

Figures 7(a) and (b) show the comparison between the moving force problem and the moving mass problem for the clamped-clamped and the clamped-free uniform elastic beams. It was observed that the deflection profile of the uniform clamped-clamped and clamped-free elastic beam traversed by moving mass is higher than that traversed by the moving force. Therefore, compared to the moving forces problem, resonance occurs earlier in the moving mass challenge.

The results also show that, in all the cases considered, the deflection profiles of clamped-clamped uniform elastic beam is higher compare to that of the clamped-free for various values of shear modulus, foundation modulus, and axial force.

CONCLUSION

Comparative study of clamped-clamped and clamped-free uniform elastic beams resting on bi-parametric subgrades is investigated in this paper. The approximate analytical solution technique is based on the generalized finite integral transformation with the beam function as the kernel, Laplace transformation, and convolution theory and finally modification of the Struble's asymptotic method. The same system, which consists of uniform clamped-clamped and clamped-free elastic beams resting on a bi-parameter foundation and traversed by a moving mass, has a critical speed that is lower than that traversed by a moving force, according to analytical and numerical solutions.

Furthermore, an increase in the foundation modulus *K* with fixed values of shear modulus *G* and axial force *N* reduces the amplitudes of vibration of the beam. Additionally, given fixed amounts of axial force and fundamental modulus, the amplitudes of vibration decrease as the shear modulus increases. Additionally, when the axial force is increased but the shear modulus and foundation modulus remain constant, the results indicate that greater shear modulus values are needed for a more pronounced effect than foundation modulus readings. Finally, it was observed that the deflection profiles of clamped-clamped uniform elastic beam are higher compare to that of the clamped-free beam.

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