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A Novel Similarity Measure for Interval Value Picture Fuzzy Environment and Extended TOPSIS

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Abstract

Correct decision-making is the most arduous task in our daily life. The decisions are hard to make in the multi-criteria decision-making (MCDM) problems due to ambiguous and unexpected information. In order to cope with such uncertainties in the data, a new decision-making approach has been developed using a newly defined similarity measure under the framework of interval-valued picture fuzzy set (IVPFS), as an extension of picture fuzzy sets (PFS). In real-life, due to insufficient data, improper knowledge or inexperience of the decision makers (DMs), the weights of attributes are either unknown or not satisfactory or partially known. To tackle such situation, the optimal weights of attributes are acquired using linear programming (LP) from the weight information that is partially known in this study. An algorithm for the TOPSIS method has been developed. Numerical examples have been executed to determine the feasibility and suitability of the suggested model. The comparison analysis shows that the suggest strategy is more effective than the prevailing methods.

Keywords: Interval-valued picture fuzzy set, similarity measure, multi-criteria decision-making, linear programming, TOPSIS

INTRODUCTION

Zadeh [1] was first to introduce the idea of fuzzy set (FS) theory, which are generally ambiguous and imprecise in our real life phenomenon. It deals with uncertain data with the help of membership degree (MD) having a value between 0 and 1. Since then it has been extensively used in several fields of practical problems and the researchers find the new horizon to solve many critical problems in various domain. The extension of FSs was started with the thought of the intuitionistic fuzzy set (IFS), established by Atanassov [2]. The notion that how much a factor is unrelated to a phenomenon has been added to the IFS by including the non-membership degree. He also developed the idea of IFS [3] into interval-valued IFS (IVIFS) to specifically include extra information which overcome The issue that occurred with the IFS was that some of the objects from [1] could not be expressed as IFS. Ashraf and Abdullah [4] presented the novel concepts of PFS with an additional degree in abstinence (AD) which is, due to the engagement of AD, a more significant framework was created, increasing the variety of information from real-life events that could be accurately acquired. The notion of PFSs did not limit to 'yes' and 'no' of human's choices but it included also 'yes', 'neutral', 'no' and 'refusal'. Vote 'for', 'neutral', 'against' or 'refusal' could be the different characteristics to cast a vote which could be an excellent example to justify the concept. The important characteristic of PFS is that these three components levels added together cannot exceed 1. Cuong [5] represents the idea of PFS, their features, and some practical operations. The membership, non-membership, and neutral degrees are the three different types of functions that we have in PFS. The PFS set-based approaches are typically offered when individual responses based on yes, absent, no, and refusal cannot be accurately communicated in

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standard fuzzy and IFSs. Following recent research works made upon PFSs include: Singh [6] estimated the correlation coefficient (CC) of PFSs, Son [7] created the distance measure and used it in the fuzzy clustering and Wei [8] provided the cross-entropy measure of PFSs and used in multi-attribute decision-making (MADM) problems. The group of DMs has a big part to play in MCDM [9–14]. For MADM utilizing picture fuzzy (PF) information, Wei [15] suggested PF cross-entropy and projection models and exhibited the PF accumulation operators' plan and implemented it to MADM for using of enterprise resource planning (ERP). Wang [16] demonstrated few PFSs operations and applied them to solve MCDM problems. Nowadays, idea of PF linguistic aggregation operators (AO) is proposed by Rahman [17]. Based on PFSs, Yang [18] defined adjustable soft discernibly matrix and implemented it in decision-making. Garg [19] developed aggregation operators on PFSs and verified it with MCDM problems. PF algorithm was proposed and evaluated on decision-making issues by Peng [20]. Wei [21] presented a technique to get the similarity measure between two PFSs. Furthermore, Jana [22] presented some AO named Dombi operators for PFSs environment and executed these activities to MADM method. Ashraf et al. [23] extended the form of PFSs to a novel concept of cubic PFSs.

Garg [24] established the idea of a linguistic Atanassov IFS, which have linguistic positive and negative degrees. Alshammari [25] outlined few operations and relations over IVPFS. PF aggregation operator were discussed and applied to address MCDM issues by Xu [26] and Garg [19]. Chinnadurai et al. [27] suggested the MCDM problem concept of using parameters to determine a unique ranking among alternatives. Teshome et al. [28] presented a procedure for controlling signals using an interval-valued neutrosophic soft set. Zulqarnain [29] proposed new score technique to address MCDM problems. Zulqarnain [30] presented the idea of intuitionistic fuzzy HSS and employed the TOPSIS approach using CC. MCDM problems were solved using the AO and the Pythagorean fuzzy hyper soft set concept by Chinnadurai [31].

The intent of the research is to establish a novel MCDM method with partially known weight criteria in context of IVPFS. The key advantages of the study are discussed. The study can tackle situations where DMs wish to convey their views via interval values that include both membership, non-membership and neutral membership values. This study can also confront such decision-making situations where the weights of the criteria are known partially. TOPSIS is a well-known decision-making method to solve MCDM problems. TOPSIS implements the "closeness to ideal solution" concept.

The proposed idea has a disadvantage. It does not work where the weights of the criteria are either completely or only partially unknown.

On various decision-making difficulties, DMs choose to communicate their judgments using interval values that include both membership and non-membership values when precise information is not available. Again, when the DMs are hesitant to express their ideas, they do so using interval-valued intuitionistic hesitant fuzzy elements (IVIHFEs). IVIHFEs have interval-valued and hesitant opinions, which enable them to handle a variety of challenging circumstances. As a result of interval-valued and hesitant perspectives, IVIHFEs are able to manage a variety of complex circumstances. Naturally, uncertainty increases with increased complexity. When the DMs take into account the frequency of occurrences of those IVIHFEs in these extremely uncertain contexts, they may no longer be useful. There are a few studies where researchers used P-IVIHFS Zhai [32], a recently introduced algorithm, to solve decisionmaking issues. Its inherent capabilities to the area of decision-making by applying the concepts of "closeness to ideal solution" and "maximum group utility for the majority," respectively, TOPSIS and VIKOR are well recognized to the MCDM researchers as decision-making methods. TOPSIS and VIKOR approaches were enhanced by FS and its numerous expansions to deal with the numerous methods of decision-making uncertainty. As a result, these compromise programming techniques have recently been used to handle numerous real-world situations involving ambiguous decision-making. De [33] suggest a strategy to making decisions that may deal with circumstances where the weights of the qualities are unknown and uncertainties are both subjective and objective in nature.

The rest of the part of this paper is set up as follows. The ground-level important ideas briefly go through in section 2 [34]. In section 3, we outline our suggested method. Two numerical instances are presented in Section 4. The comparative study with previous articles of the analysis of the corresponding results is stated in Section 5. The study concludes with the concluding remarks finally in Section 6.

PRELIMINARIES

A few essential concepts required for the study which are briefly presented in this section.

Definition 1.

Let $X = \{x_1, x_2, x_3, \dots, x_n\}$ be the universe of discourse. A PFS A on X is defined by$A = \{\langle x, \mu_A(x), \nu_A(x), \eta_A(x)\rangle | x \in X\}$ where $\mu_A(x), \nu_A(x), \eta_A(x) \in [0, 1]$ are called respectively the membership function, the non-membership function and the neutral membership function of $x \in X$ to the set A, $0 \leq \mu_A(x) + \nu_A(x) + \eta_A(x) \leq 1$.

The degree of the refusal function is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) - \eta_A(x)$.

Definition 2.

Let $X = \{x_1, x_2, x_3, \dots, x_n\}$ be the universe of discourse. An IVPFSAon X is represented by

$$A = \{\langle x, \tilde{\mu}_A(x), \tilde{\nu}_A(x), \tilde{\eta}_A(x)\rangle | x \in X\}$$

where $\tilde{\mu}_A(x)$, $\tilde{\nu}_A(x)$, $\tilde{\eta}_A(x)$ are called respectively the interval-valued membership function, the interval-valued non-membership function and the interval-valued neutral membership function of $x \in X$ to the set A,

$$\tilde{\mu}_A(x) = [\mu_A{}^l(x), \mu_A{}^u(x)],\ \tilde{\nu}_A(x) = [\nu_A{}^l(x), \nu_A{}^u(x)],\ \tilde{\eta}_A(x) = [\eta_A{}^l(x), \eta_A{}^u(x)],$$

$$0 \leq \mu_A{}^l(x) \leq \mu_A{}^u(x) \leq 1,\ 0 \leq \nu_A{}^l(x) \leq \nu_A{}^u(x) \leq 1,\ 0 \leq \eta_A{}^l(x) \leq \eta_A{}^u(x) \leq 1$$

with the condition: $0 \leq \mu_A{}^l(x) + \mu_A{}^u(x) \leq 1$, $0 \leq \nu_A{}^l(x) + \nu_A{}^u(x) \leq 1$, $0 \leq \eta_A{}^l(x) + \eta_A{}^u(x) \leq 1$. The degree of the refusal function is $\tilde{\pi}_A(x) = [\pi_A{}^l(x), \pi_A{}^u(x)]$ where $\pi_A{}^l(x) = 1 - \mu_A{}^u(x) - \nu_A{}^u(x) - \eta_A{}^u(x)$, $\pi_A{}^u(x) = 1 - \mu_A{}^l(x) - \nu_A{}^l(x) - \eta_A{}^l(x)$.

Example 1.

Consider a customer wants to buy a smart phone through the expert's ratings. For a particular model of a particular brand, the ratings are given in ranges instead of a fixed number. The experts rating is 40to 50 as a good product, 20 to 25 as not good while 10 to 15 as average one out of 100 scale. Now thisrating can be expressed as [0.40, 0.50], [0.20, 0.25], [0.10, 0.15] where the interval of acceptance MD is $\tilde{\mu}_A(x) = [\mu_A{}^l(x), \mu_A{}^u(x)] = [0.40, 0.50]$, the interval of rejection MD is $\tilde{\nu}_A(x) = [\nu_A{}^l(x), \nu_A{}^u(x)] = [0.20, 0.25]$, the interval of neutral MD is

$\tilde{\eta}_A(x) = [\eta_A{}^l(x), \eta_A{}^u(x)] = [0.10, 0.15]$ and the interval of refusal MD is$\tilde{\pi}_A(x) = [\pi_A{}^l(x), \pi_A{}^u(x)] = [0.10, 0.30]$.

Here, we define a novel similarity measure based under the context of IVPFS as an extension of [33].

Definition 3.

Let $X = \{x_1, x_2, x_3, \dots, x_n\}$ be the universe of discourse and $A = \{\langle x, \tilde{\mu}_A(x), \tilde{\nu}_A(x), \tilde{\eta}_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \tilde{\mu}_B(x), \tilde{\nu}_B(x), \tilde{\eta}_B(x)\rangle | x \in X\}$ be two IVPFS on X. Let $w = \{w_1, w_2, w_3, \dots, w_n\}$ be the weight vector of the element x where $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$. The degree of similarity between A and B denoted by $\delta(A, B)$ is defined by

$$\delta(A,B) = 1 - \sum_{j=1}^{n} \omega_j \Big[\frac{1}{12}\left(|\mu_A^l - \mu_B^l| + |\mu_A^u - \mu_B^u| + |\nu_A^l - \nu_B^l| + |\nu_A^u - \nu_B^u| + |\eta_A^l - \eta_B^l| + |\eta_A^u - \eta_B^u|\right) \tag{1}$$

$$+ \frac{1}{2} max\{|\mu Al - \mu Bl|, |\mu Au - \mu Bu|, |\nu Al - \nu Bl|, |\nu Au - \nu Bu|, |\eta Al - \eta Bl|, |\eta Au - \eta Bu|\}\Big]$$

PROPOSED METHOD

Here, we propose the concept of MCDM with IVPF information based on TOPSIS. The LP technique can be used to calculate the optimal weights of criteria. Let $P = \{P_1, P_2, P_3, \ldots, P_m\}$ be the set discrete alternatives and be the set of weights $w = \{w_1, w_2, w_3, \ldots, w_n\}$ corresponding to the criteria $\mathscr{B} = \{\mathscr{B}_1, \mathscr{B}_2, \mathscr{B}_3, \ldots, \mathscr{B}_n\}$, where $\sum_{j=1}^{n} w_j = 1$.

An IVPF decision matrix $\mathscr{R} = (\nabla ij)m\times n = ([\mu ijl, \mu iju], [\nu ijl, \nu iju], [\eta ijl, \eta iju])m\times n$ where $[\mu ijl, \mu iju]$, $[\nu ijl, \nu iju]$, $[\eta ijl, \eta iju]$ represent the respective interval of the degree of acceptance, neutral and rejection corresponding to the alternatives

P_i, $(i = 1,2,3, \ldots, m)$ respectively.

The steps to find the MCDM are as follows in order to identify the best alternative:

Step 1. Using the data provided by the DM, an IVPF decision matrix $\mathscr{R} = (\nabla_{ij})m\times n$ was developed.

Step 2. For more than one DMs we take the average in the opinion of the DMs.

Step 3. Compute the interval-valued PF positive ideal solution (IVPFPIS) ∇^+ and the interval-valued PF negative ideal solution (IVPFNIS) ∇^-,

As follows:

$$❑^+ = ([\mu_{ij}^{l+}, \mu_{ij}^{u+}], [\nu_{ij}^{l+}, \nu_{ij}^{u+}], [\eta_{ij}^{l+}, \eta_{ij}^{u+}])$$

$$= \begin{pmatrix} ([max_j \mu_{ij}^l, max_j \mu_{ij}^u], [min_j \nu_{ij}^l, min_j \nu_{ij}^u], [min_j \eta_{ij}^l, mijn\ \eta_{ij}^u]) : \mathscr{B}_j \in ✓_1 \\ ([max_j\ \mu ijl, max_J\ \mu iju], [max_J\ vijl, max_j\ \nu iju], [min_J\ \eta ijl, min_J\ \eta iju]) : Bj \in ✓2 \end{pmatrix} \tag{2}$$

$$❑^- = ([\mu_{ij}^{l-}, \mu_{ij}^{u-}], [\nu_{ij}^{l-}, \nu_{ij}^{u-}], [\eta_{ij}^{l-}, \eta_{ij}^{u-}])$$

$$= \begin{pmatrix} ([min_j \mu ijl, min_j\ \mu iju], [min_j \nu ijl, min_j\ \nu iju], [max_j\ \eta ijl, max_j\ \eta iju]) : \mathscr{B}j \in ✓1 \\ ([min_j\ \mu ijl, min_j\ \mu iju], [max_j\ \nu ijl, max_j\ \nu iju], [max_j\ \eta ijl, max_j\ \eta iju]) : Bj \in ✓2 \end{pmatrix} \tag{3}$$

where $✓_1$ represents the set of benefit criteria and $✓_2$ represents the set of the cost criteria and $✓_1 \cap ✓_2 = \phi$.

Step 4. Determine the weighted similarity δ^+ among IVPFPIS ∇^+ and each alternative, as well as the weighted similarity δ^- between IVPFNIS ∇^- and each alternative, respectively.

$$\delta i+ = 1 - \sum_{j=1}^{n} \omega j \Big[\frac{1}{12}\left(|\mu_{ij}^l - \mu jl+| + |\mu iju - \mu ju+| + |\ ijl - \ jl+| + |\ iju - \ ju+| + |5ijl - 5jl+| + |5iju - 5ju+|\right) + \frac{1}{2} max\{|\mu l - \mu jl+|, |\mu iju - \mu ju+|, |\ ijl - \ jl+|, |\ iju - \ ju+|, |5ijl - 5jl+|, |5iju - 5ju+|\}\Big]$$

where $\delta^+ \in [0,1]$.

Similarly,

(4)

$$\frac{1}{12}(\mu jl - | + |\mu iju - \mu ju - | + |\nu ijl - \nu jl - | + |\nu iju - \nu ju - | + |\eta ijl - \eta jl - | + |\eta iju - \eta ju - |)$$

$$\delta i- = 1 - \sum n_{j=1\omega j} [+^{1}_{2} \{ l - \mu jl - |, |\mu iju - \mu ju - |, |\nu ijl - \nu jl - |, |\nu iju - \nu ju - |, |\eta ijl - \eta jl - |, |\eta iju - \eta ju||_{\max |\mu ij} \quad (5)$$

where $\delta^- \in [0,1]$, $(i = 1,2,3, \dots , m)$.

Step 5. Develop the following model of objective function Z based on equations (4) and (5).

$$Z = \delta^+ - \delta^- \quad (6)$$

Step 6. The resulting objective function Z in Step 5 is maximised by resolving the LP model described in (6) to obtain the weights $\mathscr{w}_j$ of the criteria $\mathscr{B}_j$ $(j = 1,2,3, \dots , n)$.

Step 7. Obtain the degree of similarity δ^+ and $\delta^-$by putting the optimal weights in PFPIS and PFNIS based on equations (4) and (5).

Step 8. Obtain the alternative P_i's relative closeness($\checkmark\mathscr{R}_i$)as

$$\checkmark Ri = \delta^+_i {}^{\delta i^+} + \delta i^- \quad (7)$$

The maximum value of the relative closeness $\checkmark\mathscr{R}_i$of the options is that we choose the greatest option out of several possibilities P_i, where $(i = 1,2,3, \dots , m)$.

NUMERICAL EXAMPLE

We take into consideration two real-world MCDM situations from the papers [35] and [36], to demonstrate the advantages and usefulness of the suggested method.

Example 2.

The problem described in [35] has been taken and analysed with the new approach. Consider a group of alternatives $\{P_1, P_2, P_3, P_4\}$ in a scheme that require expert estimation with a set of finite criteria denoted by $\{\mathscr{B}_1, \mathscr{B}_2, \mathscr{B}_3, \mathscr{B}_4\}$. The purpose is to choose the most appropriate plan for the company. In which, P_1 represents stock purchases; P_2 represents stock awards; P_3 represents change of control, and P_4 represents bonus of the company.

We consider each criteria is associated with a partially known weight obtained in interval form as follows:

$$w_1 + w_2 + w_3 + w_4 = 1;$$
$$0.25 \le w_1 \le 0.35;$$
$$0.25 \le w_2 \le 0.35; \quad (8)$$
$$0.15 \le w_3 \le 0.25;$$
$$0.15 \le w_4 \le 0.25;$$

The DM assigned weights to the requirements for particular qualities correspondingly in order to avoid any conflicts.

Step 1. According to the data given by the DM in IVPFS situation, a matrix $\mathscr{R}$, called the decision matrix is created.

$\mathscr{R}$ corresponding to different criterion $\mathscr{B}_j$, $(j = 1, 2, 3, 4)$ is as follows:

	$\mathscr{B}_1$	$\mathscr{B}_2$	$\mathscr{B}_3$	$\mathscr{B}_4$
P_1	[0.2, 0.5] [0.1, 0.3] [0.1, 0.2]	[0.1, 0.4] [0.1, 0.2] [0.1, 0.2]	[0.2, 0.3] [0.1, 0.2] [0.2, 0.3]	[0.1, 0.2] [0.1, 0.3] [0.1, 0.2]
P_2	[0.1, 0.3] [0.1, 0.2] [0.1, 0.3]	[0.2, 0.3] [0.1, 0.4] [0.2, 0.3]	[0.1, 0.2] [0.2, 0.3] [0.1, 0.2]	[0.2, 0.3] [0.2, 0.4] [0.2, 0.3]
P_3	[0.2, 0.4] [0.2, 0.4] [0.2, 0.3]	[0.1, 0.2] [0.3, 0.4] [0.1, 0.2]	[0.2, 0.4] [0.3, 0.4] [0.2, 0.3]	[0.3, 0.4] [0.2, 0.3] [0.3, 0.4]
P_4	[0.1, 0.2] [0.1, 0.3] [0.1, 0.2]	[0.1, 0.3] [0.2, 0.3] [0.3, 0.4]	[0.1, 0.3] [0.1, 0.3] [0.1, 0.3]	[0.1, 0.3] [0.1, 0.2] [0.1, 0.4]

Step 2. As there are only one DM, this step can be omitted in this case.

Step 3. Evaluate the IVPFPIS ∇+ and IVPFNIS ∇−corresponding to different criteria $\mathscr{B}_j$, (j = 1, 2, 3, 4) based on equations (2) and (3), respectively.

∇+			
= [[0.2, 0.5] [0.1, 0.2] [0.1, 0.2]	[0.2, 0.4] [0.1, 0.2] [0.1, 0.2]	[0.2, 0.4] [0.1, 0.2] [0.1, 0.2]	[0.3, 0.4] [0.1, 0.2] [0.1, 0.2]
∇−			
= [[0.1, 0.2] [0.1, 0.2] [0.2, 0.3]	[0.1, 0.2] [0.1, 0.2] [0.3, 0.4]	[0.1, 0.2] [0.1, 0.2] [0.2, 0.3]	[0.1, 0.2] [0.1, 0.2] [0.3, 0.4]

Step 4. Use equations (4) and (5) to estimate the similarity degree δ^+ between IVPFPIS ∇^+ and individually alternative and the similarity degree δ^- between IVPFNIS $\nabla^-$and each alternative.

$$\boldsymbol{\delta}+_i = [0.9478 \quad 0.9050 \quad 0.8983 \quad 0.8983]$$

$$\boldsymbol{\delta}-_i = [0.9006 \quad 0.9244 \quad 0.8956 \quad 0.9433]$$

Step 5. Construct a model to determine the objective function Z by substituting the values of δ^+ and δ^- derived from equations (4) and (5) in equation (6), respectively.

Step 6 & 7. Solving the LP model (*8*) by maximising the objective function Z from (*6*), we have the optimal weights of the criteria, as $\boldsymbol{w}_1^* = 0.25, \boldsymbol{w}_2^* = 0.35, \boldsymbol{w}_3^* = 0.15, \boldsymbol{w}_4^* = 0.25$.

Step 8. Calculate the degree of similarity δ^+ and δ^- by putting the optimal weights corresponding to each criteria.

Step 9. Using the equation (*7*), determine the relative closeness coefficient ✓$\mathscr{R}_i$ of alternative P_i, (i = 1, 2, 3, 4); as

✓$\mathscr{R}1$ = 0.5195, ✓$\mathscr{R}2$ = 0.4934, ✓$\mathscr{R}3$ = 0.5012, ✓$\mathscr{R}4$ = 0.4806 that yields the ranking order as $P1 > P3 > P2 >$

P_4. It reveals that P_1 is the best alternative.

Example 3. Using the suggested MCDM approach in another real-world example [36] in the framework of IVPFSs to assess the best alternative. Considering each criteria is associated with a partially known weight obtained in interval form as follows:

$$\begin{aligned} &w_1 + w_2 + w_3 + w_4 = 1; \\ &0.15 \leq w_1 \leq 0.25; \\ &0.20 \leq w_2 \leq 0.30; \\ &0.25 \leq w_3 \leq 0.35; \\ &0.20 \leq w_4 \leq 0.30 \end{aligned} \tag{9}$$

The decision matrix $\mathscr{R}_1$ corresponding to different criterion $\mathscr{B}_j$, (j = 1, 2, 3, 4) is as follows:

	R_1			
P_1	[0.12,0.15] [0.34,0.36] [0.21,0.23]	[0.23,0.33] [0.48,0.49] [0.12,0.13]	[0.16,0.17] [0.21,0.31] [0.19,0.29] [	0.15,0.19] [0.21,0.25] [0.32,0.34]
P_2	[0.12,0.15] [0.34,0.36] [0.21,0.23]	[0.32,0.33] [0.34,0.35] [0.11,0.12]	[0.44,0.45] [0.42,0.43] [0.11,0.12]	[0.21,0.28] [0.15,0.17] [0.41,0.43]

P_3	[0.05,0.12] [0.15,0.18] [0.32,0.37]	[0.23,0.25] [0.31,0.35] [0.11,0.28]	[0.51,0.52] [0.12,0.19] [0.26,0.29]	[0.51,0.53] [0.22,0.23] [0.21,0.24]
P_4	[0.21,0.23] [0.31,0.32] [0.09,0.21]	0.31,0.36] [0.31,0.35] [0.27,0.28]	[0.27,0.34] [0.17,0.27] [0.32,0.35]	[0.37,0.39] [0.21,0.24] [0.12,0.15]

The decision matrix $\mathscr{R}_2$ corresponding to different criterion $\mathscr{B}_j$, (j = 1, 2, 3, 4) is as follows:

	R_2			
P_1	[0.32,0.45] [0.41,0.42] [0.12,0.13]	[0.54,0.56] [0.34,0.36] [0.04,0.07]	[0.31, 0.32] [0.21,0.24] [0.25,0.35]	[0.24,0.29] [0.31,0.34] [0.23,0.25]
P_2	[[0.32,0.39] [0.21,0.28] [0.25,0.31]	[0.21,0.22] [0.25,0.26] [0.21,0.26]	[0.54, 0.55] [0.12,0.13] [0.14,0.15]	[0.28,0.31] [0.12,0.13] [0.31,0.33]
P_3	[0.25,0.31] [0.25,0.35] [0.25,0.33]	[0.39,0.45] [0.04,0.07] [0.29,0.31]	0.47, 0.49] [0.16,0.19] [0.21,0.25]	[0.42,0.43] [0.31,0.33] [0.13,0.14]
P_4	[0.44,0.45] [0.36,0.42] [0.12,0.13]	[0.04,0.06] [0.77,0.79] [0.12,0.15]	0.39, 0.41] [0.14,0.15] [0.17,0.18]	[0.42,0.43] [0.31,0.33] [0.13,0.14]

The decision matrix $\mathscr{R}3$ corresponding to different criterion $\mathscr{B}j$, (j = 1, 2, 3, 4) is as follows:

	R_3			
P_1	0.21,0.26] [0.33,0.36] [0.31,0.37]	[0.37,0.38] [0.23,0.26] [0.29,0.31]	[0.53, 0.55] [0.03,0.04] [0.21,0.28]	[0.33,0.39] [0.12,0.14] [0.31,0.34]
P_2	[[0.24,0.26] [0.36,0.37] [0.29,0.31]	[0.48,0.49] [0.21,0.22] [0.12,0.15]	[0.49, 0.51] [0.15,0.17] [0.25,0.29]	[0.41,0.46] [0.16,0.19] [0.23,0.29]
P_3	[0.15,0.21] [0.15,0.22] [0.15,0.24]	[0.34,0.35] [0.21,0.22] [0.24,0.29]	[0.32, 0.34] [0.31,0.35] [0.21,0.31]	[0.05,0.12] [0.14,0.19] [0.45,0.49]
P_4	[0.21,0.22] [0.45,0.47] [0.12,0.14]	[0.53,0.55] [0.03,0.04] [0.21,0.28]	[0.14, 0.15] [0.49,0.59] [0.16,0.19]	[0.45,0.47] [0.12,0.17] [0.23,0.29]

The decision matrix $\mathscr{R}_4$ corresponding to different criterion $\mathscr{B}_j$, (j = 1, 2, 3, 4) is as follows:

	R_5			
P_1	[0.21,0.22] [0.47,0.49] [0.25,0.28]	[0.47,0.48] [0.29,0.31] [0.19,0.21]	[0.21, 0.22] [0.14,0.15] [0.39,0.45]	[0.48,0.49] [0.11,0.12] [0.26,0.27]
P_2	[0.34,0.36] [0.44,0.45] [0.12,0.15]	[0.58,0.59] [0.12,0.13] [0.19,0.21]	[0.58, 0.59] [0.14,0.15] [0.16,0.18]	[0.21,0.29] [0.17,0.21] [0.44,0.49]
P_3	[0.31,0.32] [0.35,0.36] [0.14,0.29]	[0.35,0.41] [0.21,0.25] [0.31,0.32]	[0.63, 0.64] [0.12,0.14] [0.13,0.14]	[0.21,0.26] [0.16,0.19] [0.22,0.23]
P_4	[0.41,0.42] [0.31,0.32] [0.14,0.25]	[0.24,0.25] [0.32,0.35] [0.27,0.34]	[0.25, 0.29] [0.23,0.31] [0.31,0.34]	[0.31,0.36] [0.42,0.44] [0.12,0.15]

Taking the average of the opinion of the different DM, the decision matrix $\mathscr{R}$ corresponding to different criterion $\mathscr{B}_j$, (j = 1, 2, 3, 4) is as follows:

	R_1			
P_1	[0.22,0.28] [0.39,0.41] [0.23,0.26]	[0.41,0.44] [0.32, 0.34] [0.17,0.19]	[0.26,0.27] 0.29,0.29], [0.25,0.31]	[0.31,0.35] [0.18,0.21] [0.28,0.30]
P_2	[0.28,0.32] [0.32,0.36] [0.22,0.27]	[0.41,0.42] [0.22,0.23] [0.16,0.19]	[0.51,0.53] [0.19,0.21], [0.17,0.19]	[0.29, 0.34] [0.15,0.18] [0.34,0.38]
P_3	[0.20,0.24] [0.22,0.28] [0.21,0.30]	[0.33,0.37] [0.19,0.22] [0.24,0.30]	[0.47,0.49] [0.19,0.23], [0.20,0.25]	[0.27,0.31] [0.20,0.23] [0.26,0.29]
P_4	[0.32,0.33] [0.36,0.39] [0.12,0.18]	[0.29,0.31] [0.34,0.37] [0.21,0.26]	[0.26,0.29] [0.27,0.35], [0.23,0.26]	[0.32,0.35] [0.28,0.31] [0.20,0.23

Following the aforesaid steps, The optimal criteria weights are determined as $w_1^* = 0.15, w_2^* = 0.30, w_3^* = 0.35, w_4^* = 0.20$ and the relative closeness coefficient as $\mathcal{CR}_1 = 0.4865,\ \mathcal{CR}_2 = 0.5163, \mathcal{CR}_3 = 0.5052, \mathcal{CR}_4 = 0.4905$.

We see, $\checkmark\mathcal{R}_2 > \mathcal{CR}_3 > \mathcal{CR}_4 > \mathcal{CR}_1$ $(0.5163 > 0.5052 > 0.4902 > 0.4865)$ which represents the alternatives' order as $P_2 > \mathcal{A}_3 > \mathcal{A}_4 > \mathcal{A}_1$.

Hence, P_2 is the fittest one to consider in the scheme.

COMPARATIVE STUDY

To demonstrate the reliability and efficacy of the recommended method, on the basis of the information we adopted from [35] and [36]. After that, the results of the suggested approach are reviewed with [35, 34] (Table 1) and [36], [37] (Table 2). The suggested approach the current piece of work deals with the IVPFS environment and the optimal weights of each criteria under the given constraints are obtained solving the LP model.

In the practical Example 2, the proposed method and other AOs or SMs described by [33] yield slightly different preference orders in terms of the desired rankings but the best option remains the same, that is, P_1 which demonstrates the practicality of the suggested method. In Example 2, however, the outcomes generated by the suggested procedure and CC technique provided by [36] the worst alternative is same that is P_4 although other arrangements are with little different while [37] were unable to rank. Both the results show the usefulness of our proposed technique when compared to other existing technique.

Table 1. The score values of IVPFNs and Ranking of CBP schemes

Score	P_1	P_2	P_3	P_4	Ranking values
IVPFHWA [35]	0.0837	0.0696	0.0679	0.0664	$P_1 > P_2 > P_3 > P_4$
IVPFHWG [35]	0.0978	0.0749	0.0802	0.0769	$P_1 > P_3 > P_4 > P_2$
IVPFWA [35]	0.0943	0.0708	0.0766	0.0754	$P_1 > P_3 > P_4 > P_2$
IVPFWG [35]	0.1013	0.0806	0.0833	0.0848	$P_1 > P_4 > P_3 > P_2$
IVPFDWA [35]	0.0819	0.0637	0.0689	0.0574	$P_1 > P_3 > P_2 > P_4$
IVPFMSM [35]	0.0865	0.0648	0.0739	0.0728	$P_1 > P_3 > P_4 > P_2$
IVPFWMSM [35]	0.5373	0.5506	0.5532	0.5511	$P_1 > P_3 > P_4 > P_2$
IVPFDMSM [35]	0.0923	0.0699	0.0783	0.0797	$P_1 > P_4 > P_3 > P_2$
IVPFDWMSM [35]	-0.0265	-0.0233	-0.0246	-0.0259	$P_2 > P_3 > P_4 > P_1$
Proposed Method	0.5194	0.4934	0.5012	0.4806	$P_1 > P_3 > P_2 > P_4$

Table 2. The score values and ranking of CBP schemes

Score	P_1	P_2	P_3	P_4	Ranking values
Liu [37]	0.50	0.49	0.49	0.49	Unable to rank
Bobin [36]	0.4000	0.5355	0.5417	0.5517	$P_4 > P_3 > P_2 > P_1$
Proposed Method	0.4865	0.5163	0.5052	0.4902	$P_2 > P_3 > P_4 > P_1$

CONCLUSION

In this paper, we develop IVPFS, as an extension of PFS, to handle the uncertainties in more convenient way. The framework is more efficient, versatile to handle positive, negative, and neutral grades to resolve real-life uncertainties. LP method is used to obtain the optimal weights of criterias from the partially known weight information. Extended TOPSIS method has been implemented in the framework of IVPFS for the ranking purpose. To validate and check the reliability of the proposed methods we solved two real life problems. We think that our research work will be beneficial to the researchers to solve a lot of challenges in real life. As a part of our future research work IVPFS can be applied in soft set, IVPFS graphs to address different problems in the real world.

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Conflict of Interest

All the authors declare that they have no conflict of interest.

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