STM JOURNALS

Recent Trends in
Fluid Mechanics

ISSN: 2455-1961
Volume 12, Issue 2, 2025
May–August
DOI (Journal): 10.37591/RTFM

https://journals.stmjournals.com/rtfm

Research RTFM

Quasi-Static Thermal Stress Analysis in a Thin Circular Cylinder Due to Internal Heat Generation Under Transient Temperature Conditions

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Abstract

This paper is concerned with inverse quasi-static thermal stress analysis in a thin circular cylinder due to internal heat generation under transient temperature conditions. The internal heat generation is modeled as a cylindrical surface heat source located in the annular region along the axial length of the cylinder. This source is placed concentrically inside the cylinder and begins releasing heat spontaneously at a specific point in time. The thin circular cylinder is subjected to a known internal heat flux, while convective heat dissipation occurs through both the upper surface and the circular boundary surface. However, the convective heat flux at the upper surface is not known and must be determined. To achieve this, an inverse thermal analysis is carried out to estimate the unknown heat flux. The problem is formulated as a non-homogeneous boundary value problem under transient thermal conditions. Analytical methods based on integral transforms are employed to derive the solution. The obtained results provide temperature and thermal stress distributions in the cylinder. These results are further illustrated graphically to visualize the effect of internal heat generation and convective boundary conditions. The approach offers a useful method for evaluating unknown thermal parameters in cylindrical structures subjected to complex internal heat sources and boundary heat exchange. This analysis is relevant to practical engineering problems where accurate thermal stress predictions are crucial for assessing material performance and structural integrity under transient thermal loading.

Keywords: Quasi-static, thermal stress, circular cylinder, heat generation, heat flux

INTRODUCTION

The quasi-static thermal stresses in a circular plate with an arbitrary beginning temperature on the upper face and a lower face at zero temperature have been calculated by Wankhede [1]. The two-dimensional transient problem for a thick disk with interior heat sources has been explored by Khalsa *et al.* [2]. Their study focuses on the thermal behavior of a thick disk subjected to internal heat generation over time. The analysis addresses the temperature distribution and heat flow patterns resulting from these sources under transient conditions [3]. This investigation provides insight into thermal responses in such geometries, which is essential for applications involving heat conduction in solid structures with internal energy generation [2]. An inverse quasi-static thermoelastic problem of a thick circular plate has been studied by Ghadle *et al.* [4]. An inverse thermoelastic problem of a thick hollow cylinder of finite length with an internal heat source has been studied by Khobragade *et al.* [5]. The study focuses on determining unknown thermal parameters based on observed thermal and mechanical responses. Using analytical methods, the authors addressed the

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Received Date: May 15, 2025
Accepted Date: June 02, 2025
Published Date: July 10, 2025

Citation: P.H. Munjankar. Quasi-Static Thermal Stress Analysis in a Thin Circular Cylinder Due To Internal Heat Generation Under Transient Temperature Conditions. Recent Trends in Fluid Mechanics. 2025; 12(2): 17–28p.

challenges of heat conduction and thermal stress distribution within the cylinder [6]. This investigation is significant for applications involving internal heating, where accurate thermal analysis is essential for ensuring structural integrity under thermal loads. An inverse thermoelastic problem of a thick hollow cylinder of finite length with an internal heat source was studied by Khobragade *et al.* [5]. The study aims to determine unknown thermal parameters from observed thermal and mechanical responses. Analytical methods were used to solve heat conduction and thermal stress distribution issues.

This research is important for applications involving internal heating, where precise thermal analysis is crucial to maintain structural integrity under varying thermal loads and ensure reliable performance [7]. The analysis of inverse quasi-static thermal stress caused by internal heat generation in a thin circular cylinder under transient temperature conditions is the focus of this paper. The heat source is modeled as a cylindrical surface within the annular region, releasing heat spontaneously. The cylinder is subjected to known internal heat flux, with heat dissipation through its upper and circular surfaces. The unknown convective heat flux at the upper surface is determined using integral transform methods, and the results are presented graphically [8]. The internal heat generation is taken as a cylindrical surface heat source in the annular region of the linear length of the cylinder and is situated concentrically inside the cylinder releases heat spontaneously at the time $t = \varsigma$. The thin circular cylinder is subjected to known internal heat flux and the convection due to dissipation takes place through the upper surface and circular boundary surface [9]. The unknown convective heat flux applied at the upper surface of the cylinder is required to be determined. To achieve this, integral transform methods are employed to obtain the analytical solution of the non-homogeneous boundary value problem [10]. This approach allows for the accurate estimation of the unknown heat flux under transient thermal conditions, enabling the analysis of temperature distribution and thermal stresses in the cylinder. The method is effective for solving inverse thermal problems in cylindrical geometries.

THE PROBLEM'S MATHEMATICAL FORMULATION IS AS FOLLOWS

The heat source, which is a cylindrical surface heat source of strength that is immediate and located concentrically inside the cylinder in an annular zone of linear length, releases heat spontaneously at a period determined by

$$g(r,z,t)=\frac{g_i}{2\pi r}\delta(r-r_1)\delta(z-z_1)\delta(t-\varsigma) \tag{1}$$

Where, δ is the Dirac-delta function.

The governing partial differential equation of non-homogeneous boundary value problem,

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{g(r,z,t)}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2}$$

with the boundary conditions

$$\frac{\partial T}{\partial z}-h_1 T=U(r,t) \text{ (unknown) at } z=-\frac{h}{2},\ 0\le r\le a \tag{3}$$

$$\frac{\partial T}{\partial z}=f(r,t) \text{ (known) at } z=\xi,\ 0\le r\le a \tag{4}$$

$$\frac{\partial T}{\partial z}+h_2 T=0 \text{ at } z=\frac{h}{2},\ 0\le r\le a \tag{5}$$

and

$$\frac{\partial T}{\partial z}+h_3 T=0 \text{ at } r=a,\ -\frac{h}{2}\le z\le\frac{h}{2} \tag{6}$$

$$T = 0 \text{ at } t = 0 \tag{7}$$

Where, h_1, h_2, h_3 be the relative heat transfer coefficients on the lower, upper and circular surface of the thin circular cylinder, respectively,

k is thermal conductivity of material of cylinder and

α is thermal diffusivity of material of cylinder.

the expressions for displacement and thermal stresses for plane stress, i.e., solid thin circular cylinder with radius *a* are

$$u = (1+\nu)a_t\left[\frac{1}{r}\int_a^r \tau r dr + \left(\frac{1-\nu}{1+\nu}\frac{r}{a^2}\right)\left(\int_0^a \tau r dr\right)\right] \tag{8}$$

$$\sigma_{rr} = a_t E\left[\frac{-1}{r^2}\int_a^r \tau r dr + \left(\frac{1}{a^2}\int_0^a \tau r dr\right)\right] \tag{9}$$

$$\sigma_{\theta\theta} = a_t E\left[\frac{1}{r^2}\int_a^r \tau r dr + \left(\frac{1}{a^2}\int_0^a \tau r dr - \tau\right)\right] \tag{10}$$

$$\sigma_{zz} = \sigma_{r\theta} = 0$$

For long solid circular cylinder in plane strain the radial displacement and thermal stresses are given by

$$u = \left(\frac{1+\nu}{1-\nu}a_t\right)\left[\left(\frac{1}{r}\int_a^r \tau r dr\right) + \left((1-2\nu)\frac{r}{a^2}\right)\left(\int_0^a \tau r dr\right)\right] \text{ for } \varepsilon_{zz} = 0 \tag{11}$$

and

$$u = \left(\frac{1+\nu}{1-\nu}a_t\right)\left[\left(\frac{1}{r}\int_a^r \tau r dr\right) + \left(\frac{1-3\nu}{1+\nu}\frac{r}{a^2}\right)\left(\int_0^a \tau r dr\right)\right] \text{ for } \varepsilon_{zz} = \varepsilon_0 \tag{12}$$

$$\sigma_{rr} = \frac{a_t E}{1-\nu}\left[\frac{-1}{r^2}\int_a^r \tau r dr + \left(\frac{1}{a^2}\int_0^a \tau r dr\right)\right] \tag{13}$$

$$\sigma_{\theta\theta} = \frac{a_t E}{1-\nu}\left[\frac{1}{r^2}\int_a^r \tau r dr + \left(\frac{1}{a^2}\int_0^a \tau r dr - \tau\right)\right] \tag{14}$$

$$\sigma_{zz} = \frac{a_t E}{1-\nu}\left(\frac{2\nu}{a^2}\int_0^a \tau r dr - \tau\right) \text{ for } \varepsilon_{zz} = 0$$

And $\sigma_{zz} = \frac{a_t E}{1-\nu}\left(\frac{2}{a^2}\int_0^a \tau r dr - \tau\right)$ for $\varepsilon_{zz} = \varepsilon_0$ (15)

The problem's mathematical formulation consists of Eqs. (1) through (15).

THE SOLUTION FOR UNKNOWN TEMPERATURE

The integral transform method has been used to find analytical solution of non-homogeneous boundary value problem (Eqs. 1–7).

The Hankel transform and its inverse developed to remove space variable *r,* defined as

$$\overline{T}\left(\alpha_n,z,t\right)=\int_0^a rJ_0\left(\alpha_n r\right)T\left(r,z,t\right)dr \tag{16}$$

$$T\left(r,z,t\right)=\sum_{n=1}^{\infty}\frac{2J_0\left(\alpha_n r\right)\overline{T}\left(\alpha_n,z,t\right)}{a^2\left[J_0^2\left(\alpha_n a\right)+J_1^2\left(\alpha_n a\right)\right]} \tag{17}$$

Where, α_n are positive roots of the transcendental equation

$h_3 J_0(\alpha a)-\alpha J_1(\alpha a)=0,$

and the finite Fourier transform and its inverse developed to remove space variable *z*, defined as

$$\overline{\overline{T}}\left(\alpha_n,\beta_m,t\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\left\{\beta_m\cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]+h_1\sin\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\}\overline{T}\left(\alpha_n,z,t\right)dr \tag{18}$$

$$\overline{T}\left(\alpha_n,z,t\right)=\sum_{n=1}^{\infty}\frac{1}{\sqrt{N}}\left\{\beta_m\cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]+h_1\sin\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\}\overline{\overline{T}}\left(\alpha_n,\beta_m,t\right) \tag{19}$$

Where, *N* is normality constant which can be determined by orthogonality of circular functions as

$$\sqrt{N}=\frac{4}{2\beta_m^2\left(\xi+\frac{h}{2}\right)h+\beta_m\sin\left[2\beta_m\left(\xi+\frac{h}{2}\right)\right]}$$

and β_m are positive roots of transcendental equation

$\beta h_2\cos\left(\beta h\right)-\beta^2\sin\left(\beta h\right)=0\cdot$

On applying the Hankel transform as defined in Eq. (16) and the Finite Fourier transform as defined in Eq. (18), along with their respective inverse transforms, the temperature distribution within the circular cylinder is obtained.

These transforms simplify the solution of the governing differential equations, allowing for the analytical determination of temperature variation inside the cylinder under the given thermal conditions [4]. The method provides an effective approach for solving complex thermal problems involving cylindrical geometries with internal heat generation.

$$T\left(r,z,t\right)=\left(\frac{8}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{J_0\left(\alpha_n r\right)}{J_0^2\left(\alpha_n a\right)+J_1^2\left(\alpha_n a\right)}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right.$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{20}$$

Since $T=0$ at $t=0$

$\Rightarrow$ The temperature change $\tau=T-T_i=T$ (21)

The unknown convective heat flux applied on the lower surface of cylinder is given by

$$U(r,t)=\left(\frac{8}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{J_0(\alpha_n r)}{J_0^2(\alpha_n a)+J_1^2(\alpha_n a)}\right)\left(\frac{-h_1\beta_m}{\sqrt{N}}\right)$$
$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{22}$$

DISPLACEMENT AND THERMAL STRESSES IN THIN SOLID CIRCULAR CYLINDER

The expressions for displacement and thermal stresses for plane stress, i.e., solid thin circular cylinder can be obtained by using Eqs. (8) to (11) as

$$u(r,z,t)=\left(\frac{8(1+\nu)a_t}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{a(1+\nu)J_1(\alpha_n r)+r(1-\nu)J_1(\alpha_n a)}{\alpha_n a\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)$$
$$\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$
$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{23}$$

$$\sigma_{rr}(r,z,t)=\left(\frac{8a_tE}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{rJ_1(\alpha_n a)-aJ_1(\alpha_n r)}{\alpha_n a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$
$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{24}$$

$$\sigma_{\theta\theta}(r,z,t)=\left(\frac{8a_tE}{ka^2}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{aJ_1(\alpha_n r)+rJ_1(\alpha_n a)-a\cdot rJ_0(\alpha_n a)}{\alpha_n\cdot a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{25}$$

$$\sigma_{zz}=\sigma_{rz}=0 \tag{26}$$

DISPLACEMENT AND THERMAL STRESSES IN LONG SOLID CIRCULAR CYLINDER

For long solid circular cylinder in plane strain the radial displacement and thermal stresses can be obtained by using Eqs. (12) to (15) as

$$u(r,z,t)=\left(\frac{8(1+\nu)a_t}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{aJ_1(\alpha_n r)+(1-2\nu)rJ_1(\alpha_n a)}{\alpha_n a\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\}\text{ for } \varepsilon_{zz}=0 \tag{27}$$

$$u(r,z,t)=\left(\frac{8(1+\nu)a_t}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{a(1+\nu)J_1(\alpha_n r)+(1-3\nu)rJ_1(\alpha_n a)}{\alpha_n\cdot a\cdot(1+\nu)\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\}\text{ for } \varepsilon_{zz}=\varepsilon_0 \tag{28}$$

$$\sigma_{rr}(r,z,t)=\left(\frac{8a_tE}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{rJ_1(\alpha_n a)-aJ_1(\alpha_n r)}{\alpha_n a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{29}$$

$$\sigma_{\theta\theta}(r,z,t)=\left(\frac{8a_tE}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{aJ_1(\alpha_n r)+rJ_1(\alpha_n a)-a\cdot r\cdot\alpha_n\cdot J_0(\alpha_n a)}{\alpha_n\cdot a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\} \tag{30}$$

$$\sigma_{zz}(r,z,t)=\left(\frac{8a_tE}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{2\nu\cdot J_1(\alpha_n a)-aJ_0(\alpha_n r)}{\alpha_n\cdot a\cdot r\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\}\text{for }\varepsilon_{zz}=0 \quad (31)$$

$$\sigma_{zz}(r,z,t)=\left(\frac{8a_tE}{ka^2(1-\nu)}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{2\cdot J_1(\alpha_n a)-a\cdot\alpha_n\cdot J_0(\alpha_n r)}{\alpha_n\cdot a\cdot\left[J_0^2(\alpha_n a)+J_1^2(\alpha_n a)\right]}\right)\left(\frac{\beta_m cos\left[\beta_m\left(z+\frac{h}{2}\right)\right]}{\sqrt{N}}\right)$$

$$\times\left\{\left(\frac{Q_0\beta_m\overline{f}(\alpha_n)}{\alpha_n^2+\beta_m^2}\right)\left(1-e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)t}\right)+\alpha\int_0^t\overline{\overline{g}}(\alpha_n,\beta_m,\tau)e^{-\alpha\left(\alpha_n^2+\beta_m^2\right)(t-\varsigma)}d\tau\right\}\text{ for }\varepsilon_{zz}=\varepsilon_0 \quad (32)$$

NUMERICAL CALCULATION SPECIAL CASE

Setting $f(r)=\left(r^2-a^2\right)^2$

$$g(r,z,t)=\frac{g_i}{2\pi r}\delta(r-r_1)\delta(z-0)\delta(t-\varsigma)$$

$$\overline{f}(\alpha_n)=\frac{8a}{\alpha_n^5}\left[\left(8-a^2\alpha_n^2\right)J_1(\alpha_n a)-49\alpha_n J_0(\alpha_n a)\right]$$

$$\overline{\overline{g}}(\alpha_n,\beta_m,\tau)=\frac{g_i}{2\pi r}r_1J_0(\alpha_n r_1)\left[\beta_m\cos\left(\beta_m\frac{h}{2}\right)+h_1\sin\left(\beta_m\frac{h}{2}\right)\right]\delta(t-\varsigma)$$

Where, r_1 is the radius of annular region situated $0<r_1<r$ and δ is the Dirac-delta function.

Located in the center of the circular cylinder in a radial orientation, the heat source is an instantaneous cylindrical heat source of strength that releases heat instantly.

DIMENSION

Radius of a circular cylinder $a=1ft$,

Central circular path of cylinder $r_1=0.5ft$,

Thickness of a circular cylinder $z=0.1ft$,

Level surface where known heat flux is situated inside circular cylinder $\xi=0.05ft$.

MATERIAL PROPERTIES

The numerical calculation has been carried out for an Aluminum (Pure) hollow circular cylinder with the material properties as,

Density $\rho=169lb/ft^3$,

Specific heat $c_p=0.208Btu/lb^0F$,

Thermal conductivity $k=117Btu/\left(hr\cdot ft\cdot{}^0F\right)$,

Thermal diffusivity $\alpha=3.33ft^2/hr$, Poisson ratio $\nu = 0.35$,

Coefficient of linear thermal expansion $a_t = 12.84 \times 10^{-6} \frac{1}{F}$,

Lamé constant $\mu = 26.67$

Young's modulus $E = 130GPa$.

ROOTS OF TRANSCENDENTAL EQUATION

The $\alpha_1 = 0.471448, \alpha_2 = 1.07908, \alpha_3 = 1.696798, \alpha_4 = 2.312416, \alpha_5 = 2.928661$ are the positive roots of transcendental equation $h_3 J_0(\alpha a) - \alpha J_1(\alpha a) = 0$ and the $\beta_1 = 2.627675, \beta_2 = 5.307325, \beta_3 = 8.067136, \beta_4 = 10.90871, \beta_5 = 13.819192,$ heat transfer coefficient $h_1 = h_2 = h_3 = 10$ are the positive roots of transcendental equation $\beta h_2 \cos(\beta h) - \beta^2 \sin(\beta h) = 0$.

The numerical calculation has been carried out by setting for convenience,

$$\chi = \frac{8}{ka^2}, \ \gamma = \left(\frac{16(1+\nu)\alpha}{ka^2}\right), \ \lambda = \left(\frac{8\alpha E}{ka^2}\right) \text{ and } \xi = \left(\frac{8\alpha E}{ka^2(1-\nu)}\right).$$

RESULTS ANS DISCUSSION

In this problem, a circular cylinder is considered under a transient temperature field, and expressions are determined for the unknown temperature, displacement, and thermal stress functions due to convective heat transfer and internal heat generation modeled as a cylindrical surface heat source. The internal heat source is situated concentrically and acts along the axial length of the cylinder [1]. The convective heat transfer leads to time-dependent temperature variations, which in turn induce displacement and thermal stresses within the solid thin circular cylinder. The analysis accounts for the influence of both the internal heat generation and the heat dissipation through convection at the boundary surfaces. This coupling between thermal and mechanical fields is essential for understanding the behavior of materials exposed to dynamic thermal environments [6]. The results provide insight into how heat transfer mechanisms affect the structural response of cylindrical bodies, which is crucial in many engineering applications involving thermal loading.

- *From Figure 1:* The temperature changes due to convective heat transfer with time and the convection due to dissipation through circular boundary surface is observed.
- *From Figure 2:* The temperature changes cause heat transfer within circular cylinder which result displacement at circular boundary surface.
- *From Figure 3:* The radial thermal stresses are zero at center and at circular boundary. For long circular cylinder, the radial thermal stresses are less as compared to thin cylinder. The radial stresses develop tensile stresses in both thin and long cylinder.
- *From Figure 4:* The development of compressive angular thermal stresses is observed. The angular thermal stresses are zero at center.
- *From Figure 5:* The compressive axial thermal stresses seen around the center where internal cylindrical surface heat source is situated.

So, expansion is observed in both radial and axial directions, resulting in the bending of the thin circular cylinder. In contrast, for a long circular cylinder, the thermal stresses developed are absorbed along the thickness, minimizing deformation. Therefore, the displacement becomes inversely proportional to the length of the cylinder [6]. This indicates that the shorter cylinders experience more noticeable bending due to thermal expansion, while longer cylinders exhibit less displacement as the stresses are more effectively distributed through their thickness. The length of the cylinder plays a significant role in determining the extent of displacement under thermal loading conditions.

The thermoelastic problems due to surface heat sources have wide applications in engineering fields such as surface coatings, dielectric barriers, anti-reflective coatings on glass, and solar cells. These problems are significant for understanding thermal effects on structural behavior. Analytical formulas for temperature distribution are employed to generate specific examples of interest [8]. These examples help in evaluating the resulting displacement and thermal stresses within the material. Such analysis is essential for predicting material performance and ensuring the reliability of engineering components subjected to surface heating under various operational conditions.

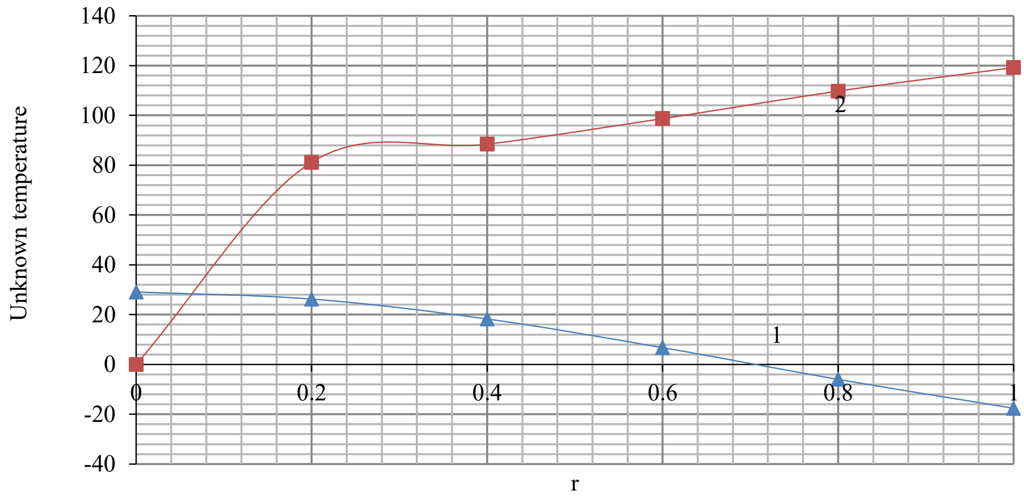

Figure 1. Unknown temperature distribution $\frac{U(r,z,t)}{\chi}$ along radial direction.

Line 1 represents unknown temperature distribution $\frac{U(r,z,t)}{\chi}$ *for thin solid circular cylinder.*

Line 2 represents unknown temperature distribution $\frac{U(r,z,t)}{\chi}$ *for long solid circular cylinder.*

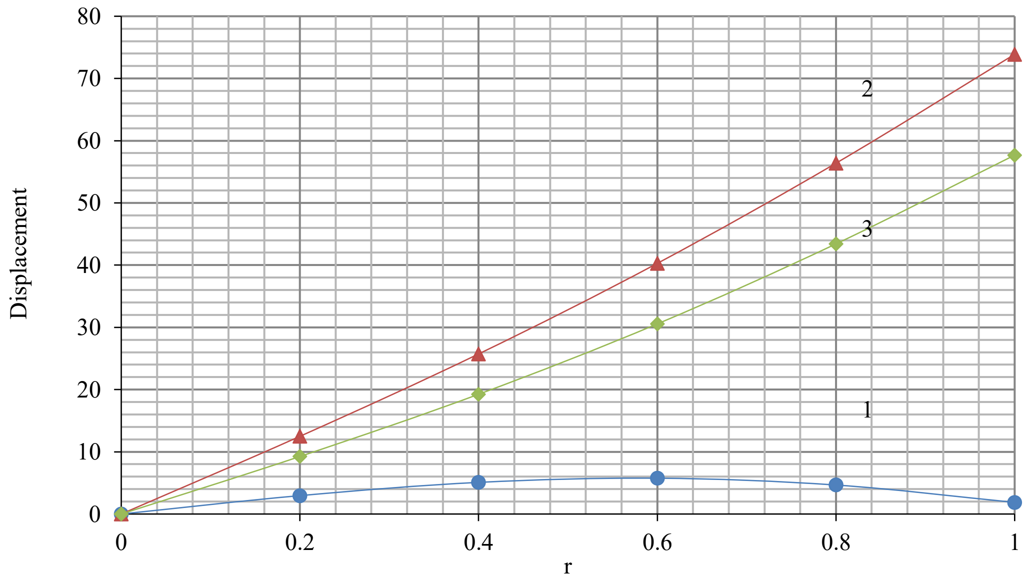

Figure 2. Displacement function $\frac{u(r,z,t)}{\gamma}$ along radial direction.

Line 1 represents displacement function $\frac{u(r,z,t)}{\gamma}$ *for thin solid circular cylinder.*

Line 2 represent displacement function $\frac{u(r,z,t)}{\gamma}$ *for long solid circular cylinder when* $\varepsilon_{zz} = 0$.

Line 3 represent displacement function $\frac{u(r,z,t)}{\gamma}$ *for long solid circular cylinder when* $\varepsilon_{zz} = \varepsilon_0$.

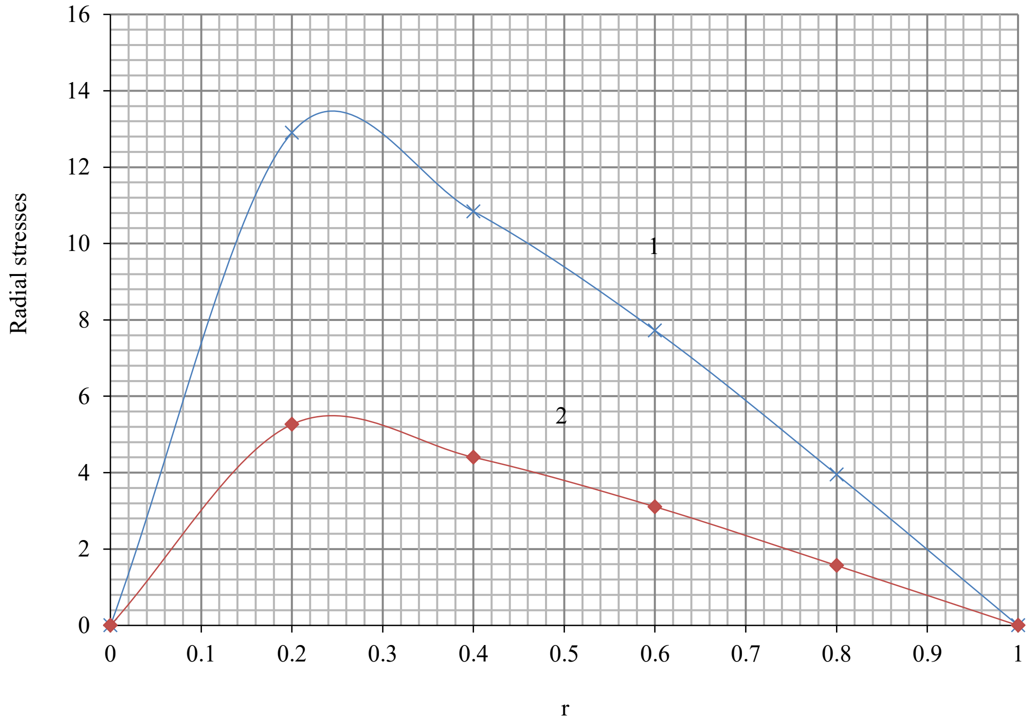

Figure 3. Radial stress function $\frac{\sigma_{rr}(r,z,t)}{\lambda}$ along radial direction.

Line 1 represents radial stress function $\frac{\sigma_{rr}(r,z,t)}{\lambda}$ *for thin solid circular cylinder.*

Line 2 represents radial stress function $\frac{\sigma_{rr}(r,z,t)}{\lambda}$ *for long solid circular cylinder which is almost zero.*

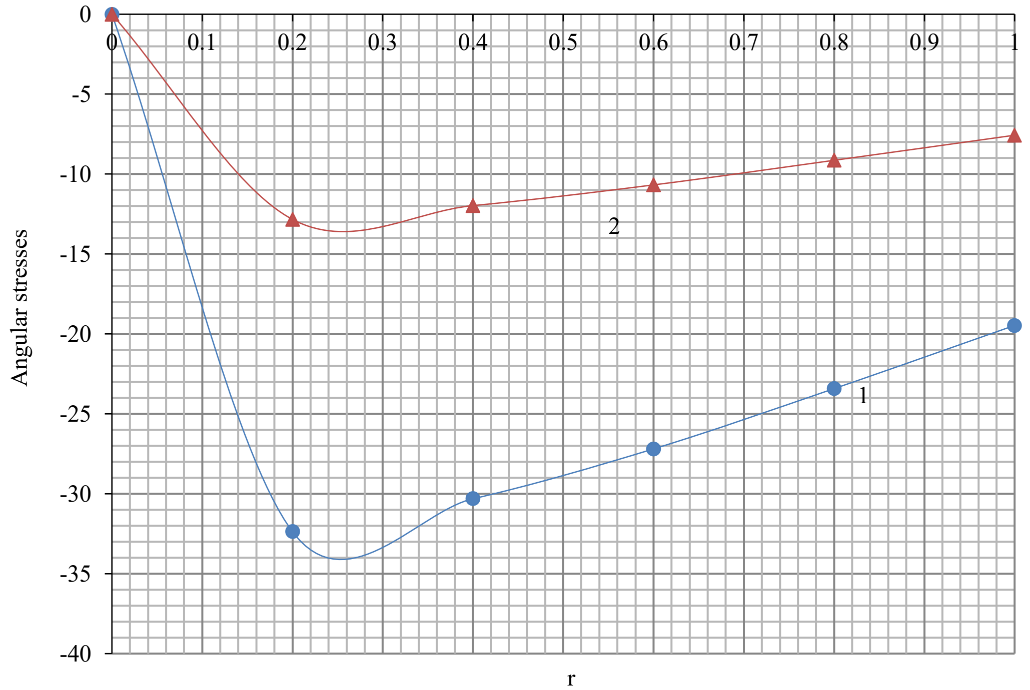

Figure 4. Angular stress function $\frac{\sigma_{\theta\theta}(r,z,t)}{\lambda}$ along radial direction.

Line 1 represents angular stress function $\frac{\sigma_{\theta\theta}(r,z,t)}{\lambda}$ *for thin solid circular cylinder.*

Line 2 represents angular stress function $\frac{\sigma_{\theta\theta}(r,z,t)}{\lambda}$ *for long solid circular cylinder which is almost zero.*

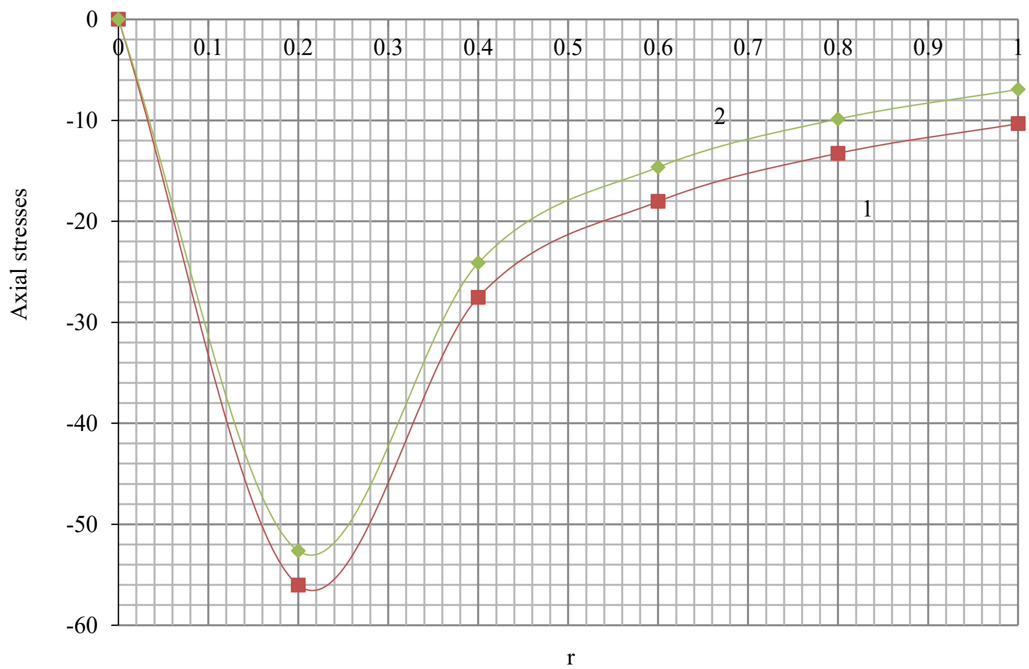

Figure 5. Axial stress function $\frac{\sigma_{zz}(r,z,t)}{\xi}$ along radial direction.

The axial stress function $\frac{\sigma_{zz}(r,z,t)}{\xi}$ *for thin solid circular cylinder in plane state is zero.*

Line 1 represent axial stress function $\frac{\sigma_{zz}(r,z,t)}{\xi}$ *for long solid circular cylinder when* $\varepsilon_{zz} = 0$.

Line 2 represent axial stress function $\frac{\sigma_{zz}(r,z,t)}{\xi}$ *for long solid circular cylinder when* $\varepsilon_{zz} = \varepsilon_0$.

CONCLUSION

This paper presents an analytical approach to solving an inverse quasi-static thermal stress problem in a thin circular cylinder subjected to transient thermal loading from an internal cylindrical heat source. By employing integral transformation methods, the study determines unknown temperature distributions, convective heat flux, displacement, and thermal stress functions.

The results reveal that the thermal and mechanical responses are significantly influenced by the cylinder's geometry, particularly its length. Thin cylinders exhibit noticeable deformation due to thermal expansion, while long cylinders distribute stress more uniformly, reducing displacement. The findings are applicable to various engineering systems where thermal management and structural reliability under transient heating are critical.

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