STM JOURNALS

Trends in
Opto-electro & Optical Communication

ISSN: 2231-0401 (Online)
ISSN: 2347-9957 (Print)
Volume 14, Issue 3, 2024
September–December
DOI (Journal): 10.37591/TOEOC

https://journals.stmjournals.com/toeoc

Research TOEOC

Self-similarities in Hilbert Envelope Energy Regions on Motion Waveforms Application in Detecting Motion Irregularities in Video Frames

E. Cui[1], J.F. Peters[1,*]

Abstract

This paper introduces self-similar Hilbert envelope energy regions that are commonly found in oscillating spectral curves. The time-varying amplitudes and peak value frequencies in typical spectral curves reflect the response of a system to external stimuli. A Hilbert envelope is a smooth curve connected between spectral curve peak values. Each peak value $x(t)$ on an envelope curve x at time t identifies an envelope-bounded region with interior area (called Hilbert envelope energy region (denoted byE_{env_x})) which provides a measure of the size of a system response to a time-varying stimulus. A planar curve tangent to the peak values of an oscillating waveform is called a Hilbert envelope. Such an envelope is a series of pathways that extend between a nonlinear waveform's maximum values. In other words, a Hilbert envelope provides a 'Fingerprint' of the spectral flow of an oscillating waveform $x(t)$ in as much as an envelope is tangent to every maximal value of the waveform. This paper includes an application of Hilbert energy envelope regions in tracking the self-similarities of either a walker or runner motion recorded in infrared videos.

Keywords: Aperiodic waveform, envelope, Hilbert energy region, infrared (IR), motion waveform, peak value, self-similarity, signal energy, spectral curve

INTRODUCTION

In this study, IR camera recordings of surface radiation appear as an oscillatory waveform resulting in various forms of surface motion, such as vibrations of an operational machine part or the up and down movements of a walker or runner. A recorded motion waveform signal is typically aperiodic such that oscillations in one temporal interval are seldom duplicated in other temporal intervals. A Hilbert envelope is a planar curve tangent to the peak values in an oscillating waveform. In effect, such an envelope is a sequence of paths that stretch between the maximal values of a nonlinear waveform. In this study, Hilbert envelopes encapsulate a motion waveform, making it possible to identify, measure, and compare motion oscillations that are self-similar. Earlier work on time-varying systems focused on the self-similarities of temporally close moving objects [1] and the oscillating lumen flux emanating from moving objects [2].

***Author for Correspondence**
J.F. Peters
E-mail: james.peters3@umanitoba.ca

[1]Student (Ph.D.), Department of Electrical & Computer Engineering, University of Manitoba, Winnipeg, Manitoba R3T 5V6, Canada

Received Date: September 21, 2024,
Accepted Date: October 22, 2024
Published Date: December 07, 2024

Citation: E. Cui, J.F. Peters. Self-similarities in Hilbert Envelope Energy Regions on Motion Waveforms Application in Detecting Motion Irregularities in Video Frames. Trends in Opto-electro & Optical Communication. 2024; 14(3): 34–49p.

HILBERT ENVELOPE

A Hilbert envelope (H_{env}) is a curve that covers (intersects) the peak values in a time-varying waveform, such as a motion wave. The H_{env} is a recent tool that makes it possible to construct a curve that intersects the peak values on either a continuous- or discrete-time oscillatory spectral curve. A spectral curve displays the time-varying responses of a system to a stimulus, such as varying lumen-reflected light values emanating from an oscillating object. The spectral peak value is the maximal system response to a stimulus at a particular time. Each peak value on a spectral curve $\mathrm{x(t)}$ at time t has associated energy [3]. $\mathrm{H_{env}}$ is commonly used to measure the size of a nonlinear vibration system. Identifying the energy levels in vibrating systems is useful for analyzing, identifying, and simulating vibrating systems [4, 5]. Modulating $\mathrm{H_{env}}$ peak values (also called non-uniform sampling points) is a basic step in smoothing the excessive oscillatory system motion.

Each Hilbert envelope H_{env} results from the application of a Hilbert transform $H_{env}(x(t))$ to an oscillatory waveform signal $x(t)$, defined by

$x(t) =$ oscillating motion waveform

$$g(\tau) = \frac{1}{\pi\tau}$$

$$g(t-\tau) = \frac{1}{\pi}\left(\frac{1}{t-\tau}\right)$$

$$H_{env}\big(x(t)\big) = \int_{-\infty}^{\infty} x(t)g(t-\tau)dt$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(t)}{t-\tau}dt$$

Remark 1. Benefit of time-shifted Hilbert envelope

Time-shifting $\mathrm{g(t-\tau)}$ by τ seconds in a Hilbert envelope so that $\mathrm{H_{env}\big(x(0)\big)}$ is defined at $\mathrm{t} = 0$. The benefit of time-shifting is that the envelope. $\mathrm{H_{env}\big(x(0)\big)}$ is causal, i.e., $\mathrm{H_{env}\big(x(t)\big)}$ is defined for all values of t such that $0 \le \mathrm{t} \le \tau$. For instance, if we construct a Hilbert envelope on the motion waveform of an oscillating object such as a runner or walker, the envelope will include the rest state at the onset of motion when $\mathrm{t} = 0$. For details on how a Hilbert envelope is constructed on an oscillating waveform, see Appendix A.

The Hilbert transform $H_{env}\big(x(t)\big)$ is used to construct an upper and lower envelope from a time-domain signal $x(t)$ [6]. The two waveforms (envelope $H_{env}\big(x(t)\big)$ and motion wave $x(t)$) are orthogonal to each other (i.e., differ in phase by $90°$) [7].

Example 1. A sample Hilbert envelope on the self-similar upper and lower parts of a sound pulse train is shown in Figure 1. The pulse train is a non-sinusoidal waveform. This example illustrates the two basic parts of a Hilbert envelope: an upper envelope (red wave portion) that intersects the positive part of the sound wave and a lower envelope (blue wave portion) that intersects the negative part of the sound wave. In this example, the upper and lower parts of the pulse train have closed areas in the positive and negative directions.

Example 2. Another example of upper and lower Hilbert envelopes that intersect the peak values on an aperiodic motion signal is shown in Figure 2. In this example, two quite different self-symmetries can be observed, namely, contours of diametrically opposite impulses and opposite bounded areas above and below the left of the middle of the horizontal axis. An instance of self-similarity is evident in the Hilbert envelope on the far side of this motion waveform, as shown in Figure 2. For further evidence of the self-similarities in this bounded by the upper and lower envelopes of the motion waveform, see example 3.

A self-similar motion signal contains approximately the same energy levels. Self-similar objects appear approximately the same on any scale [8]. Here, we call attention to motion signal energy a highly accessible source of measurable self-similar parts in motion signals. This is because the area bounded by a motion curve facilitates the measurement of self-similarity. A motion curve (also called 'motion signal') $x(t)$ area in a temporal interval $[t_a, t_b]$ is also called motion signal energy, demoted by E_x, i.e.,

$$E_x = \int_{t_a}^{t_b} |x(t)^2| dt\,.\,(Motion\ signal\ energy)$$

Remark 2. Measure of the Size of a Time-Varying Waveform

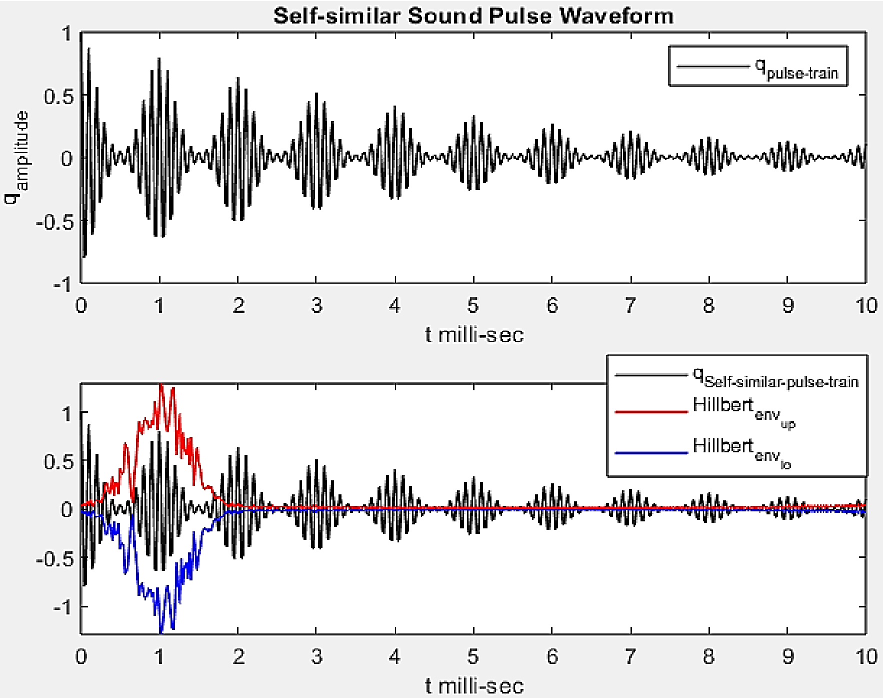


Figure 1. Hilbert envelope on a self-similar aperiodic pulse train.

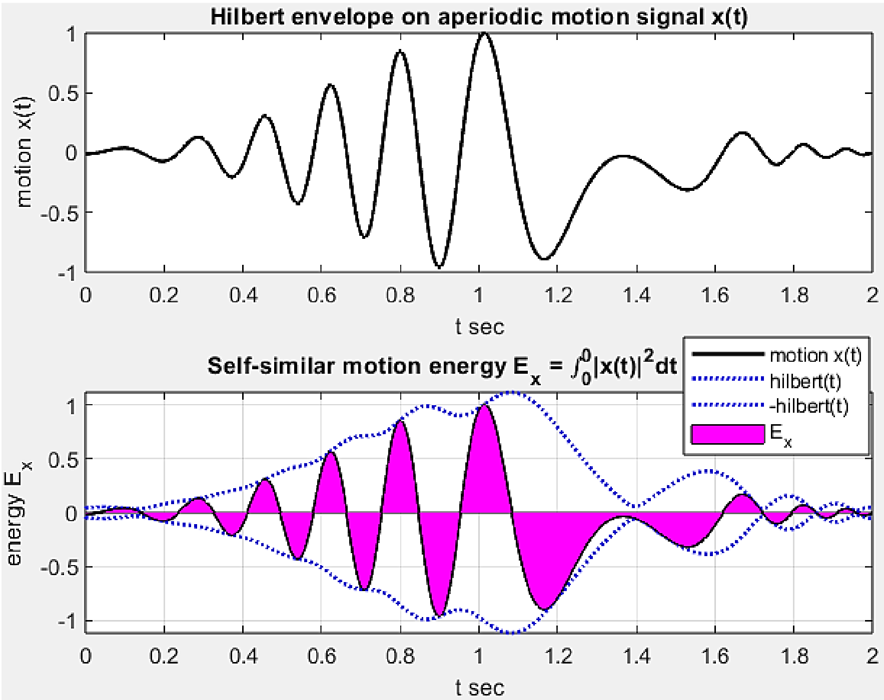


Figure 2. Self-similar energy levels in the interior of a Hilbert envelope on a detected motion signal in an IR video.

Over time, signal waveforms emanating from an oscillating object typically vary in amplitude and peak value frequency. Typical examples of varying signal peak value amplitudes and frequencies are sound waves from a musical instrument, wind ocean waves, tornado trajectories, heat waves such as those from a volcano, and the varying movements of a walker or long-distance runner. The area bounded by a signal waveform provides a measure of the size of a signal, termed its energy (see [9] P. 65), which is a variation of a Teager-Kaiser squared sinusoidal signal $(A\omega)^2$ with amplitude A and frequency ω(see, e.g., [10, 11]). For a continuous-time signal $x(t)$ over a time interval $t_1 \leq t \leq t_2$ such as the one we have in mind here (e.g., motion amplitude variation of a runner), we use the Oppenheim-Willsky definition of energy E_x [12], namely,

$$E_x = \int_{t_1}^{t_2} |x(t)^2|\, dt$$

where $|x(t)|$ denotes the magnitude of $x(t)$ at time t and the integral represents the area of a planar region bounded by $x(t)$ Above and below a horizontal axis passing through the origin at $(0,0)$.

Example 3. Motion signal with Self-Similar Motion Energy Levels.

Quite a few self-similar energy levels (represented by magenta-shaded areas bounded by a motion curve and the horizontal axis in a temporal interval ranging from $\mathrm{t} = 0\mathrm{sec}$ to $\mathrm{t} = 2\mathrm{sec}$) are in Figure 2. For example, observing the self-similar energy levels in

Figure 3.= x_{11}, x_{12} energy levels, i.e.,

$$E_{x_{11}} = \int_{0.2}^{0.32} |x_{11}(t)^2| dt \approx E_{x_{12}} = \int_{0.32}^{0.41} |x_{12}(t)^2| dt. \text{ (See Figure 3)}$$

Figure 4.= x_{21}, x_{22} energy levels, i.e.,

$$E_{x_{21}} = \int_{0.69}^{0.74} |x_{21}(t)^2| dt \approx E_{x_{22}} = \int_{0.74}^{0.775} |x_{22}(t)^2| dt. \text{ (See Figure 4)}$$

Figure 5.= x_{31}, x_{32} energy levels, i.e.,

$$E_{x_{31}} = \int_{0.775}^{0.825} |x_{31}(t)^2| dt \approx E_{x_{32}} = \int_{0.825}^{0.97} |x_{32}(t)^2| dt. \text{ (See Figure 5)}$$

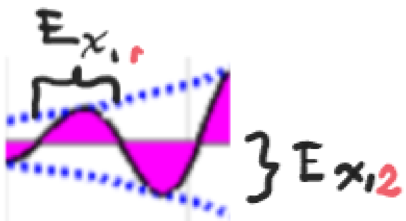


Figure 3. Self-similar energy levels in the interior of a Hilbert envelope on a detected motion signals x_{11}, x_{12} in an IR video.

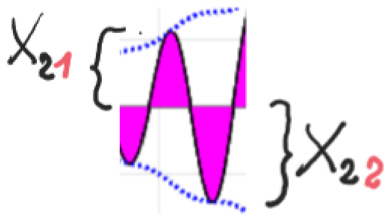


Figure 4. Self-similar energy levels in the interior of a Hilbert envelope on a detected motion signals x_{21}, x_{22} in an IR video

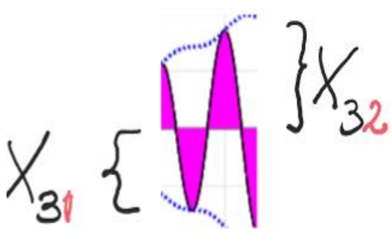


Figure 5. Self-similar energy levels in the interior of a Hilbert envelope on a detected motion signals x_{31}, x_{32} in an IR video.

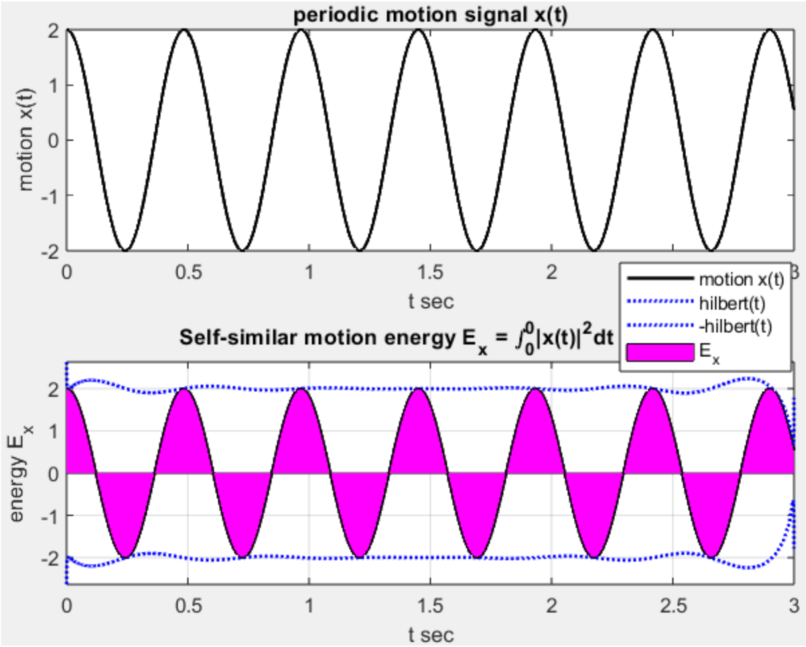


Figure 6. $E_{x(t)} = E_x(t + T_0)$ energy levels, i.e., $x(t)$ is a periodic motion signal with period T, i.e., the signal value $x(t)$ repeats after T_0 for every t

Remark 3. With self-similar energy levels in the regions of a motion signal with peak values intersecting a Hilbert envelope, regional similarity preserves the motion spectrum. Although we do not consider it here, one can also pinpoint the motion spectrum regions with energies that are quite similar. Let p, q be the signal $x(t)$ peak values in a Hilbert envelope $H(x(t))$ with energies $E_p \neq E_q$ that are quai-similar, provided there are $k, k' \in \mathcal{R}$ such that $kE_p = k'E_q$ (see, e.g., [13], P. 3). The detection of self-similar as opposed to quasi-similar motion spectrum energy regions is preferred because self-similar motion spectra regions closely reflect energy regions susceptible to smoothing. By modifying the source of the perturbed non-uniform signal oscillation (such as those of a runner who moves non-periodically up and down), the motion signal peak values become more uniform and periodic.

PRELIMINARIES

The motion signal peak values that intersect a Hilbert amplitude envelope spectrum (first identified in [14], pp.69-70) constitute a particularly useful Hilbert amplitude spectrum. There are two basic types of signals to consider, here, namely,

Definition 1. Periodic motion signal

For each time t, a periodic signal repeats after T_0 s, that is, $x(t) = x(t + T_0)$, where T_0 is the fundamental period of the signal.

Example 4. For the motion signal $x(t)$ in Figure 6, $T_0 = 0.5$sec. for each time t, that is, $x(t) = x(t + 0.5)$ every T_0 s.

Definition 2. Aperiodic Motion Signal

For each time t, an aperiodic signal does not repeat itself after T_0 seconds, that is, $x(t) \neq x(t + T_0)$, where T_0 is the fundamental period of the signal.

Example 5. For the motion signal $x(t)$ in Figure 2, for each time t, $x(t) \neq x(t + 0.5)$ for any T_0 s.

For a periodic motion signal $x(t)$, notice that the energy E_x repeats after T_0 s. This observation provides the following useful results.

Lemma 1. Let $x(t)$ be a periodic signal with a period T_0. Then $E_{x(t)} = E_{x(t+T_0)}$.

Proof. For simplicity, let $x(t)$ be a signal peak value, and let $E_{x(t)}$ equal the single energy (region area with the peak value on its border. From Def. 1), E_x repeats itself after T_0 seconds, that is, $E_x = E_{x(t+T_0)}$. This provides the desired results.

Theorem 1. The energy level at time t in a periodic signal x(t) repeats itself after T_0 seconds.
Proof. The desired result is immediate from Lemma 1.
A useful consequence of Theorem 1 is

Corollary 1. A continuous periodic signal contains repeated self-similar regions.

The smoothing effect of the Hilbert envelope on a periodic signal is absent in an envelope on an aperiodic signal. That is, the energy levels of different bounded regions are usually not exactly repeated in an aperiodic signal. From Def. 2, we obtain the following result.

Theorem 2. The energy level at time t in an aperiodic signal x(t) is usually not exactly repeated.

Remark 4. Theorem 2 does not exclude the self-similar energy regions in the Hilbert envelope $\mathrm{H_{env}}(\mathrm{x(t)})$ on an aperiodic waveform. If a pair of energy regions $\mathrm{E_x}$, $\mathrm{E_y}$ for peak values $\mathrm{x, y}$ in $\mathrm{H_{env}}(\mathrm{x(t)})$ have approximately the same area, then of self-similarity [8], $\mathrm{E_x}$, $\mathrm{E_y}$are self-similar.

Lemma 2. The self-similarity in a waveform can be increased by modulating the signal sufficiently.

Proof. Let $x(t)$ be a signal containing segment $\{x(t), x(t+t_0)\}$ and let $x_{mod}(t) = x(t)e^t$, which smooths the original $x(t)$ waveform. Hence, segment $\{x(t), x(t+t_0)\}$ is smoothed, which results in an increase in the self-similarity of parts of the smoothed segment of the signal. This increase in self-similarity of the segments occurs in all segments of the signal.

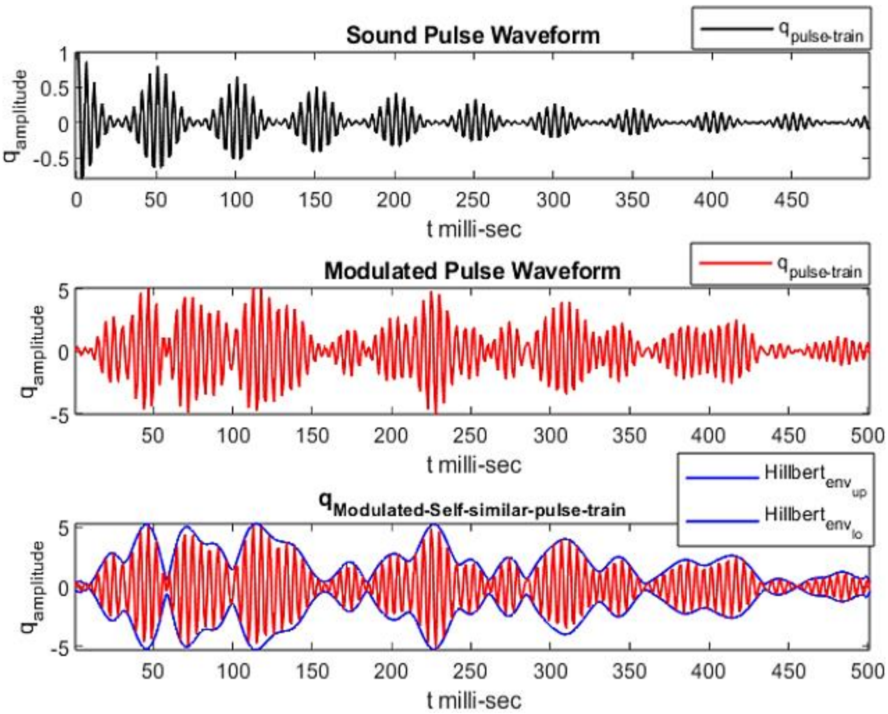


Figure 7. Hilbert envelope on an exponentially modulated waveform $x(t) = H_{env}(x(t)e^{2.2\pi T-2.3})$.

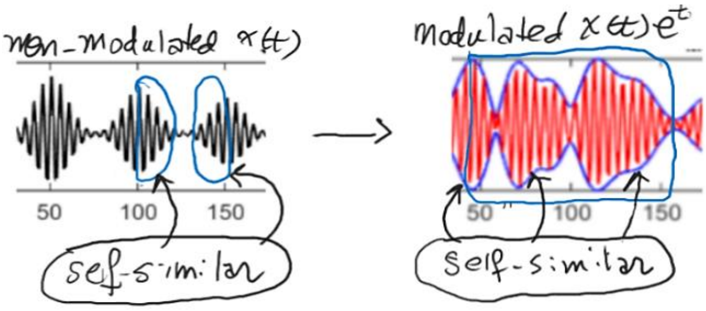


Figure 8. Increased self-similarities in Hilbert envelope on the exponentially modulated waveform in the interval $50 \leq t \leq 150$.

Example 6. The sound signal in Figure 1 was modulated, as shown in Figure 7. After modulating the waveform $\mathrm{x(t)}$ with the exponential $\mathrm{e}^{2\pi t-2.1}$, a significant increase in the self-similar regions in $\mathrm{x(t)e^t}$ can be observed, for example, in the interval $50 \leq \mathrm{t} \leq 150$ in Figure 8.

Definition 3. Hilbert Energy Region

Let $\mathrm{x(t)}$ be a signal peak value that intersects a Hilbert envelope $\mathrm{env_x}$ that identifies a Hilbert energy region (denoted by $\mathrm{E_{env_x}}$), which is defined as

$$E_{env_x} = \int_{t_1}^{t_2} |x(t)^2| dt$$

Definition 4. A lobe in the Hilbert energy region $\mathrm{E_{env_x}}$ is an energy region that includes at most one $\mathrm{x(t)}$ peak value.

Example 7. A sample energy region lobe with a peak value at $\mathrm{x}(0.8) = 1$ is shown in the aperiodic motion signal in Figure 2. A sample lobe with a peak value at $\mathrm{x}(0.0182) = -1$ is shown in the periodic motion signal in Figure A.16.

A practical consequence of Def. 3 is the following result.

Theorem 3. Let $\mathrm{E_{env_x}}$ be a Hilbert energy region on lobes of $\mathrm{H}(\mathrm{x(t)})$ with approximately equal areas on a signal $\mathrm{x(t)}$. Then, the number of self-similar Hilbert envelope energy regions is non-zero.

Proof. Let E_{env_x} be a Hilbert energy region containing at least one pair of lobes L_1, L_2 with approximately equal areas. By definition, L_1, L_2 were self-similar. Hence, the desired result is as follows.

Self-similar Signal Energy Regions

A practical result of Theorem 3 is that can be used to modulate a vibrating object with an oscillating waveform, such as a walker or runner motion signal, such that the aperiodicity of the motion waveform is diminished, resulting in a smoother signal. A Hilbert envelope offers a means of isolating the signal peak values. After identifying self-similar Hilbert energy regions with identifiable signal peak values, it is possible to modulate those peak values on non-self-similar energy regions so that the number of self-similar Hilbert energy regions increases, as shown in Figure 9.

Application: Self-Similar Energy Regions in Walker Motion Waveforms

This section briefly presents evidence of self-similar energy regions in a walker signal recorded in a sequence of video frames using an infrared (IR) camera. An IR camera records low-frequency electromagnetic radiation in the form of surface heat and transforms the radiation emitted by the object surfaces into images. Unlike reflected visible light with wavelengths in the 400-700 nanometer (nm) range recorded by ordinary cameras, an IR camera records radiated heat measured in the 700-1,000 nm range (Figure 9). The typical somewhat glaring, diffusive (spread over a wide area) character of reflected daylight from various surfaces, IR images are consistently non-diffusive, non-noisy, and incisive renditions of time-varying radiated surface heat from moving objects. The radiation from a moving object varies, while the radiation from an outdoor, stationary object during either a low wind or windless time is unvarying. The IR videos of the walkers reported herein were obtained under low-light, windless conditions.

Both modulated and non-modulated motion waveforms were recorded. Here is a brief overview of the application of the Hilbert transform to derive a modulated IR waveform:

Modulated Motion Waveform Envelope Extraction Method

Let $\mathrm{fr}[\mathrm{x},:,\mathrm{t}]$ be a column: of discrete radiated heat values in row x detected at time t in a moving object in an IR video. Then $\mathrm{fr}(\mathrm{x},:)$ is exponentially modulated using

$$x(fr[x,:]) = fr[x,:]e^{-j2\pi fr[x,:]}$$

To obtain $\mathrm{H}_{\mathrm{env}}\big(\mathrm{x}(\mathrm{fr}[\mathrm{x},:])\big)$, a Hilbert envelope on the modulated waveform $\mathrm{x}(\mathrm{fr}[\mathrm{x},:])$ (see, e.g., Figure 2) defined by

$$H_{env}\big(x(fr[x,:])\big) = \big|sin\big(\omega x(fr[x,:])\big)\big|$$

For the derivation of $\mathrm{H}_{\mathrm{env}}\big(\mathrm{x}(\mathrm{fr}[\mathrm{x},:])\big)$, see Appendix A.

Figure 9. (a) Sample motion signal SI value and (b) Pure and modulated IR motion waveforms.

Remark 5. Unlike the single image target detection methods in [15], IR object motion detection is based on the extraction of IR radiation emanating from a vertical line that passes through the centroid of an oscillating object in a sequence of IR video frames. This IR vertical line is a source of a nonlinear waveform that changes from frame to frame. The Hilbert envelope on this modulated waveform provides a smooth sinusoidal waveform (Figure 2) useful for walker and runner training purposes, i.e., diminishing the vertical oscillations or either a walker or a runner result in smoother movements requiring less physical exertion.

Example 8. Self-similar Walker Motion in IR Motion Waveforms

A sample IR video frame (with IR units on the vertical scale and finite impulse response (FIR) Newton's ($N \cdot s$) horizontal scale) is shown in Figure 9. In this video frame, the sample IR waveform emanates from a walker's vertical vector after approximately 600 $N \cdot s$. The modulated form of the IR waveform is shown in Figure 10. For example, in the 500–580 $N \cdot s$ range, the modulated waveform has many more hills and valleys, which leads to a greater number of self-similar lobes in the Hilbert envelope on this IR signal.

What we mean by IR Motion Waveform Energy

The energy associated with the area of the lobes in the interior of a Hilbert envelope on IR waveform $\mathrm{x(t)}$ at time t (either modulated or non-modulated) provide us with a means of measuring the size of an IR waveform, i.e., the area $\int_0^{t+\epsilon} |x(t)^2| dt$ bounded by a lobe attached to a peak value on a Hilbert envelope is a useful measure of the energy associated with the radiation $\mathrm{x(t)}$ in the interval $0 \leq t \leq t + \epsilon$ (i.e., in the short interval of time for a signal that begins at $t = 0$ with a duration with a minuscule upper bound $\epsilon > 0$).

i. The Hilbert IR envelope lobe energy serves as a means of pinpointing moving object surfaces with the highest radiation.

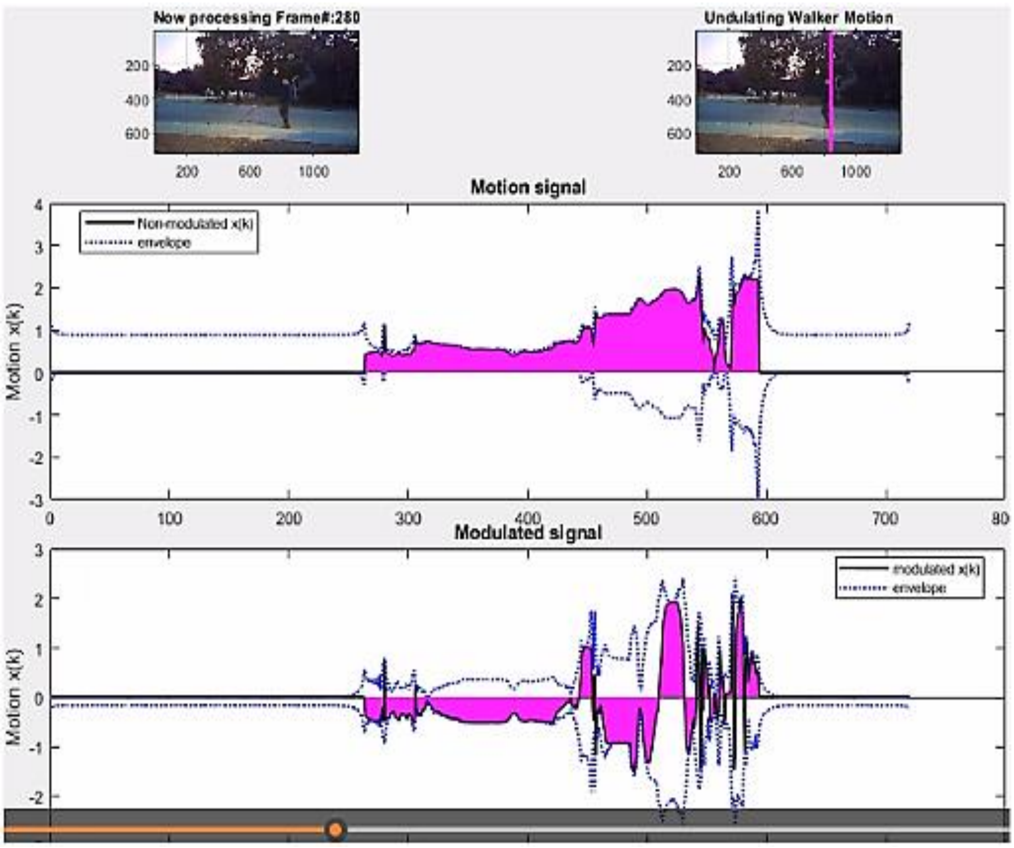


Figure 10. Hilbert IR energy lobes.

ii. Usually, IR waveform lobes do not have equal sizes. As a result, the relative size (energy) of varying waveform envelope oscillations in relation to neighboring waveform lobes provides a useful measure of the parts of a waveform that are the source of the greatest variation.
iii. From Lemma 2, the self-similarity in a sufficiently modulated Hilbert IR envelope leads to an increase in the self-similar envelope lobe. Hence, an increase in the self-similar IR envelope lobes is concomitant with an increase in the self-similar energy levels (i.e., the size of each Hilbert envelope lobe area equals an energy level).

Example 9. No.1 Self-similar walker motion Hilbert IR energy lobes

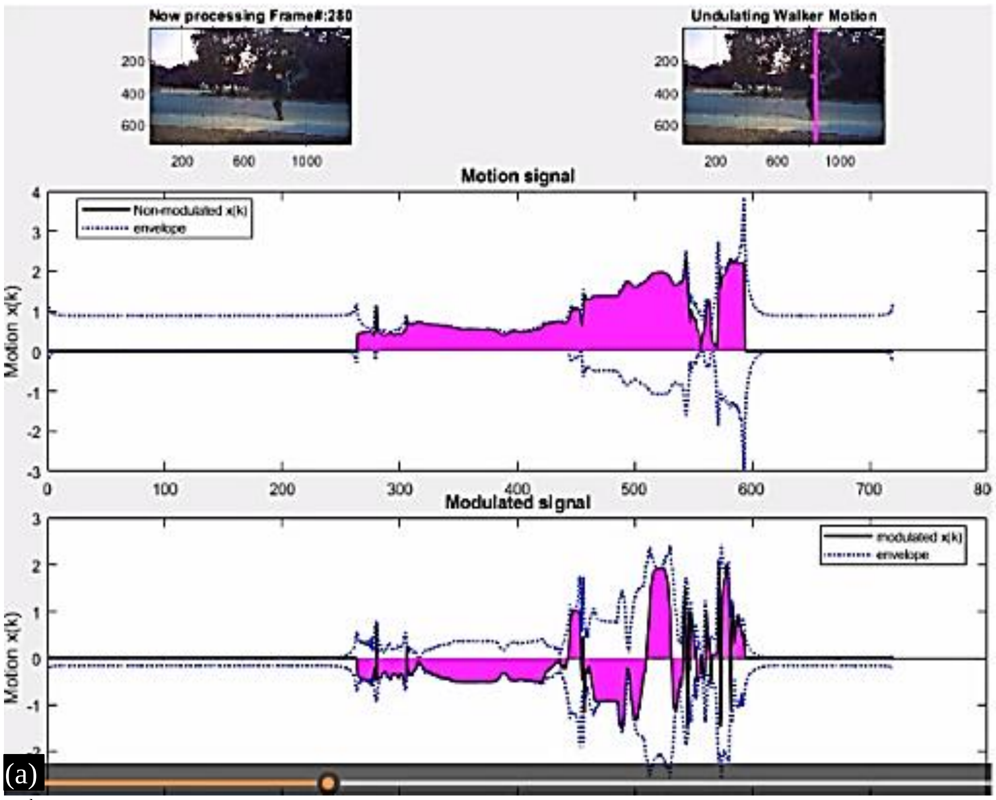
Sample walker motion Hilbert IR energy lobes emanating from a pure IR signal and a modulated IR waveform in a video are shown in Figure 10. Additional sample IR energy lobes attached to modulated waveforms are shown in Figure 11. Consistent with the observation in Lemma 2, there is an increase in the number of self-similar energy levels. This increase in self-similar energy levels is consistent with the observation in Lemma 2.

Example 10. No.2 Self-similar Walker Motion Hilbert IR Energy Lobes

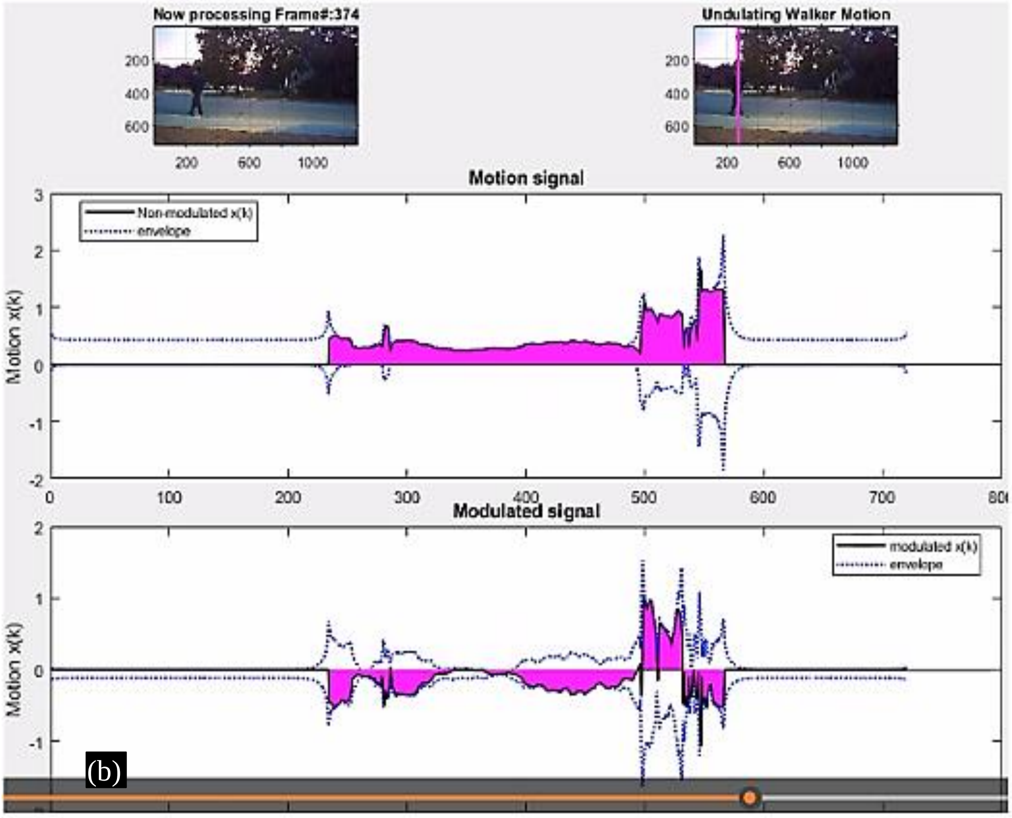
A second IR video frame in Figure 12 is another source of walker motion Hilbert IR energy lobes emanating from a pure IR signal and a modulated IR waveform. In this second IR video, additional IR energy lobes attached to modulated IR waveforms are shown in Figure 13. Again, consistent with the observation in Lemma 2, there is a significant increase in the number of self-similar IR envelope lobes with a corresponding increase in the number of self-similar energy levels.

Example 11. No.3 Self-similar Walker Motion Hilbert IR Energy Lobes

A frame from the third IR video in Figure 14 is yet another source of the Hilbert IR energy lobes emanating from a pure IR signal and a modulated IR waveform. In the third IR video, additional IR energy lobes attached to the modulated waveforms are shown in Figure 15. Again, consistent with the observation in Lemma 2, there is a significant increase in the number of self-similar IR envelope lobes, with a corresponding increase in the number of self-similar energy levels.



2nd Modulated IR.



3rd Modulated IR

Figure 11. (a) and (b) Two modulated IR motion waveforms

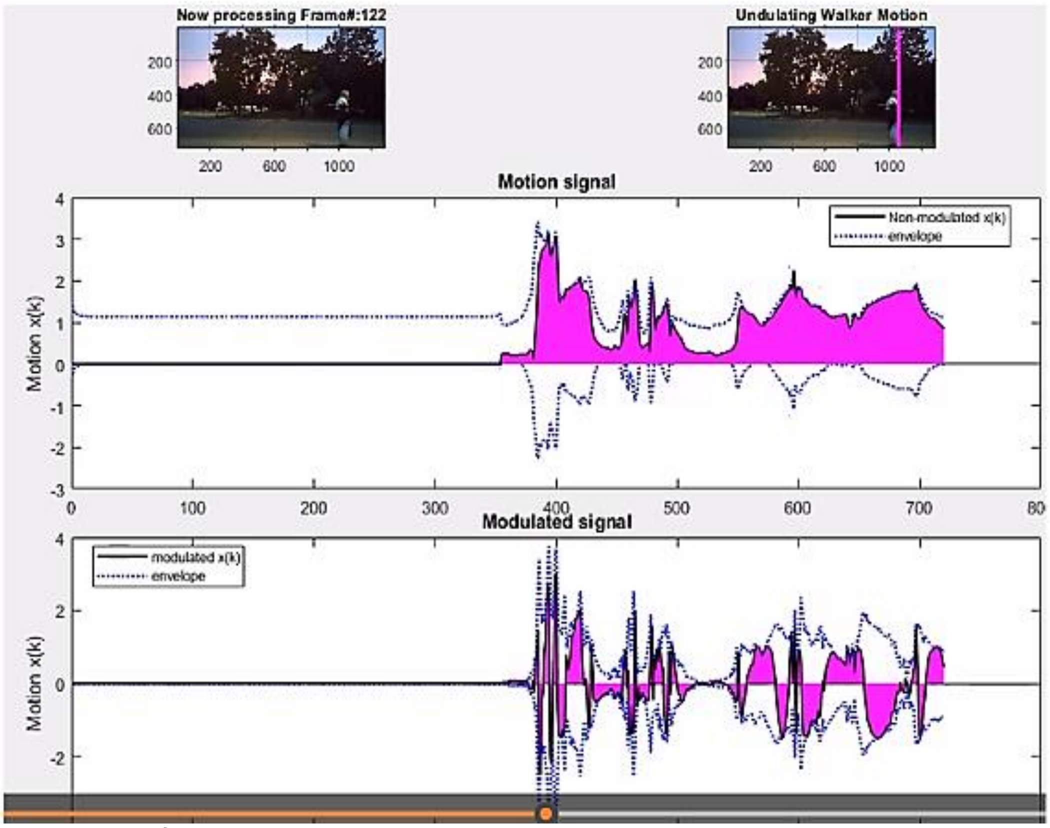
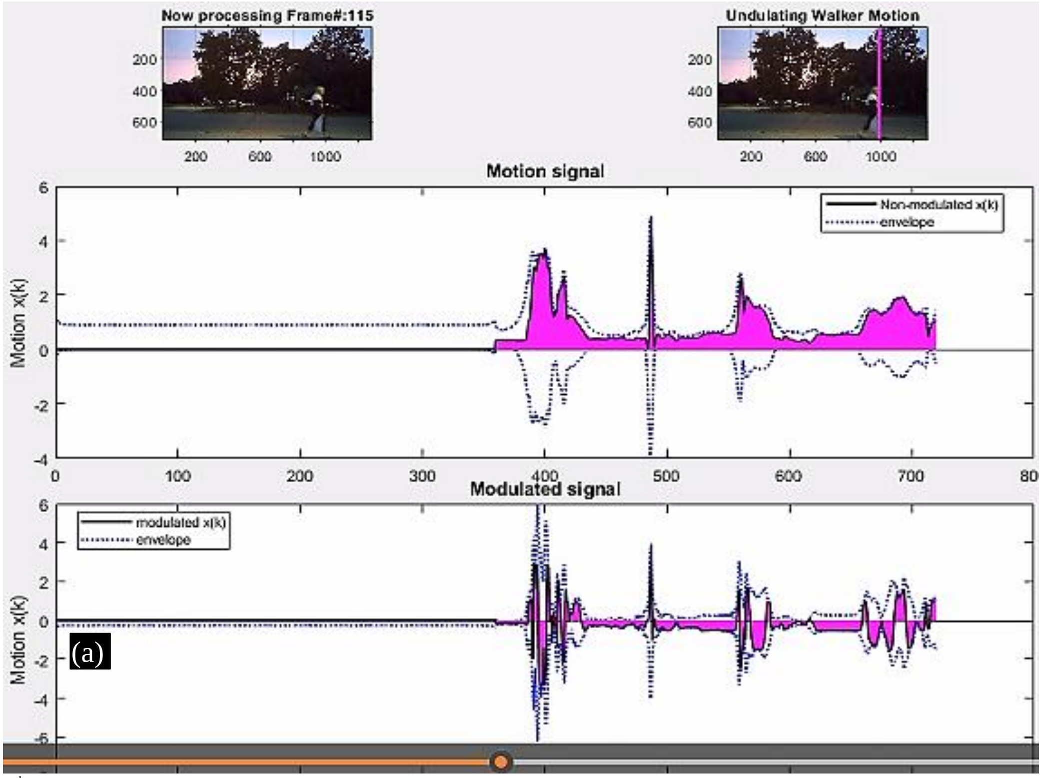
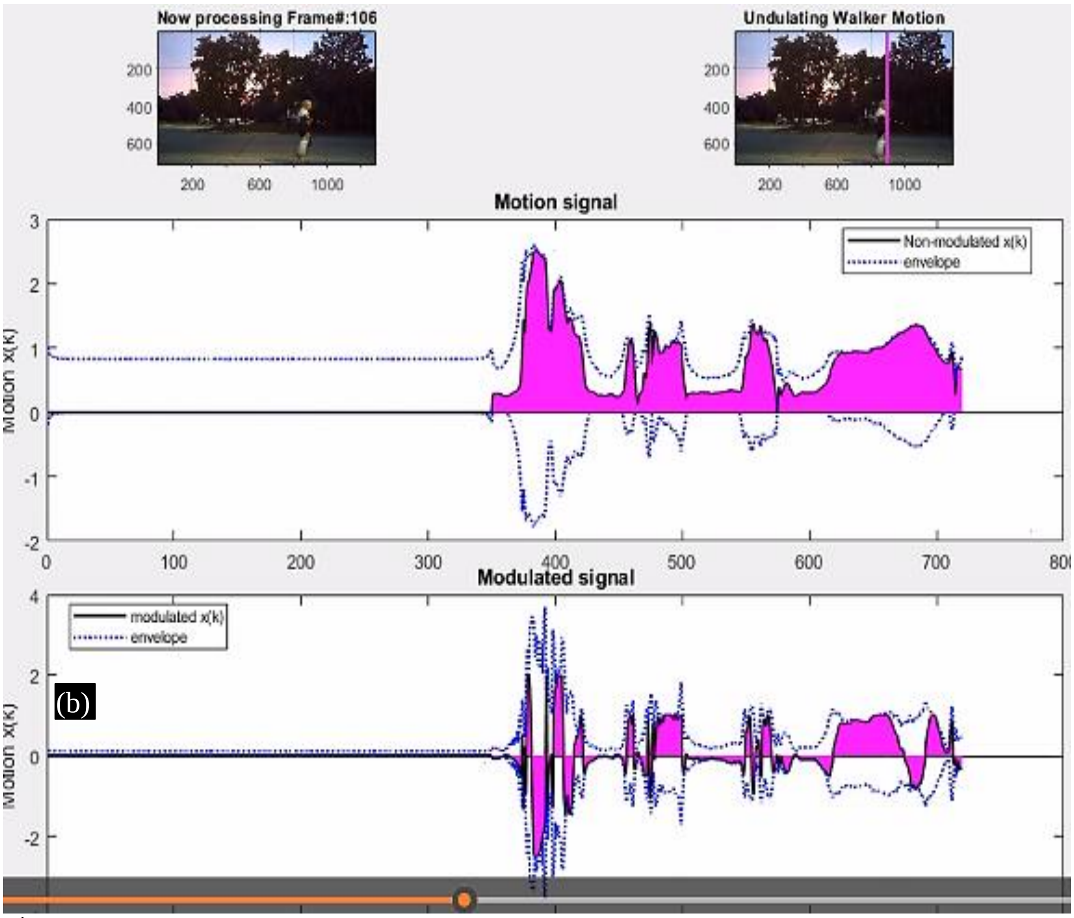


Figure 12. 2nd Hilbert IR energy lobes.



5th Modulated IR



6th Modulated IR

Figure 13. (a) and (b) Another sample: 2 modulated IR signals.

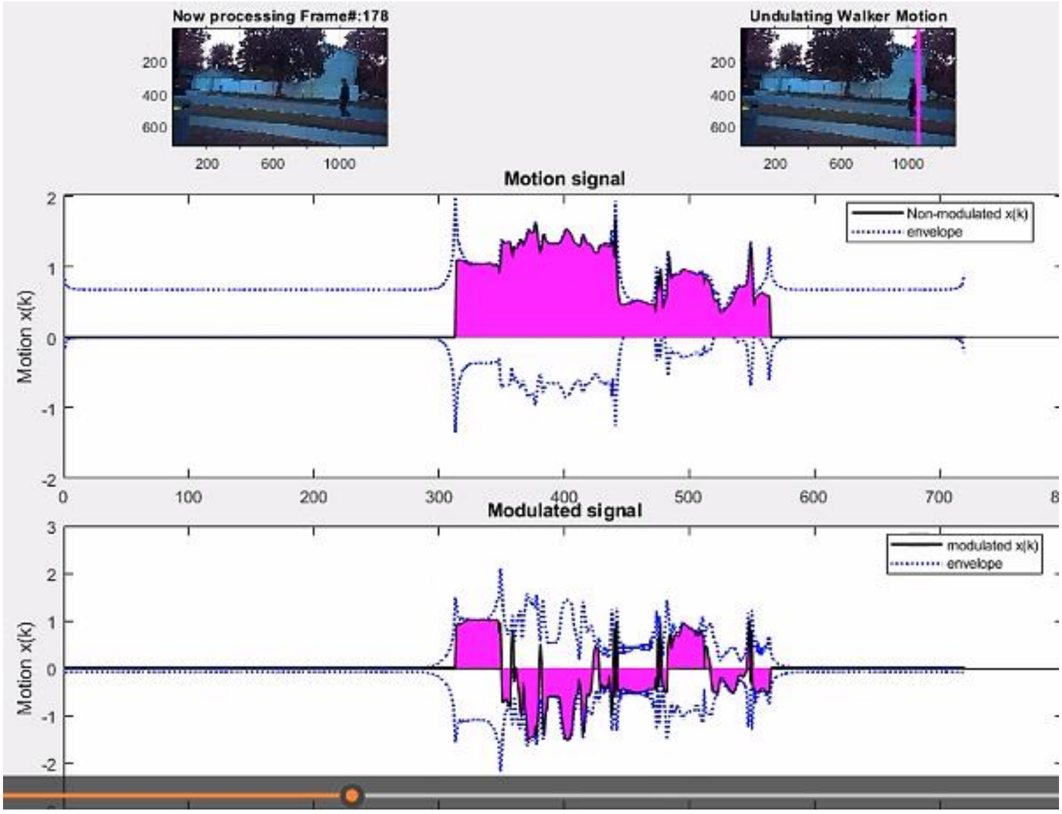
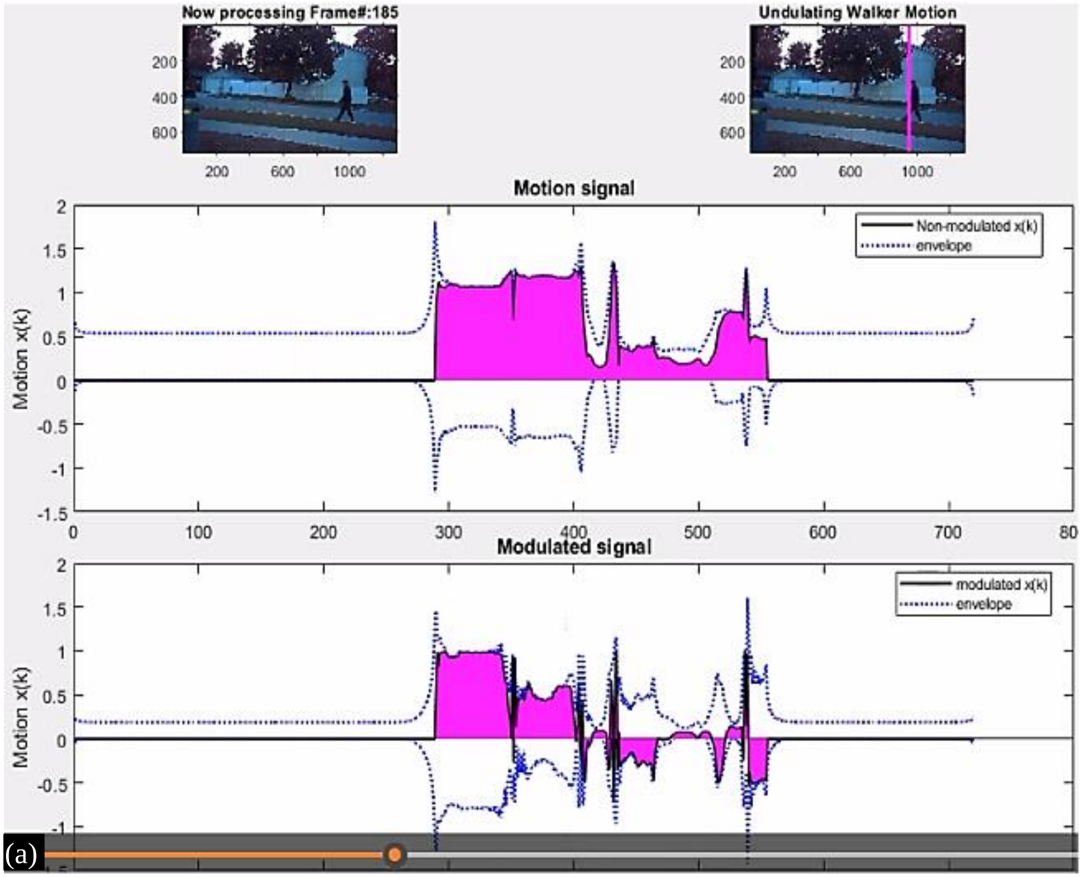
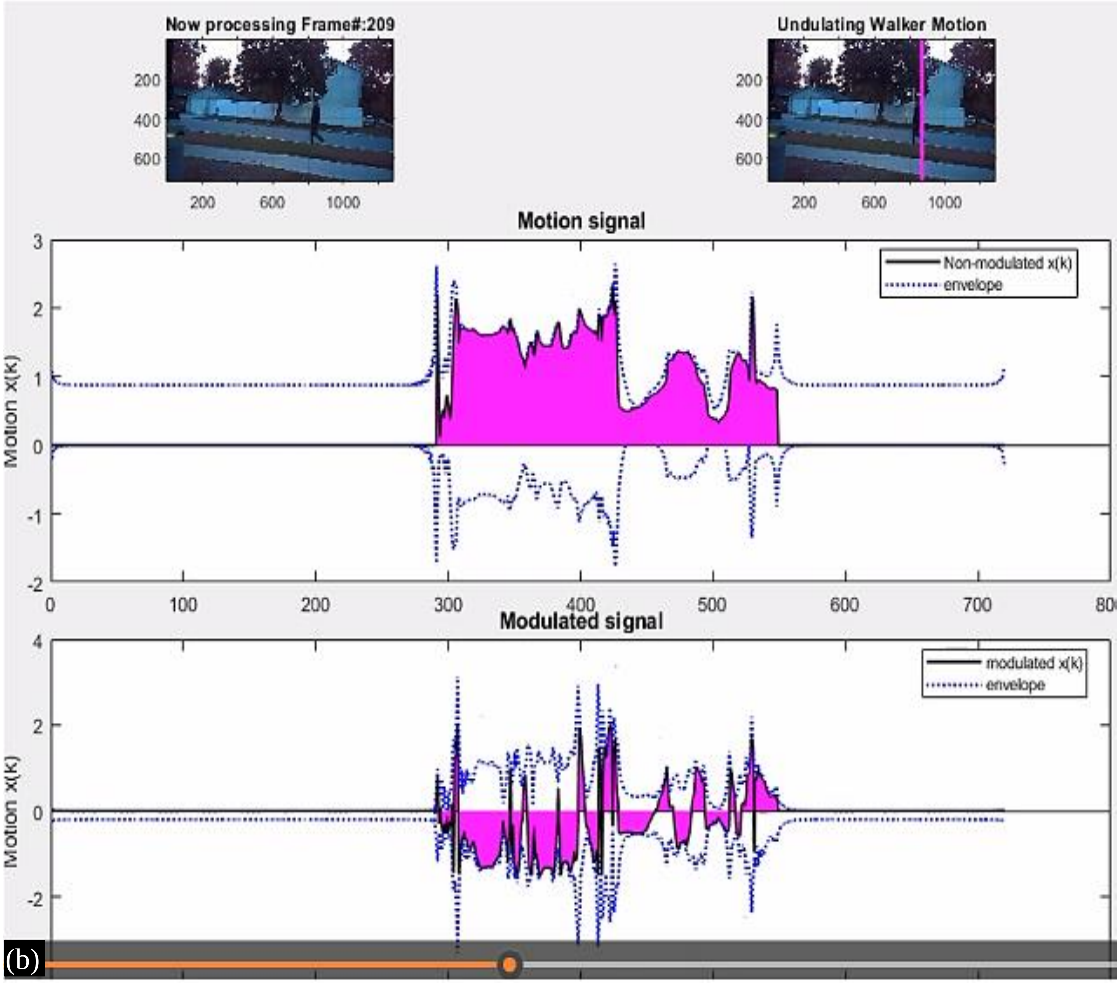


Figure 14. Hilbert IR energy lobes.



(a)

Modulated IR



8th Modulated IR.

Figure 15. (a) and (b) Another sample: 2 modulated IR signals.

RESULTS

This section introduces the results based on observations made regarding Hilbert envelopes on oscillatory waveforms.

Theorem 4. Let $\mathrm{H_{env}}\big(\mathrm{x(t)}\big)$ Be an envelope on the peak values of a modulated signal x(t). then

i. The number of self-similar $\mathrm{H_{env}}\big(\mathrm{x(t)}\big)$ energy lobes will increase.
ii. Every pair of Hilbert energy region lobes on a modulated signal with equal areas is self-similar.
iii. Every pair of Hilbert energy region lobes on a modulated signal with approximately the same area is self-similar.

Proof

i. From Lemma 2, even if the number of self-similar lobes in $H_{env}\big(x(t)\big)$ is zero, after sufficient modulation, the number of self-similar lobes will be non-zero.
ii. By definition, $H_{env}\big(x(t)\big)$ energy lobes with the same area are self-similar.
iii. Immediate from Theorem 3. This is an immediate consequence of Lemma 2.

DECLARATION OF COMPETING INTEREST

This statement certifies that no conflicts of interest exist in the submission of this manuscript. The manuscript has been approved for publication by all authors. The work described here is original research that has not been published previously and is not under consideration for publication elsewhere.

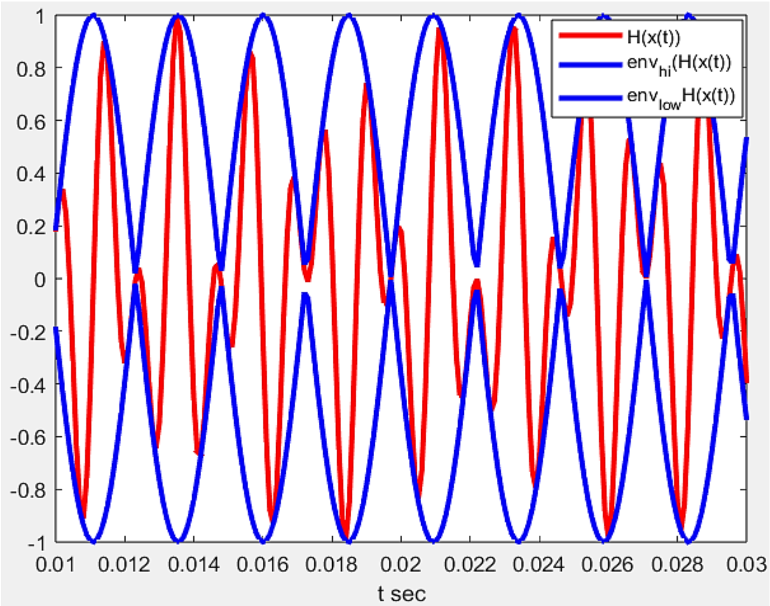


Figure 16. Hilbert energy $E_{env_{x(t)}} = \int_0^{0.03} H\big(x(t)\big)$ on a Hilbert envelope on the waveform $x(t) = sin(2\pi 203t)sin(2\pi 721t)$

Credit Authorship Contribution Statement

James F. Peters: Conceptualization, Investigation, Formal Analysis, Supervision, original draft. Enze Cui: Methodology, Validation, Software, and editing.

Acknowledgments

This research was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) discovery grant 185986; Instituto Nazionale di Alta Matematica (INdAM) Francesco Severi, Gruppo Nazionale per le Strutture Algebriche, Geometriche e Loro Applicazioni grant 9 920160 000362, n.prot U 2016/000036; and Scientific and Technological Research Council of Turkey (TÜBİTAK) Scientific Human Resources Development (BIDEB) under grant no. 2221-1059B211301223.

Appendix A. Hilbert Envelope Derivation

In this section, we derive the Hilbert transform envelope for a planar modulated waveform, as shown in Figure 16.

$x(t) = sin(2\pi 203t)sin(2\pi 721t)$

The Peak value in an oscillating planar waveform $x(t)$ identifies a family of curves $F(x, y, t) = 0$ and $\frac{\partial F}{\partial t}(x, y, t) = 0$, where t is the instant $x(t)$ at locations (x, y). An envelope is a planar curve tangent to every peak value on $x(t)$ [16] p. 150. To derive the Hilbert envelope $H_{env}\big(x(t)\big)$ for the $x(t)$ waveform, a sinusoidal form of $H\big(x(t)\big)$ from and [17] are used:

$a(t) = \frac{1}{\pi}\int_{-inf}^{inf} x(t)cos(t\tau)$

$b(t) = \frac{1}{\pi}\int_{-inf}^{inf} x(t)sin(t\tau)$

$H\big(x(t)\big) = -\int_0^{inf}[a(t)sin(xt) - b(t)cos(xt)]dt = -sin(\Omega t)cos(\omega t).$

Obtain $H_{env}\big(x(t)\big)$ using

$$H_{env}\big(x(t)\big) = \sqrt{sin^2(\omega t)sin^2(\Omega t) + H^2\big(x(t)\big)} = \sqrt{sin^2(\omega t)sin^2(\Omega t) + sin^2(\Omega t)cos^2(\omega t)} = |sin(\Omega t)|\sqrt{sin^2(\omega t) + cos^2(\omega t)} = |sin(\Omega t)|$$

Hence, $H_{env}\big(x(t)\big) = |sin(\Omega t)|$.

CONCLUSION

This study demonstrates how to follow and analyze movement patterns from videos, such as a person walking or running, by using a mathematical tool called the Hilbert envelope. Using this method on video frames from infrared (IR) cameras, researchers have identified self-similarities or movement patterns that repeat similarly. These patterns can be used to detect and smooth erratic movements. Modifying these signals may enhance movement homogeneity, resulting in smoother and more reliable motion, according to the research. Sports training, medical rehabilitation, and tracking the operation of machinery to identify and address anomalies are some fields in which this method may be helpful.

REFERENCES

1. Kaiser JF. On a simple algorithm to calculate the ‘energy’ of a signal. International Conference on Acoustics, Speech, and Signal Processing, Albuquerque, vol.1, NM, USA, 1990, pp. 381-384. DOI: 10.1109/ICASSP.1990.115702.
2. Cui E, Peters JF. Self-similarities in optical flows. Chaos Solitons Fractals. 2022;164:112722. doi: 10.1016/j.chaos.2022.112722.
3. Andrade AO, Kyberd P, Nasuto SJ. The application of the Hilbert spectrum to the analysis of electromyographic signals. Inf Sci. 2008;178:2176–93. doi: 10.1016/j.ins.2007.12.013.
4. Feldman M, Braun S. Nonlinear vibrating system identification via Hilbert decomposition. Mech Syst Signal Process. 2017;84:65–96. doi: 10.1016/j.ymssp.2016.03.015.
5. Feldman M. Hilbert transform applications in mechanical vibration. John Wiley & Sons; 2011. doi: 10.1002/9781119991656.
6. Al-Ahmari A, Sinha JK, Asnaashari E. A comparison between Hilbert transform and a new method for signal enveloping. In: Sinha JK, editor. Vibration engineering and technology of machinery: Proceedings of VETOMAC X 2014, held at the University of Manchester, UK, September 9-11, 2014. Springer International Publishing; 2015. p. 163–72. doi: 10.1007/978-3-319-09918-7_14.
7. Kim AH, Kim IH. Essential spectra of quasisimilar (p,k)-quasihyponormal operators. J Inequal Appl. 2006;2006:1–7. doi: 10.1155/JIA/2006/72641.
8. Krantz S, Rosen KH, Zwillinger D. Standard Mathematical Tables and Formulae. Boca Raton (FL): Chapman & Hall/CRC Press; 2003.
9. Oppenheim AV, Willsky AS, Nawab SH. Signals & Systems. 2nd ed. Upper Saddle River (NJ): Prentice Hall; 2003.
10. Kido KI. Digital Fourier analysis: Advanced techniques. Springer; 2015. doi: 10.1007/978-1-4939-1127-1.
11. Boudraa AO, Salzenstein F. Teager–Kaiser energy methods for signal and image analysis: A review. Digit Signal Process. 2018;78:338–75. doi: 10.1016/j.dsp.2018.03.010.
12. Råde L, Westergren B. Mathematics Handbook for Science and Engineering. 5th ed. Berlin (Germany): Springer-Verlag Berlin Heidelberg; 2004. 562 p. DOI: 10.1007/978-3-662-08549-3.
13. Lathi BP, Green RA. Linear systems and signals. New York: Oxford University Press; 2005.
14. Haider MS, Peters JF. Temporal proximities: Self-similar temporally close shapes. Chaos Solitons Fractals. 2021;151:111237. doi: 10.1016/j.chaos.2021.111237.
15. Deng H, Liu J, Li H. EMD based infrared image target detection method. J Infrared Millim Terahertz Waves. 2009;30:1205–15. doi: 10.1007/s10762-009-9548-9.
16. Ulrich T. Envelope calculation from the Hilbert transform. Los Alamos National Laboratory, Los Alamos, NM, USA; 2006.
17. Weisstein EW. Self-similarity. MathWorld--A Wolfram Web Resource. 2024. Available from: https://mathworld.wolfram.com/Self-Similarity.html.

