

Numerical Investigation of Slip Conditions in Nanofluid Flow Induced by a Nonlinearly Stretching Sheet: A bvp4c Approach

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Abstract

The present work describes a numerical analysis of the steady, laminar flow over a nonlinear stretching sheet in a nanofluid medium. Slip conditions in terms of velocity, temperature, and concentration have been introduced into the mathematical model to capture the impact of interfacial mass transfer on the stretching surface. The problem is modeled to include nanoparticle transportation in terms of Brownian diffusion and thermophoresis, thereby affecting the temperature and concentration profiles. The linked set of ordinary differential equations has been obtained by applying appropriate similarity transformations. The subsequent boundary value problem is solved by a fourth–fifth order adaptive RK45-shooting scheme. The numerical results are then validated by the MATLAB bvp4c solver, which follows a collocation technique, and the similarity between the two results indicates the reliability of the computational technique. The impact of the velocity, thermal, and concentration slip factors, as well as the Prandtl number on the velocity, temperature, and nanoparticle concentration profiles is analyzed. An increase in velocity slip will result in the weakening of the momentum transfer between the stretching surface and the fluid; hence, the velocity in the boundary layer will be decreased. Similarly, both the temperature profile and the thickness of the thermal boundary layer will decrease as thermal slip increases. Furthermore, an increase in the concentration slips parameter results in a decrease in both the local Sherwood number and the thickness of the concentration boundary layer for a fixed Lewis number. In summary, the study reveals how the effects of different types of slips can greatly affect the momentum, heat, and mass transfer in nanofluids.

Keywords: Nanofluid; boundary layer; slip effect; shooting method; Bvp4c solver

INTRODUCTION

The study of two-dimensional flow over a stretching sheet has gained considerable attention in the field of fluid mechanics owing to the pioneering contributions of Crane [1]. This area has acquired considerable importance owing to various applications involving stretching surfaces, such as plastic sheet extrusion, production of glass fibers, cooling of metallic sheets, aerodynamics, and the production of paper and glass. Another important contribution to thermal-fluid studies is the definition of the concept nanofluid [2] as a base fluid carrying nanoscale particles homogeneously dispersed within the base fluid and having sizes usually below 50 nm. Some examples of base fluids are water, oil, and ethylene glycol, while some examples of suspended particles are metal, oxide, or carbon nanotubes. Since nanoparticle addition can enhance the thermal conductivity, nanofluids are becoming attractive fluids to enhance the

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transfer of heat and mass. Some areas where nanofluids can be used include manufacturing processes, heat exchangers, microchannels, thermal management of electronics, and biomedicine. Figure 1 shows some applications of nanofluids.

In the last few years, convective heat transfer in nanofluids has increased.

Nanofluids can provide better thermal conductivity and heat transfer performance compared to traditional base fluids. This feature is important because conventional fluids have comparatively low thermal conductivity and heat transmission capacity. Common base fluids used in industry include water, oil, and ethylene glycol. According to the authors in [3], nanofluids possess good stability, rheological characteristics, increased thermal conductivity, and no adverse effect on pressure drop. Consequently, numerous studies [4–8] have conducted research on nanofluids and mathematical modeling of these fluids in the context of natural convection heat transfer.

Considering the pioneering work [9] on flow over a moving surface, many studies have attempted to apply the models for flow over stretching surfaces in more complex settings. The work [10] focused on mixed-convection micropolar fluid flow near a linearly stretching surface under uniform suction. Later, [11] analyzed the problem for micropolar fluid near a linearly stretching surface, while [12] extended their study to a nonlinearly stretching surface. The Marangoni effects and interfacial phenomena on the flow of hydrogen oxide nanofluids over a stretching sheet have been studied [13] by applying comparative machine learning algorithms for the prediction of fluid flow and heat transfer features. The use of an artificial neural network algorithm [14] for the analysis of non-linear hemodynamics with fractional Jeffrey ternary hybrid nanofluid flow in multi-stenotic arteries has demonstrated the efficiency of ANN models in modeling complex biofluid dynamics phenomena. Heat and mass transfer were examined in a viscous fluid over a nonlinear sheet, ignoring energy dissipation due to viscosity [15]. An MHD hybrid nanofluid with bioconvection flow in the presence of gyrotactic microorganisms over an inclined cylinder focused on magnetic and bioconvection effects on thermosolutal transport [16]. In addition, [17] analyzed numerically heat and flow over a nonlinear stretching surface considering two different thermal boundary conditions.

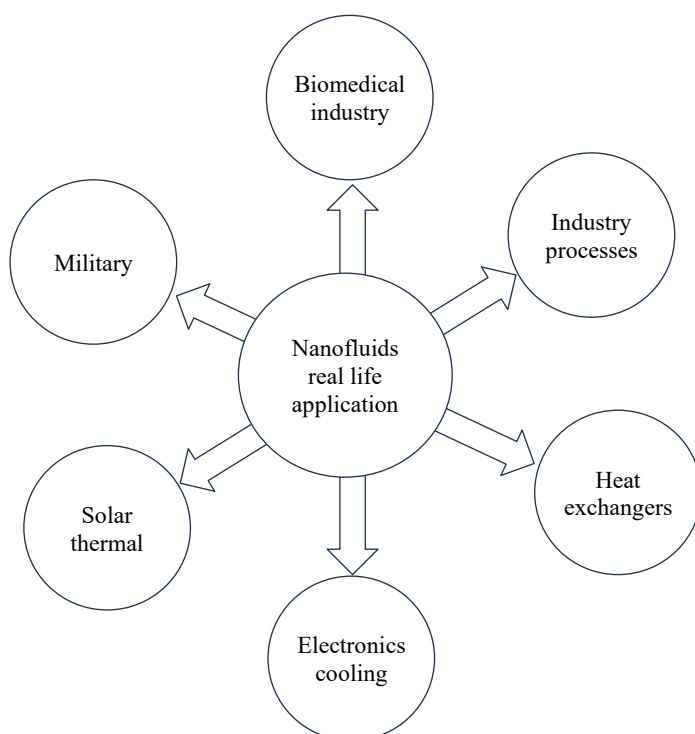


Figure 1. Application of nanofluids.

Fractional boundary value problems were solved using the collocation and shooting approaches [18]. The idea of using an artificial neural network approach to predict heat and mass transfer in Cu–Al₂O₃–TiO₂–SiO₂–MWCNT hybrid nanofluids intended for use in solar thermal applications [19]. Their findings validated the applicability of machine learning algorithms in predicting the thermal performance of hybrid nanofluids [20].

Multilayer neural networks investigate entropy generation in penta-hybrid nanofluid flow relevant to rocket-engine nozzle cooling, emphasizing the emerging role of artificial intelligence in improving sophisticated nanofluid-based thermal systems. Similarly, [21] a nonlinear shooting technique was used to deal with two-point boundary-value problems. The shooting technique was used to find solutions to a system of coupled nonlinear dimensionless ordinary differential equations [22]. The foregoing research efforts are helpful for our current study in terms of computational and modeling perspectives.

The current study aimed to investigate the impact of velocity, thermal, and concentration slip conditions on the boundary layer flow and transport in a nanofluid through a nonlinear stretching sheet. Although slip phenomena have been widely incorporated into related models, the combined role of these three slip mechanisms in the current configuration has received comparatively limited attention. The nonlinear partial differential equations that govern the system were converted through suitable similarity transformations into a non-linear system of ODEs. Numerical solutions were obtained independently using an adaptive RK45 shooting technique and MATLAB's bvp4c boundary-value solver. bvp4c uses the Lobatto IIIA collocation method, whereas the shooting method solves the original boundary value problem by first converting it into an initial value problem. Agreement between the two numerical solutions, as well as comparisons with available benchmark results, provides evidence of the accuracy and numerical robustness of the adopted approach. The impact of controlling dimensionless numbers on velocity, temperature, and nanoparticle concentration profiles is then discussed with the help of graphical illustrations and tabulated data.

MODEL CONFIGURATION

In the present model, the stretching sheet velocity is defined as $u_w = ax^n$, where a is a positive constant and $n \geq 0$ represents the power-law index governing the nonlinear stretching behavior. Consequently, the surface velocity depends nonlinearly on the streamwise distance x . The physical model is shown schematically in Figure 2, and it is accompanied by the coordinate system used in the model is shown. The stretching wall is kept at constant temperatures and concentrations T_w and C_w , respectively, which are greater than those of the ambient fluid T_∞ and C_∞ . The governing boundary-layer equations describing momentum, thermal energy, mass transport, and nanoparticle concentration, including slip effects, are written in the two-dimensional Cartesian framework (x, y) in accordance with Refs. [23] and [24].

GOVERNING EQUATIONS

Equation of Mass Conservation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Equation of momentum

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

Equation of Temperature Distribution

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (3)$$

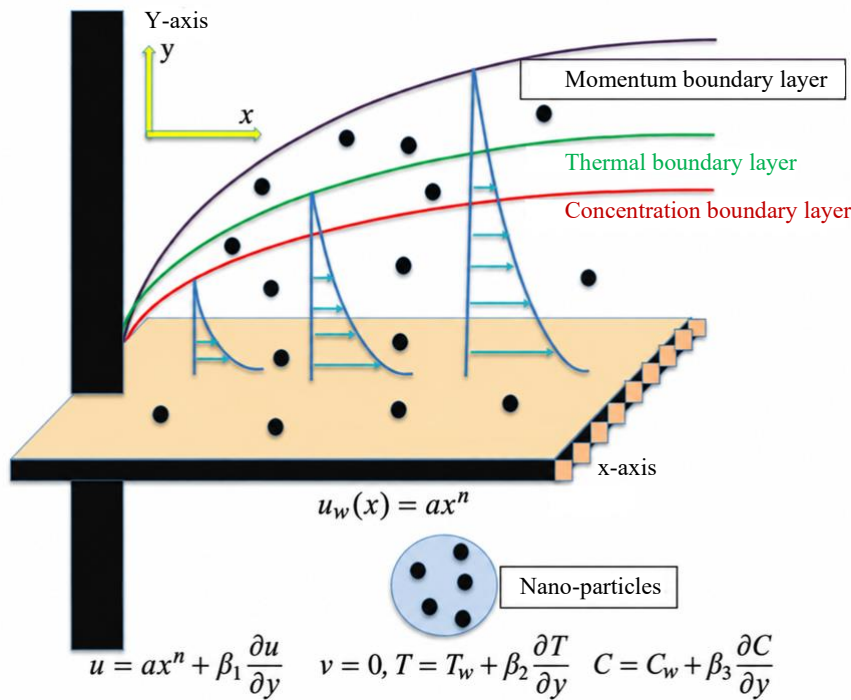


Figure 2. Physical model and cartesian co-ordinate system.

Equation of Concentration Distribution

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right), \quad (4)$$

where

$$\alpha = \frac{k}{(\rho C)_f}, \tau = \frac{(\rho C)_p}{(\rho C)_f}$$

The model is based on the classical Buongiorno nanofluid approach, where Brownian motion and thermophoresis are the key transport mechanisms for nanoparticles. Therefore, these effects are included in the energy and concentration equations, whereas nanoparticle-related force terms are excluded from the momentum equation. Such a treatment is appropriate for a dilute nanoparticle suspension, where the direct contribution of nanoparticles to the momentum balance is negligible. However, their indirect effects continue via the interlinked mechanisms of heat and mass transport.

Boundary Conditions

The boundary conditions for Equations (1)–(4) were suggested by researchers in previous studies.

$$\begin{cases} u = ax^n + \beta_1 \frac{\partial u}{\partial y}, v = 0, T = T_w + \beta_2 \frac{\partial T}{\partial y}, C = C_w + \beta_3 \frac{\partial C}{\partial y}, & \text{at } y = 0, \\ u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty & \text{as } y \rightarrow \infty, \end{cases} \quad (5)$$

The streamwise and transverse velocity components are denoted by u and v , while T and C represent the temperature and concentration of nanoparticles, respectively. The physical properties of the base fluid are represented using subscripts f such that their density is ρ_f , thermal diffusivity is α , and kinematic viscosity is ν . The parameter a follow $a > 0$, while n is used to describe the power-law dependence of the stretching velocity. D_B and D_T represent the coefficients due to Brownian diffusion and thermophoretic diffusion of nanoparticles, respectively. In addition, $\tau = \frac{(\rho C)_p}{(\rho C)_f}$ is the ratio between

the heat capacity of the nanoparticle material and that of the base fluid, where ρ_p is the density of the particles. Parameters β_1 , β_2 , and β_3 refer to the slip effects in terms of velocity, thermal, and concentration, respectively. In the case when $\beta_1 = \beta_2 = \beta_3 = 0$, slip effects vanish. Under this condition, the fluid in immediate contact with the stretching wall has no relative motion with respect to the surface, as commonly assumed in conventional viscous-flow models.

Transformation of Equations

To obtain a self-similar representation of the governing equations, the following similarity transformations are adopted [25]:

$$\begin{cases} u = ax^n f'(\eta), v = -\sqrt{\frac{va(n+1)}{2}} x^{\frac{n-1}{2}} \left(f(\eta) + \frac{n-1}{n+1} \eta f'(\eta) \right), \\ \eta = \sqrt{\frac{a(n+1)}{2v}} x^{\frac{n-1}{2}} y, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \end{cases}$$

With the expressions provided above, Equation (1) is automatically satisfied, while Equations (2)–(4) transform into the following:

$$f''' + ff'' - \left(\frac{2n}{n+1} \right) f'^2 = 0 \quad (6)$$

$$\theta'' + Pr[f\theta' + Nb\theta'\phi' + Nt\theta'^2] = 0 \quad (7)$$

$$\phi'' + Le f\phi' + \left(\frac{Nt}{Nb} \right) \theta'' = 0 \quad (8)$$

After applying the transformations based on similarity, the basic equations can be transformed into nonlinear ordinary differential equations, as shown in Equations (6)–(8). The boundary value problem is then solved numerically using the transformed boundary conditions.

$$\begin{cases} f(0) = 0, f'(0) = 1 + \delta f''(0), \theta(0) = 1 + \beta \theta'(0), \\ \phi(0) = 1 + \lambda \phi'(0), f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0, \end{cases} \quad (9)$$

Here, primes represent derivatives with respect to the similarity variable η , with the dimensionless thermophysical numbers defined as follows:

$$\begin{aligned} Pr &= \frac{v}{\alpha}, Nb = \tau \frac{D_B(C_w - C_\infty)}{v}, Nt = \tau \frac{D_T(T_w - T_\infty)}{T_\infty v}, Le = \frac{v}{D_B}, \\ \delta &= \beta_1 \sqrt{\frac{a(n+1)}{2v}} x^{\frac{n-1}{2}}, \beta = \beta_2 \sqrt{\frac{a(n+1)}{2v}} x^{\frac{n-1}{2}}, \lambda = \beta_3 \sqrt{\frac{a(n+1)}{2v}} x^{\frac{n-1}{2}}, \end{aligned} \quad (10)$$

The dimensionless parameters δ , β , and λ should be regarded as local similarity parameters because they explicitly involve the streamwise coordinate x . In accordance with the local similarity formulation [25], the numerical analysis was performed at a fixed location along the stretching surface. Consequently, these parameters were held constant while applying the similarity transformation, which allowed the original PDEs to be reformulated into a self-similar ODE system depending only on η . This treatment follows the standard local similarity framework frequently adopted for nonlinear stretching-sheet boundary-layer problems involving slip conditions.

The dimensionless parameters used in the present formulation are Pr , the Prandtl number; Le , the Lewis number; Nb , the Brownian motion parameter; and Nt , the thermophoresis parameter. In addition, δ , β , and λ characterize the local velocity, thermal, and concentration slip effects, respectively. A special limiting case is obtained by setting $n = 1$ and $Nt = Nb = 0$ in Equations (7) and (8), for which the present boundary-value problem recovers the conventional flow and heat-transfer problem over a stretching surface in a viscous fluid. Subsequently, attention is directed toward the engineering characteristics,

namely, the local skin-friction coefficient C_f , local Nusselt number Nu_x , and local Sherwood number Sh_x , whose definitions are provided below [26, 27].

The relevant engineering quantities, namely, the local skin friction coefficient, local Nusselt number, and local Sherwood number, are expressed as

$$C_f = \frac{\tau_w}{\rho u_w^2}, Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, Sh_x = \frac{x h_m}{D_B(C_w - C_\infty)}$$

The surface shear stress τ_w , wall heat flux q_w , and wall mass flux h_m are expressed as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, h_m = -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}.$$

Using the above expressions, we obtain the following relations:

$$\begin{aligned} C_f(Re_x)^{\frac{1}{2}} &= \sqrt{\frac{n+1}{2}} f''(0) \\ Nu_x(Re_x)^{-\frac{1}{2}} &= -\sqrt{\frac{n+1}{2}} g'(0), \\ Sh_x(Re_x)^{-\frac{1}{2}} &= -\sqrt{\frac{n+1}{2}} h'(0), \end{aligned}$$

The local Reynolds number is defined as $Re_x = \frac{U_w x}{\nu}$. For convenience and consistency with the validation results presented in Table 1, the skin-friction coefficient is reported throughout the manuscript in terms of its magnitude, i.e., $C_f(Re_x)^{\frac{1}{2}} = \sqrt{\frac{n+1}{2}} f''(0)$, since $f''(0)$ is negative for the present flow configuration. Note that this reduces to $-f''(0)$ in the special case $n = 1$, which corresponds to the validation comparison in Table 1.

Numerical Solution Methodology

The governing equations, as defined in Equations (6)–(8), as well as their corresponding boundary conditions defined in Equation (9), were numerically solved. To solve these equations numerically, similarity transformations were used to reduce the non-linear partial differential equations to a system of non-linear ordinary differential equations. The nonlinearity of the system is such that there are no closed-form analytical solutions available for it; hence, numerical solutions were obtained using the shooting technique combined with the MATLAB function ode45 [28, 29]. An independent verification was then performed using the MATLAB bvp4c routine, which solves boundary-value problems through a Lobatto IIIA collocation scheme.

To apply the shooting method, the unknown parameters $f''(0)$, $\theta'(0)$, and $\phi'(0)$ are considered shooting parameters; thus, the original boundary value problem is converted into an initial value problem. Trial values are first assigned to these unknown slopes, after which the corresponding first-order equations are integrated numerically from the wall, $\eta = 0$, to a large finite approximation of the far-field boundary, $\eta = \eta_\infty$, using the adaptive ode45 solver. The numerical solution obtained at the outer boundary is then checked against the required far-field conditions through the residuals of $f'(\eta_\infty)$, $\theta(\eta_\infty)$, and $\phi(\eta_\infty)$. These residuals are provided to fsolve, which systematically updates the trial slopes until the far-field conditions are fulfilled within the specified convergence tolerance. The iteration is continued until a converged solution is obtained. Before carrying out the computation, the governing system is transformed into its first-order form following the standard methodology reported in Refs. [30–32]. The overall computational algorithms for the shooting and bvp4c methods are illustrated in Figures 3 and 4, respectively.

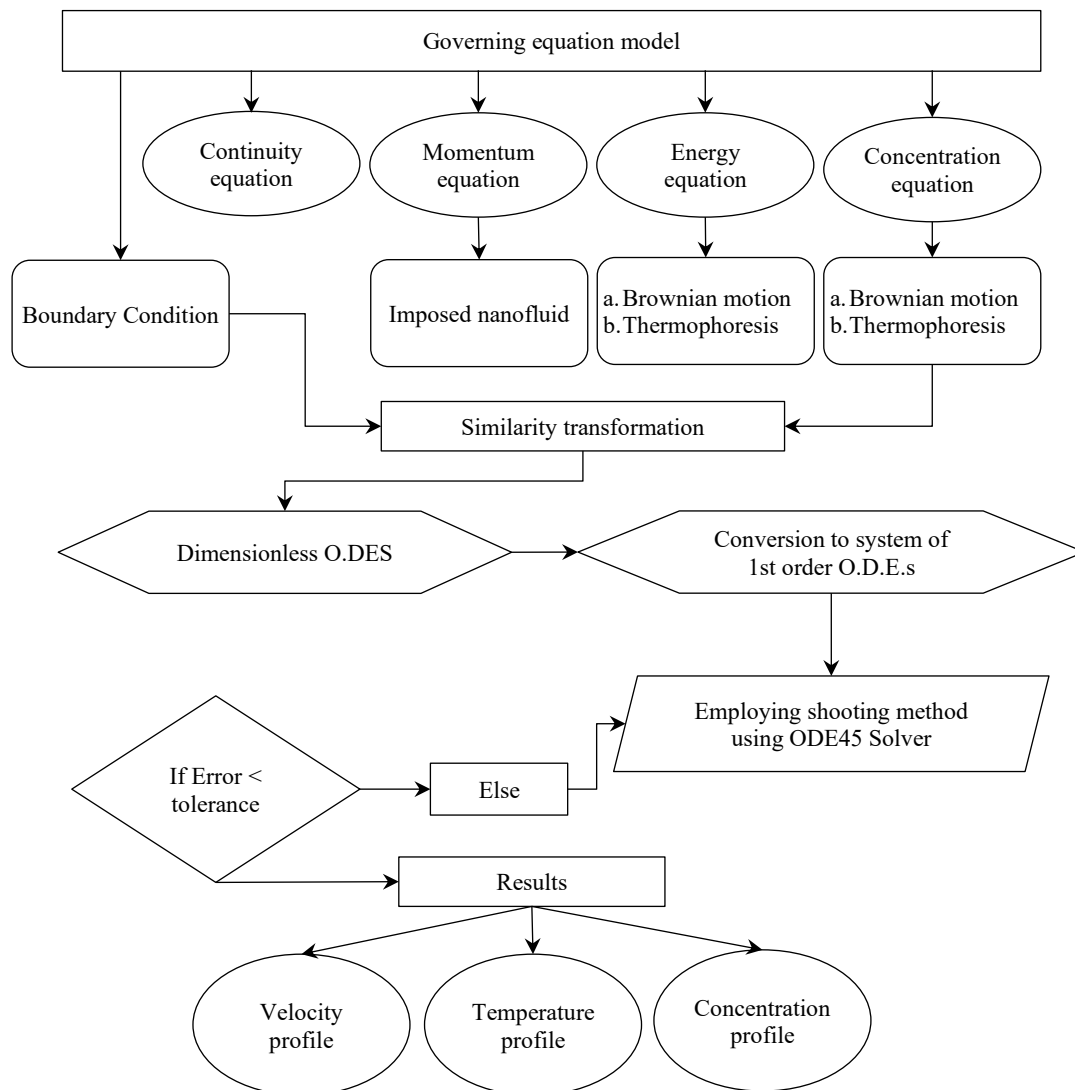


Figure 3. Shooting method schematic diagram.

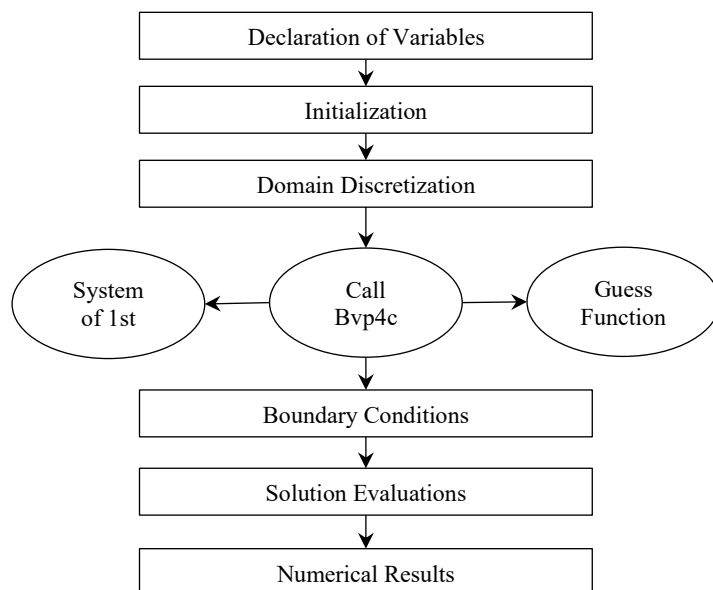


Figure 4. Bvp4c flow diagram.

An equivalent initial-value problem was adopted to solve the boundary-value problem numerically. The governing equations were first transformed into a system of first-order equations, and the conditions specified at infinity were applied at a sufficiently large computational boundary, η_∞ . The unknown initial slopes s_1, s_2 , and s_3 served as shooting parameters and were iteratively modified using MATLAB's `fsolve` routine until the residuals associated with the far-field conditions met the specified convergence criterion.

The transformation is defined as

$$\begin{aligned} z_1' &= z_2, \\ z_2' &= z_3, \\ z_3' &= -z_1 z_3 + \frac{2n}{n+1} z_2^2, \\ z_4' &= z_5, \\ z_5' &= -\text{Pr} + (z_1 z_5 + N_b z_5 z_7 + N_t z_5^2), \\ z_6' &= z_7, \\ z_7' &= -Le z_1 z_7 - \frac{N_t}{N_b} z_5'. \end{aligned}$$

The associated initial conditions at the wall, ($\eta = 0$) are

$$\begin{aligned} z_1(0) &= 0, \\ z_2(0) &= 1 + \delta z_3(0), \\ z_3(0) &= s_1, \\ z_4(0) &= 1 + \beta z_5(0), \\ z_5(0) &= s_2, \\ z_6(0) &= 1 + \lambda z_7(0), \\ z_7(0) &= s_3, \end{aligned}$$

where $s_1 = f''(0)$, $s_2 = \theta'(0)$, and $s_3 = \phi'(0)$ are the unknown initial slopes to be determined such that the asymptotic boundary conditions are satisfied.

$$z_2(\eta_\infty) \rightarrow 0, z_4(\eta_\infty) \rightarrow 0, z_6(\eta_\infty) \rightarrow 0,$$

are enforced at a suitably large finite value of η_∞ . During the shooting procedure, the unknown initial slopes s_1, s_2 , and s_3 are iteratively updated using the MATLAB routine `fsolve` until all the far-field boundary requirements are simultaneously fulfilled within the prescribed convergence tolerance.

The transformed problem was treated as an initial-value system in which the unknown starting values $z_3(0), z_5(0), z_7(0)$ were adjusted to ensure satisfaction of the boundary conditions in the far-field region $\eta = \eta_\infty$. Carefully selected convergence settings were applied to both numerical schemes to obtain reliable and repeatable results. In the shooting procedure, MATLAB's adaptive `ode45` integrator was used with relative and absolute error tolerances of 10^{-8} . The corresponding unknown initial slopes were successively corrected through the `fsolve` routine until the far-field residuals satisfied the required tolerance and the numerical solution reached convergence. The boundary-value problem was also numerically solved independently by the MATLAB routine `bvp4c` with relative tolerance of 10^{-8} and absolute tolerance of 10^{-10} . The semi-infinite physical domain used in these calculations was truncated to the finite domain $0 \leq \eta \leq 13$, discretized by an equidistant grid of 200 points. The initial solution supplied to the solver was $f(\eta) = 1 - e^{-\eta}$, $f'(\eta) = e^{-\eta}$, and $f''(\eta) = -e^{-\eta}$ while the default adaptive

mesh refinement of MATLAB was retained. The influence of the computational boundary was examined by changing η_∞ between 11 and 14. Since the velocity profiles and engineering quantities remained essentially unchanged, $\eta_\infty = 13$ was adopted in the remaining simulations. The strong consistency between the shooting method, the bvp4c solutions, and the available reference results demonstrates the reliability and numerical stability of the adopted methodology.

Graphical Results and Discussion

The nonlinear system, as per Equations (6)–(8), was solved numerically while satisfying the conditions as per Equation (9). A shooting approach was adopted to handle the boundary-value formulation owing to its effectiveness and convenience for nonlinear problems of this type. MATLAB's ode45 solver was used to integrate the resulting system after its conversion into an appropriate initial-value problem. As an additional validation step, solutions were also obtained using the built-in bvp4c solver in MATLAB. A very close agreement between the two sets of numerical results was obtained, providing confidence in the accuracy and reliability of the adopted computational procedure.

The accuracy of the numerical scheme was verified using the case where the skin friction coefficient behaved under zero velocity slip limit ($\delta = 0$). As presented in Table 1, the obtained numerical results for the RK45 shooting approach and bvp4c algorithm in MATLAB were compared [17, 33]. To carry out the error analysis, the results [33] were chosen as the benchmark solution. The comparison reveals a very good agreement between the results because the relative percentage errors obtained are smaller than 0.23% for each value of the stretching parameter. This result clearly shows the reliability of the adopted method of solving.

The values of skin friction, Nusselt number, and Sherwood number are provided in Tables 2 and 3 because they are important indices for momentum, heat, and mass transfer, respectively. Table 2 lists the influences of the velocity slip, thermal slip, and concentration slip parameters, which are denoted as (δ) , (β) , and (λ) , respectively. Table 3 lists the influence of the power law index (n), Prandtl number (Pr), and thermophoresis parameter (Nt). The tabulated findings provide an extensive parametric analysis of the transport properties of the surface and can be considered a valuable addition to the findings depicted graphically later.

Table 1. Validation of the present numerical methods through comparison of the skin-friction coefficient $-f''(0)$ with the published results [17] (Runge–Kutta method) and [33] (Homotopy Analysis Method). The relative errors are calculated with respect to the published results [33].

n	Cortell (RK)	Adem & Kishan (HAM)	Present (shooting)	Rel. error (%)	Present (bvp4c)	Rel. error (%)
0	0.627547	0.628316	0.627557	0.1208	0.627562	0.1200
0.2	0.766758	0.768579	0.766838	0.2265	0.766845	0.2256
0.5	0.889477	0.889230	0.889547	0.0356	0.889552	0.0362
0.75	0.953786	0.953811	0.953958	0.0154	0.953966	0.0163
1	1.000000	1.000000	1.000001	0.0001	1.000008	0.0008
1.5	1.061587	1.061690	1.061601	0.0084	1.061609	0.0076
3	1.148588	1.148540	1.148594	0.0047	1.148592	0.0045
7	1.216847	1.216500	1.216852	0.0289	1.216850	0.0288
10	1.234875	1.234420	1.234876	0.0369	1.234875	0.0369
20	1.257418	1.256830	1.257424	0.0473	1.257424	0.0473
100	1.276768	1.276050	1.276774	0.0567	1.276774	0.0567

Table 2. Effects of the velocity slip parameter (δ), thermal slip parameter (β), and concentration slip parameter (λ) on the skin-friction coefficient, local Nusselt number, and local Sherwood number.

δ	β	λ	$-C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$	$Sh_x Re_x^{-1/2}$
0.00	0.20	0.30	0.000001	0.601026	0.455317
0.20			0.776379	0.556288	0.424508
0.40			0.641149	0.524479	0.402369
0.60			0.549093	0.499938	0.385150
0.00	0.00	0.30	1.000001	0.671770	0.426654
	0.20		1.000001	0.601026	0.455317
	0.40		1.000001	0.542287	0.479469
			1.000001	0.493095	0.499933
0.00	0.20	0.00	1.000001	0.585437	0.593234
		0.20	1.000001	0.596680	0.493586
		0.40	1.000001	0.604760	0.422544
		0.60	1.000001	0.610845	0.369353

Table 3. Effects of the power-law index (n), Prandtl number (Pr), and thermophoresis parameter (Nt) on the skin-friction coefficient, local Nusselt number, and local Sherwood number.

n	Pr	Nt	$-C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$	$Sh_x Re_x^{-1/2}$
0.0	2.0	0.2	0.443749	0.447852	0.348424
0.5			0.770369	0.528711	0.403791
1.0			1.000001	0.601026	0.455317
1.5			1.186907	0.666119	0.502327
1.0	1.0	0.2	1.000001	0.445188	0.531464
	2.0		1.000001	0.601026	0.455317
	3.0		1.000001	0.687382	0.411751
	5.0		1.000001	0.778096	0.367554
1.0	2.0	0.1	1.000001	0.635189	0.574604
		0.2	1.000001	0.601026	0.455317
		0.3	1.000001	0.568944	0.355339
		0.4	1.000001	0.538848	0.272372

Effect of the Stretching Parameter n

The influence of the stretching parameter n on the velocity profile $f'(\eta)$ in nondimensional form is illustrated in Figures 5 and 6 for the no-slip and slip conditions, respectively. In both situations, increasing n suppresses the velocity profile and reduces the thickness of the momentum boundary layer. This phenomenon can be attributed to the non-linear stretching velocity of the sheet, which is governed by the power-law index n . As n increases the stretching motion becomes increasingly nonlinear, producing stronger velocity variations near the wall and a more rapid reduction in velocity away from the surface. The momentum field is therefore confined to a narrower region. The same qualitative response to n is maintained when velocity slip is considered; however, slip reduces the amount of momentum transferred directly from the moving surface to the neighboring fluid. In consequence, the velocity profile for slip flow remains lower than its corresponding no-slip counterpart.

Effect of the Velocity Slip Parameter δ

Figure 7 shows the effect of changes in the velocity slip parameter δ on the velocity field. With an increase in the value of δ , there is a continual decrease in the fluid velocity and momentum boundary layer thickness. Physically, the slip condition weakens the interaction between the stretching wall and the neighboring fluid by allowing a finite relative velocity at the surface. Consequently, only a reduced portion of the wall-induced motion is communicated to the fluid, which reduces the transport of momentum in the boundary layer.

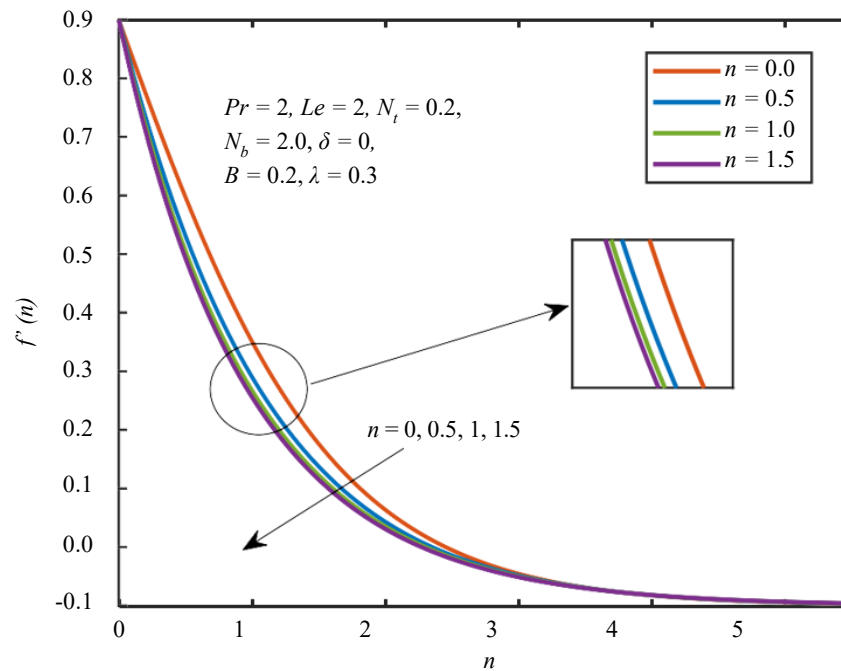


Figure 5. Velocity distribution with no velocity slip ($\delta = 0$) for $N_t = N_b = 0.2$, $Le = Pr = 2$.

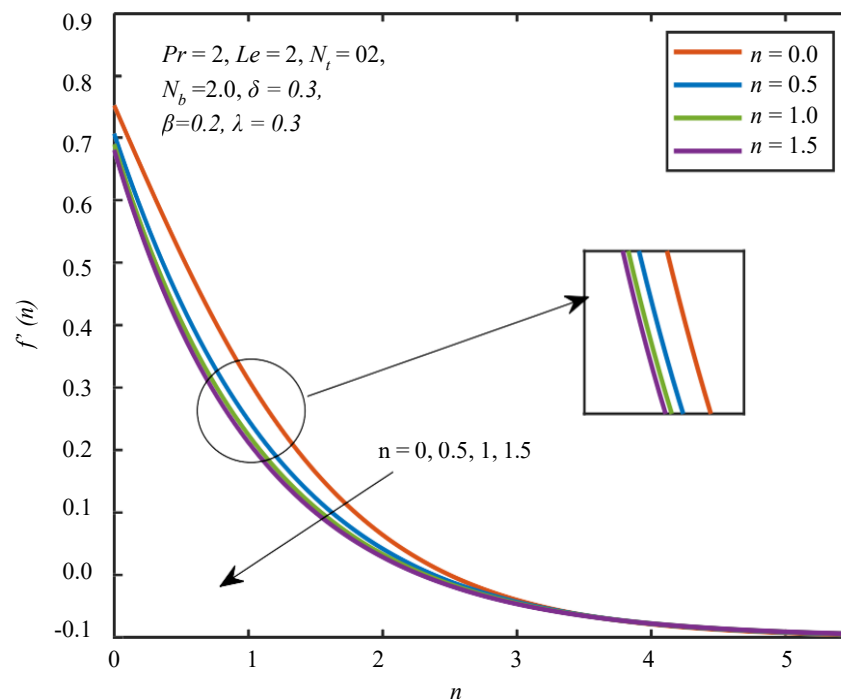


Figure 6. Velocity distribution with velocity slip ($\delta = 0.3$) for $N_t = N_b = 0.2$, $Le = Pr = 2$.

Therefore, the velocity distribution decays more rapidly toward its free-stream value. This effect is consistent with the engineering quantities reported in Table 2, which exhibit a reduced skin friction coefficient, local Nusselt number, and local Sherwood number with increasing δ , reflecting reduced surface transport characteristics.

Effect of the Prandtl Number Pr

The effect of the Prandtl number (Pr) on the temperature and nanoparticle concentration profiles is illustrated in Figures 8 and 9, respectively. As Pr increases, the temperature profile decreases, resulting

in the shrinking of the thermal boundary layer near the stretching surface. In physical terms, a high Prandtl number indicates weak thermal diffusivity, which implies that heat cannot diffuse effectively in the fluid. Therefore, a sharper temperature gradient develops near the wall, leading to an increase in the local Nusselt number, as demonstrated in Table 3. Because the energy and concentration fields are coupled through thermophoretic effects, this modification of the temperature field also influences nanoparticle migration. The restricted thermal region limits particle dispersion away from the wall. Thus, the concentration boundary layer thickness is reduced, and the mass transfer rate increases.

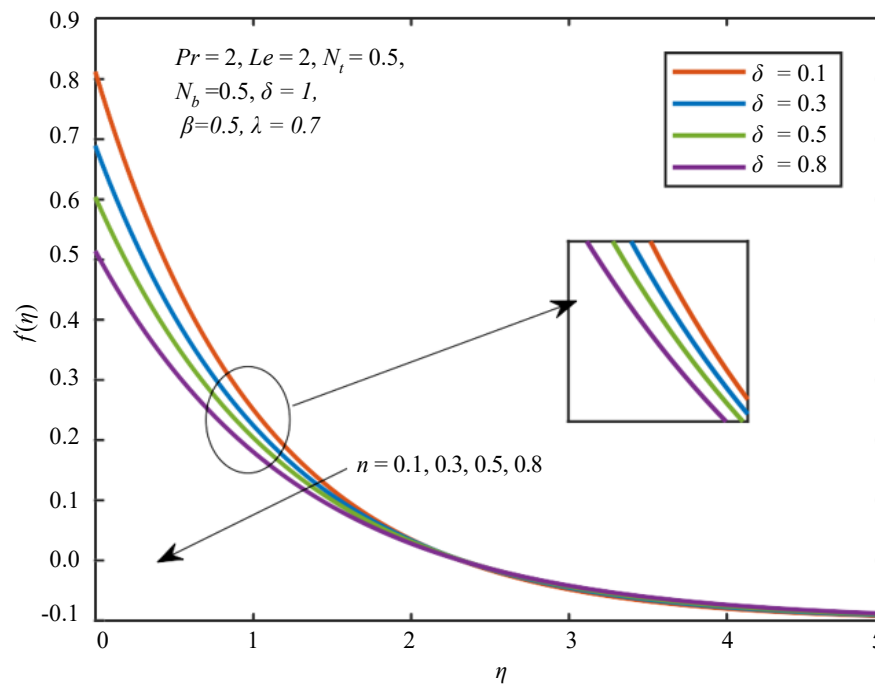


Figure 7. Effect of velocity slip parameter δ on $f'(\eta)$ for $n = 1$, $Nt = Nb = 0.5$, $Le = Pr = 2$.

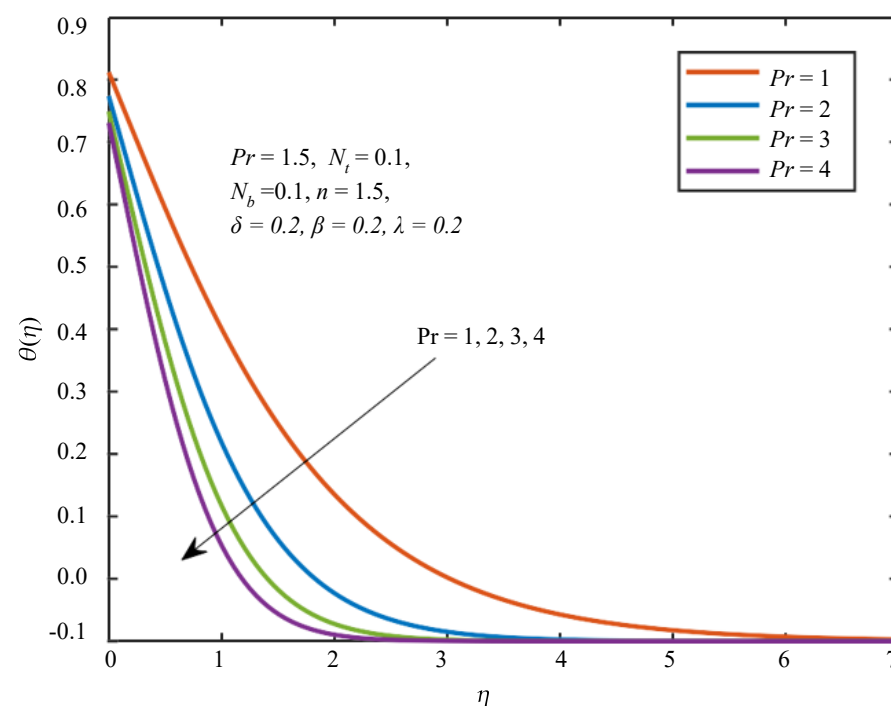


Figure 8. Effect of Prandtl number Pr on the temperature profile $\theta(\eta)$ for $\delta = \beta = \lambda = 0.2$, $Nt = Nb = 0.1$, $Le = n = 1.5$.

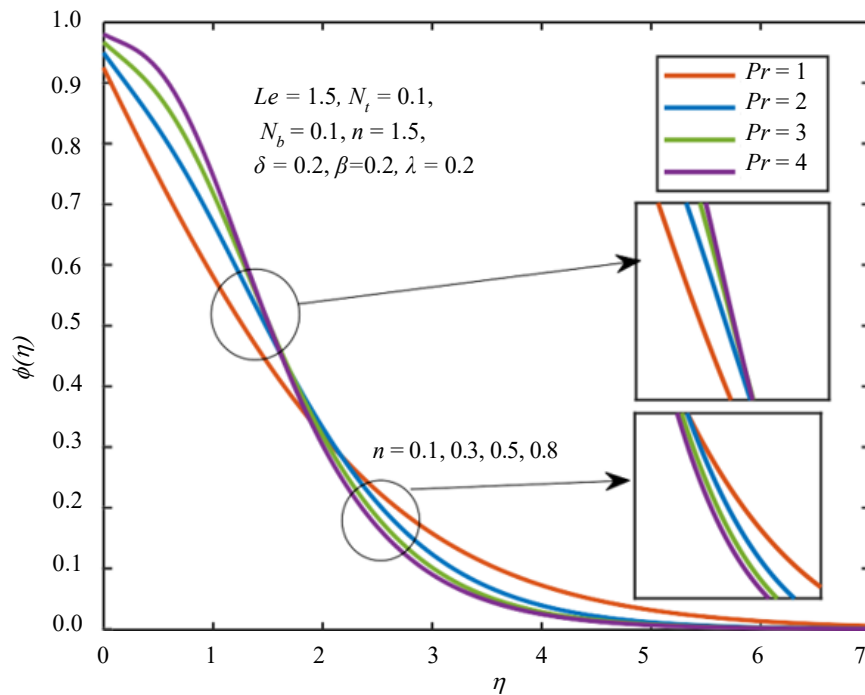


Figure 9. Effect of Prandtl number Pr on the concentration profile $\phi(\eta)$ for $\delta = \beta = \lambda = 0.2$, $Nt = Nb = 0.1$, $Le = n = 1.5$.

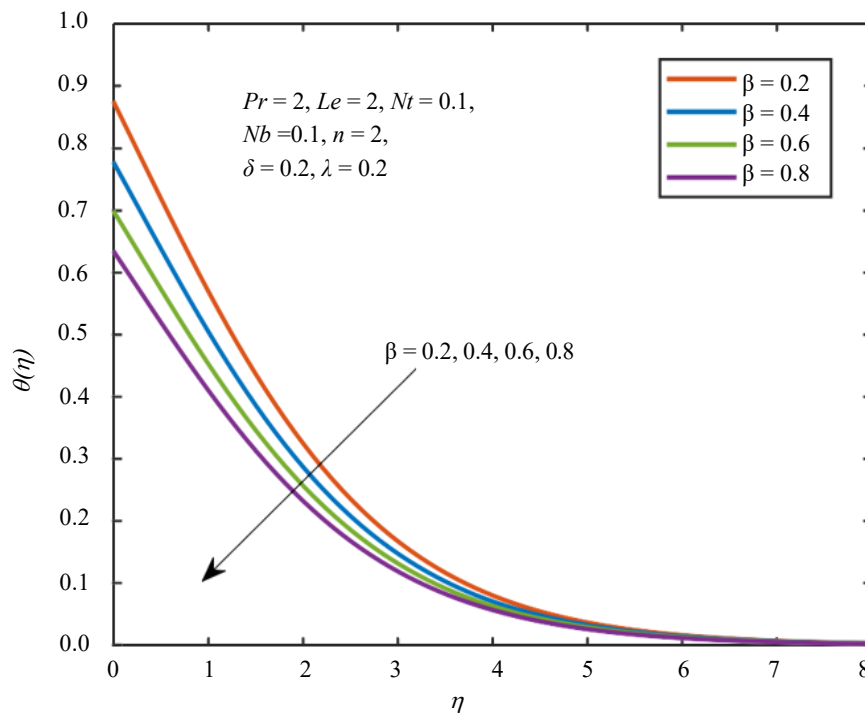


Figure 10. Effect of thermal slip parameter β on the temperature profile for $\delta = \lambda = 0.2$, $Nt = Nb = 0.1$, $Le = n = Pr = 2$.

Effect of the Thermal Slip Parameter β

Figures 10 and 11 show the effect of the parameter β on the profiles of temperature and nanoparticle concentration distribution. An increase in β results in a decrease in both. This behavior is associated with the temperature discontinuity introduced at the stretching surface by thermal slip, which weakens the transfer of thermal energy from the wall to the adjacent fluid. Consequently, the fluid receives less

heat, resulting in a lower temperature field and a thinner thermal boundary layer. Because the energy and concentration fields are coupled through Brownian diffusion and thermophoretic effects, the reduction in temperature also influences nanoparticle transport, causing the concentration profile and concentration boundary layer to decrease. Table 2 validates these findings, whereby the local Nusselt number decreases, suggesting low thermal energy transfer, while the local Sherwood number increases, suggesting high mass transfer at the surface.

Effect of the Concentration Slip Parameter λ

Figure 12 depicts the effect of the concentration slip factor λ on the nanoparticle concentration distribution. As λ increases, the concentration distribution decreases and the corresponding concentration boundary layer thickness decreases. This is because of the slip condition in the stretching surface, as a result of which the concentration difference at the wall is affected.

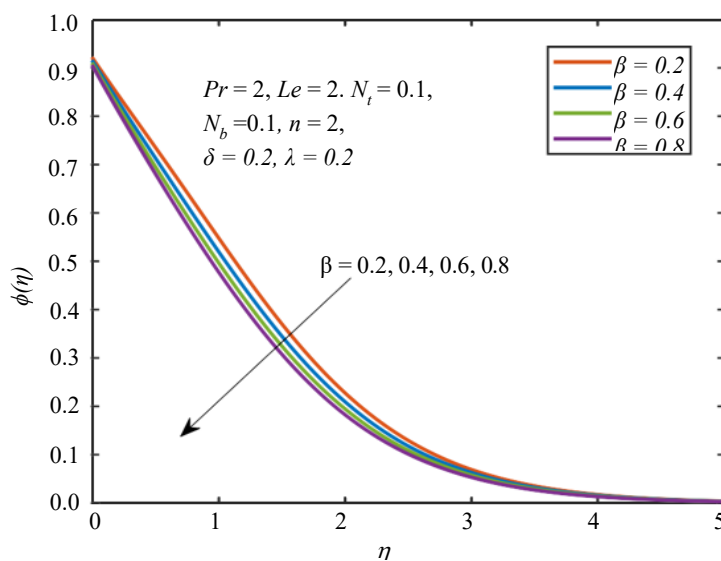


Figure 11. Effect of thermal slip parameter β on the concentration profile for $\delta = \lambda = 0.2$, $Nt = Nb = 0.1$, $Le = n = Pr = 2$.

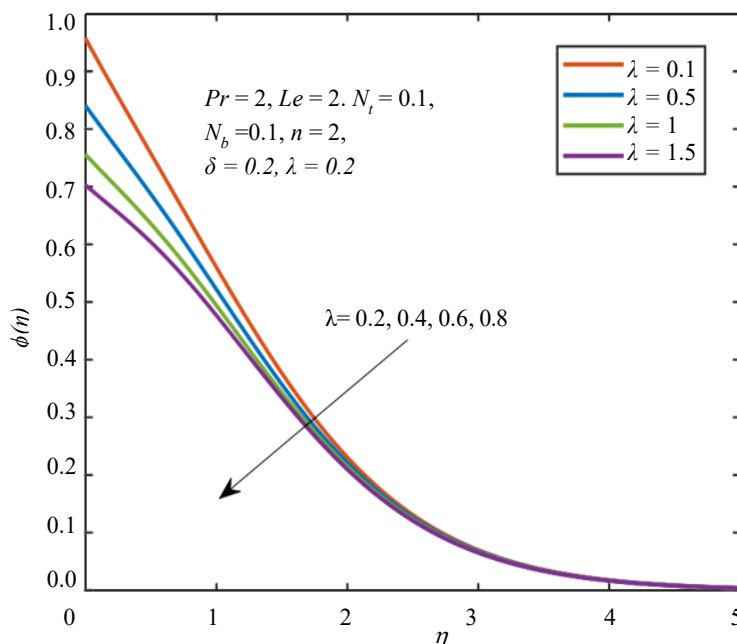


Figure 12. Effect of concentration slip parameter λ on nanoparticle concentration profile $\phi(\eta)$ for $\delta = \beta = 0.2$, $Nt = Nb = 0.1$, $Le = Pr = n = 2$.

Consequently, the concentration gradient responsible for nanoparticle diffusion is weakened, leading to lower mass transport from the surface. Therefore, the nanoparticles become less concentrated in the region away from the wall. This observation is also supported by the engineering results presented in Table 2, where increasing λ produces a lower local Sherwood number, indicating a reduction in the surface mass-transfer rate.

CONCLUSION

In this study, a mathematical model of the boundary layer flow of a nanofluid with a nonlinear stretching sheet under the effect of velocity, thermal, and concentration slip conditions was considered. The effects of thermophoresis and Brownian motion were also considered. By utilizing the appropriate similarity transformation, the governing PDEs were transformed into nonlinear ODEs and solved by using the shooting technique. The numerical solutions were further validated using the MATLAB built-in solver bvp4c, revealing excellent correspondence and supporting the validity and robustness of the numerical technique used.

The key numerical findings of this analysis can be summed up as follows:

1. The increase in velocity slip diminishes the momentum exchange from the stretching sheet to the surrounding fluid, thereby causing a reduction in the velocity of the fluid.
2. An increase in the thermal slip parameter weakens the temperature field, resulting in a narrower thermal boundary layer. The nanoparticles' concentration at the boundary layer reduces considerably with increasing thermal and concentration slip parameters.
3. If the value of concentration slip increases with a constant Lewis number (Le),
4. it reduces both the thickness of the concentration boundary layer and local Sherwood number.
5. If all three parameters, namely, velocity slip, temperature slip, and concentration slip, are considered simultaneously, their combined influence on the transport behavior can be observed. This will make the physical modeling of momentum, energy, and mass transfer more realistic.
6. The good agreement between the shooting method and the bvp4c solver validates the present numerical framework to analyze nonlinear boundary-layer flow problems with multiple slip effects.
7. The results of this study are insightful for slip-dominated nanofluid transport and provide a reliable numerical benchmark for future investigations of nonlinear boundary layer flows involving complex interfacial transport mechanisms.

Nomenclature

Roman symbols	Dimensionless quantities
$f(\eta)$	Dimensionless stream function
$\theta(\eta)$	Dimensionless temperature
$\phi(\eta)$	Dimensionless nanoparticle volume fraction
Le	Lewis number
N_b	Brownian motion parameter
N_t	Thermophoresis parameter
n	Power-law stretching index
Nu_x	Local Nusselt number
Pr	Prandtl number
Re_x	Local Reynolds number
Sh_x	Local Sherwood number

Dimensional quantities

a	Positive constant
C	Nanoparticle volume fraction
C_w	Nanoparticle volume fraction at the wall
C_∞	Ambient nanoparticle volume fraction
D_B	Brownian diffusion coefficient (m^2/s)
D_T	Thermophoretic diffusion coefficient (m^2/s)
h_m	Wall mass flux ($\text{kg m}^{-2} \text{s}^{-1}$)
q_w	Wall heat flux (W m^{-2})
T	Fluid temperature (K)
T_w	Wall temperature (K)
T_∞	Ambient temperature (K)
u, v	Velocity components along the x - and y -directions (m s^{-1})
u_w	Stretching sheet velocity (m s^{-1})
x, y	Cartesian coordinates (m)

Greek Symbols**Dimensionless quantities**

δ	Velocity slip parameter
β	Thermal slip parameter
λ	Concentration slip parameter
η	Similarity variable
τ	Ratio of heat capacities, $(\rho c)_p/(\rho c)_f$

Dimensional quantities

α	Thermal diffusivity (m^2/s)
β_1	Velocity slip coefficient (m)
β_2	Thermal slip coefficient (m)
β_3	Concentration slip coefficient (m)
μ	Dynamic viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
ν	Kinematic viscosity (m^2/s)
ρ_f	Density of base fluid (kg m^{-3})
ρ_p	Density of nanoparticles (kg m^{-3})
ψ	Stream function (m^2/s)

Abbreviations

PDEs	Partial differential equations
ODEs	Ordinary differential equations 2D, Two-dimensional
BVP	Boundary value problem
bvp4c	MATLAB boundary value problem solver

Subscripts

w	Wall condition
∞	Ambient (free-stream) condition

Superscripts

$(\cdot)'$	Differentiation with respect to η
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