

Modeling Galaxy Formation in a Hierarchical Universe: A Fiducial Approach and Comparison with Observational Data

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Abstract

We have developed a detailed model to understand how galaxies form in the framework of hierarchical theories of structure formation. Our model accounts for key processes like the formation and merging of dark matter halos, the heating and cooling of gas inside these halos, the regulation of star formation driven by energy from evolving stars and supernovae, galaxy mergers, and the changes in star populations over time. This approach is very flexible and can be used with any hierarchical clustering theory. We base our star formation and galaxy merging models on insights from numerical simulations. With this, we can predict things like the number of galaxies, their brightness, color, and how fast they rotate. The study focuses on the standard cold dark matter (CDM) model, and we also examine how different assumptions—like star formation rates or galaxy mergers—affect the results. We compare our predictions to a wide range of observational data, including galaxy luminosity functions in the B and K bands, galaxy colors, the Tully-Fisher relation, faint galaxy counts, and redshift distributions at $B \sim 22$. By doing this, we can narrow down the most important physical processes and get a better idea of how galaxies form. Our model performs quite well when we use a reasonable set of parameters. It produces a more accurate galaxy luminosity function than previous models and explains the observed counts of faint galaxies in both the B and K bands, as well as their redshift distributions. However, the model has its shortcomings. It does not quite match the color of many observed elliptical galaxies.

Keywords: Galaxy formation, dark matter halos, star formation, galaxy mergers, cold dark matter (CDM)

INTRODUCTION

The evolution of dark matter halos and stellar populations in galaxies is the least uncertain aspect of galaxy formation. Over the years, we have gained significant insights into how dark matter halos form and evolve, thanks to detailed simulations and analytical models (such as those from Press and Schechter in 1974 and others). Additionally, the spectrophotometric evolution of stars, when given an initial mass function, can now be predicted reliably, at least for stars with solar metallicity. However,

when it comes to understanding the gas dynamics and star formation processes, things become more uncertain. Radiative cooling, first identified by Rees and Ostriker in 1977, plays a crucial role in this process. Without external heating sources, the gas inside the galactic halos can cool rapidly, losing pressure and collapsing toward the center, where it eventually forms stars. The energy released by stars as they evolve and explode in supernovae regulates the cooling of gas, affecting the rate at which new stars can form. Recent hydrodynamic simulations are beginning to shed light on these complex processes, helping us to understand how gas

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behaves within dark matter halos and how galaxies evolve. Despite this progress, reliable predictions about galaxy mergers and the dynamics of galaxies within different environments are still beyond our reach. Although many aspects of galaxy formation remain uncertain, the knowledge we have gained thus far is sufficient to create detailed models of how galaxies form. These models have their roots in pioneering work by scientists such as White and Rees (1978) and Larson (1974) and have been refined with the advent of more advanced simulations and observational data. These early models laid the groundwork for predicting the key properties of galaxies, such as their star formation rates, luminosities, and the relationships between galaxy size, mass, and brightness. In the 1990s, models by Lacey, Cole, White, and Frenk advanced the field by introducing new techniques and frameworks. These models can simulate galaxy formation in a hierarchical universe, where smaller structures merge to form larger ones, while incorporating complex processes, such as gas cooling, star formation, and the effects of stellar evolution. Although these early models had some successes, they also showed limitations, such as difficulties in matching the faint end of the galaxy luminosity function and the Tully-Fisher relation (which links galaxy brightness to its rotation speed). More recently, Kauffmann, White, and Guiderdoni (1993) took things a step further by using Monte Carlo simulations of galaxy mergers, integrating star formation histories, and exploring how different environmental factors shape galaxy morphology and properties. Their work provided new insights into how galaxies of different types of form, helping explain the observed Hubble sequence (the classification of galaxies by shape) and other trends related to galaxy colors, luminosities, and stellar ages [1–5].

The Interstellar Medium and Star Formation

While our understanding of dark matter and its evolution is relatively advanced, the behavior of baryonic (ordinary) matter, especially gas within galaxies, is still not fully understood. However, several key processes must be considered to accurately describe the evolution of galaxies. The most important of these are the ability of the gas to cool (radiative cooling), star formation, and the energy released by stars and supernovae, which can influence the surrounding gas. We explore how these processes are connected to the gravitational potential of the dark matter halo of a galaxy.

Radiative Cooling

Radiative cooling is a crucial process for gases inside dark matter halos. It determines the amount of gas that can cool and eventually collapse to the center of the halo to form stars. In a system without cooling, numerical simulations showed that the gas simply followed the distribution of dark matter. When shock waves hit the gas, it heats rapidly to the halo's virial temperature, a state where the gas is essentially in thermal equilibrium with the surrounding gravitational potential. The cooling process depends on factors such as the density and temperature of the gas and determines the "cooling time" (the time required for the gas to lose enough energy to condense and form stars).

We can compute the amount of gas cooled by calculating the cooling time at different points within the halo. The gas at the center of the halo cools fastest and can form stars; however, if cooling occurs too quickly (as at a high redshift), the cooling radius may extend beyond the halo's virialized region. In this case, the entire gas mass of the halo can cool, meaning that the entire halo contributes to star formation.

Radiative Cooling

Radiative cooling is a fundamental process for gas inside dark matter halos, as it determines the amount of gas that can cool and eventually collapse to form stars at the center of the halo. In the absence of cooling, the simulations show that the gas simply follows the distribution of dark matter. When shock waves hit the gas, they quickly heat up to the halo's virial temperature, where the gas is in thermal equilibrium with the surrounding gravitational potential.

Cooling efficiency depends on factors such as gas density and temperature. The time required for gas to cool and condense into stars is called the "cooling time." We can compute the amount of gas cooled

by calculating the cooling time at various points within the halo. The gas at the center of the halo cooled the fastest, leading to star formation. However, if cooling occurs too quickly, particularly at a high redshift, the cooling radius might extend beyond the halo's virialized region, causing the entire gas mass to cool and contribute to star formation.

Star Formation and Feedback

Once the gas at the center of the halo of a galaxy cools and condenses, it starts to form stars. In this process, gas is divided into three components: hot diffuse gas, cold dense gas (which forms stars), and the stars themselves. The balance between these components is regulated by the impact of the star formation and stellar evolution on the surrounding gas.

Simulations by Navarro and White (1993) showed that stars form in a gaseous disk within the dark matter halo. As stars form, they release energy primarily through supernovae, which affect the surrounding gas. This energy can heat the gas and even eject it from the system. Such feedback plays a critical role in regulating star formation: if supernovae release sufficient energy, they can push gas back into the hot, diffuse phase, thereby limiting further star formation. The amount of energy injected is governed by a factor called ν (nu), which represents the fraction of supernova energy converted into the kinetic energy of the gas.

Simulations revealed that even a modest value of ν (approximately 0.1) 10% of the energy from stars couples to the gas can significantly influence the evolution of gas, pushing some of it back into the hot phase and regulating star formation. The energy from supernovae is closely linked to the halo's gravitational potential, meaning that galaxies with more massive halos (deeper potential wells) will have different star formation histories from galaxies with shallower halos.

To model this process, we use equations that link the star formation rate to the amount of cooled gas in the system and the fraction of gas ejected back into the hot phase. The star formation rate at any given time is driven by the availability of cold gas, which depends on the amount of gas that has cooled down as well as the history of mergers (where different galaxies combine and share gas).

In this framework, the star formation rate is determined by a combination of the cooled gas mass and feedback energy from stars. The key equation for the star formation rate is [6–10]:

$$\dot{m}_* = (\text{cold gas, feedback energy})$$

Here, the amount of cold gas available is influenced by both the cooling processes and stellar feedback. The energy from supernovae and stellar evolution regulates how much gas is pushed back into the hot phase, making it available for future cooling and subsequent star formation.

MODELING STRATEGY

The Algorithm for Galaxy Formation

We outline the complete algorithm used to simulate the formation and evolution of a galaxy population within a hierarchical universe. The framework integrates the concepts discussed earlier—including halo formation and merging—with the physics of gas heating and cooling, star formation, and galaxy merging.

To specify the model, several key parameters must be set, which fall into distinct categories:

- *Cosmological parameters:* These include Ω_0 , Ω_b , H_0 , and σ_8 , which define the cosmological model. In this study, we fixed most of these parameters and adopted the standard cold dark matter (CDM) model, except for Ω_b , which can be varied.
 $\Omega_0 \rightarrow$ Total matter density (baryonic + dark matter)
 $\Omega_b \rightarrow$ Baryonic matter fraction, constrained by primordial nucleosynthesis.

H_0 → Hubble constant (current expansion rate).

σ_8 → RMS density fluctuation on $8 h^{-1}$ Mpc scales.

- *Star formation and feedback model*: The star formation and feedback model is based on numerical simulations by Navarro and White (1993). The key parameter is v , which quantifies the fraction of energy from supernovae (SNe) and evolving stars that is transferred as kinetic energy to intragalactic gas. This energy regulates both the star formation rate and the amount of gas ejected from the galaxy. Parameters α , F_{hot} , and α_{hot} were chosen to match the results of these simulations. The zero-point x_0 of the star formation rate equation was treated as a free parameter.
 v → Star formation efficiency; fraction of stellar/SNe energy heating gas.
 α → Faint-end slope of the stellar mass function or star formation scaling.
 F_{hot} → Fraction of gas heated/reheated by feedback.
 α_{hot} → Strength/scaling of supernova feedback.
 x_0 → Zero-point of the star formation rate; treated as a free parameter.
- *Initial mass function (IMF)*: We adopt a standard observationally determined IMF for stars with masses between 0.1 and 125 solar masses. Additionally, we assume that brown dwarfs form along with visible stars, so the total mass in stars is scaled by a factor of T (total stellar mass) compared to the mass of visible stars alone.
- *Galaxy merger timescale*: The timescale for the merger of galaxies within a single dark matter halo is determined by the parameters α_{mrg} and $\alpha_{\text{mrg}2}$, which are set based on simulations by Navarro, Frenk, and White (1994).
 α_{mrg} → primary scaling factor controlling merger timescale.
 $\alpha_{\text{mrg}2}$ → secondary factor for corrections due to orbital or halo properties.

Together, they determine how quickly galaxies coalesce within a halo.

Computational Steps

The computational process for evolving a realization of the block model follows these steps:

1. *Parameter setup*: First, we set all the model parameters, including the output redshift.
2. *Block model realization*: We generate a realization of the block model, typically using a total mass $M_{\text{block}} = 8 \times 10^{15} M_{\odot}$ and 20 levels of subdivision. We select the density perturbation for the top level of the model from a Gaussian distribution with width $\sigma(M_{\text{block}})$, ensuring that the realizations fairly sample both underdense and overdense regions of the large-scale structure.
 $M_{\text{block}} = 8 \times 10^{15} M_{\odot}$ → sets the mass scale of the largest region being modeled.
20 levels → defines the hierarchical subdivision to capture finer structures
 $\sigma(M_{\text{block}})$ → determines the amplitude of initial density fluctuations, ensuring realistic sampling of cosmic density variations
3. *Halo selection*: Starting with the lowest level of the hierarchy, we identify halos that formed earlier than the required output redshift.
4. *Halo parameter calculation*: For each selected halo, we compute its mass M_{H} based on its position in the hierarchy. We then calculate the halo formation redshift, which provides the physical parameters required for the model.
First, the halo's mass M_{H} is determined from the merger tree structure

Then, the formation redshift is used to calculate its density, virial radius, and other properties, which feed into galaxy formation or luminosity modeling.

This approach allows for an efficient simulation of galaxy formation, capturing the hierarchical nature of structure growth while incorporating key physical processes, such as gas cooling, star formation, and feedback mechanisms.

Fiducial Model

We evaluate our fiducial galaxy formation model by comparing its properties with observed data. We list the parameter values that define the fiducial model, grouped according to the different physical processes simulated.

Cosmology

Cold dark matter power spectrum.

- $\Omega_0 = 0.0, \Omega_\Lambda = 1, H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}, \sigma_8 = 0.67.$
- $\Omega_b = 0.06.$
 $\Omega_0 = 0.0$ (matter density), $\Omega_\Lambda = 1$ (cosmological constant),
 $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Hubble constant),
 $\sigma_8 = 0.67$ (normalization of density fluctuations).
Baryon fraction: $\Omega_b = 0.06$, consistent with primordial nucleosynthesis constraints

Star Formation and Feedback

- $\nu = 0.2$, which leads to the following parameters: $\alpha^* = -1.5, L_{\text{hot}} = 140 \text{ km s}^{-1}$, and $\alpha_{\text{hot}} = 5.5.$
- $z_0 = 2.0 \text{ Gyr.}$
 $\alpha^* = -1.5$ (faint-end slope of stellar mass function), $L_{\text{hot}} = 140 \text{ km s}^{-1}$
 $L_{\text{hot}} = 140 \text{ km s}^{-1}$ (threshold circular velocity for reheating),
 $\alpha_{\text{hot}} = 5.5$ (efficiency of supernova feedback).
Characteristic star formation timescale: $z_0 = 2.0 \text{ Gyr.}$

Stellar Population

- Salpeter Initial Mass Function (IMF), and $T=2.7.$
Parameter controlling the bright-end cutoff of the luminosity function: $T=20$

Galaxy Merging

- $r_{\text{ig}} = 0.5 r_{\text{dyn}}$, and $\alpha_{\text{mrg}} = 0.25.$
Interaction radius $r_{\text{ig}} = 0.5 r_{\text{dyn}}$, i.e., galaxies interact when separated by half the dynamical radius.
Merger efficiency: $\alpha_{\text{mrg}} = 0.25$, controlling the probability and timescale of galaxy mergers

The fiducial model is based on standard CDM cosmology, with a baryon fraction $\Omega_b = 0.06$, which is consistent with the constraints from primordial nucleosynthesis. Parameter values were selected to produce a model that fits well with the observed 5-band galaxy luminosity function.

$\Omega_b = 0.06$ ensures that the amount of normal matter in the model matches observational cosmology constraints

Luminosity Functions

We compare the blue and infrared galaxy luminosity functions of our fiducial model with observational data. The 5-band data were taken from Loveday et al. (1992) and adjusted to Johnson B using a color correction increases the numeric magnitude, meaning slightly fainter in flux, depending on the convention. The III-band data are from Mobasher, Sharples, and Ellis (1993).

The model luminosity function follows a power-law form at faint magnitudes and breaks at the bright end. This break results from the cooling criterion outlined earlier. Its position depends on the choice of the IMF, as well as the mass locked in non-luminous stars. In our fiducial model, we set $T = 2.7$, as a balance between fitting the bright, exponentially declining end of the 5-band luminosity function and the III-band data. The chosen value of T implies a mean stellar mass-to-light ratio of $15 h M_\odot/L_\odot$, which is slightly high but still consistent with observations.

$T = 2.7$ is a tuned value to fit both the bright end of the 5-band luminosity function and the III-band data.

$15 h M_\odot/L_\odot \rightarrow$ implied stellar mass-to-light ratio from your model, slightly high but acceptable

The faint-end slope of the luminosity function in our model is steeper than the estimate of Loveday et al. (1992), who found a faint-end slope of $\alpha = 0.97 \pm 0.15$. Our model has a slope of $\alpha \sim 1.5$, which is a marked improvement over other models that predict steeper slopes, such as $\alpha \sim 2$ (e.g., WF, Cole 1991, and Kauffmann et al. 1993). This improvement arises because we adjusted the feedback and merger parameters to produce a shallower faint-end slope. After these adjustments, our model fits the III-band luminosity function well. Interestingly, some studies of cluster luminosity functions have found steeper faint-end slopes (In galaxy clusters, there are relatively more faint galaxies than in the general field) than those observed in the field. A better match to both field luminosity functions could be achieved if the brightest galaxies were redder.

$\alpha = 0.97 \pm 0.15$ → indicates good alignment with observed data, showing that galaxy number density does not rise too sharply at the faint end

$\alpha \sim -1.5$ or -2 , the slope is steeper, meaning many more faint galaxies are predicted.

Colors and Star Formation Rates

One of the biggest challenges for galaxy formation models is reproducing the broad color distribution observed in galaxies, which reflects a wide variety of star formation histories. The distribution of B–K colors while also illustrating the spread in star formation rates, as measured by H α luminosities, across different galaxies.

The color distribution and range of star formation histories are largely independent of galaxy magnitude, except for the faintest galaxies. This implies that both older, redder galaxies (such as ellipticals) and younger, bluer galaxies (such as spirals) can be equally bright in the B band. In hierarchical models, more massive galaxies generally form later, making this observation challenging for traditional models.

Hierarchical models, such as ours, have typically struggled to produce luminous red elliptical galaxies. However, our model represents an improvement in this area compared with previous models (e.g., Lacey et al. 1993). Despite this progress, fully resolving the issue of luminous red elliptical galaxies remains an unsolved problem for this type of galaxy formation model.

CONCLUSION

In this study, we developed and analyzed a fiducial model of galaxy formation within the framework of hierarchical CDM cosmology. By combining analytical prescriptions with insights from numerical simulations, the model integrates key physical processes—radiative cooling, star formation, stellar feedback, and galaxy merging—into a coherent framework.

The fiducial model successfully reproduces several important observational features:

- *Luminosity functions*: The model produces blue and infrared luminosity functions with realistic shapes, including a break at the bright end and a significantly improved faint-end slope compared with earlier models. The faint-end slope ($\alpha \sim 1.5$) is steeper than the observational estimates but still represents progress over earlier predictions.
 $\alpha \sim 1.5$ → indicates a higher abundance of faint galaxies than typically observed, but still closer to reality compared to older models
- *Stellar mass-to-light ratios*: The chosen parameter values yield a stellar mass-to-light ratio consistent with observational constraints, indicating a reasonable balance between visible and non-luminous stellar populations.
- *Star formation and galaxy colors*: This model reproduces a broad distribution of galaxy colors and star formation rates, capturing the diversity observed in the local universe. However, it still faces challenges in generating bright red elliptical galaxies, highlighting the ongoing difficulty in hierarchical formation scenarios.
- *Feedback and merger effects*: Adjustments to feedback strength and merger timescales are critical in improving the match between model predictions and observational data. These parameters play a key role in regulating star formation and shaping galaxy properties.

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