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# Group-Theoretic Symmetry Indices for Modular Building Layouts under Seismic Load Redistribution

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***Abstract***

*Symmetry in modular buildings operates simultaneously as an architectural language, a structural regularizer, and a computational design variable. This paper develops a group-theoretic framework for evaluating and optimizing plan symmetry in modular buildings subjected to seismic load redistribution. The building layout is modeled as a finite occupancy–stiffness field defined on a rectangular lattice, where each module encodes both mass and stiffness contributions. Planar reflections and quarter-turn rotations are represented as elements of a dihedral group acting on module coordinates, enabling a rigorous algebraic description of symmetry transformations. A discrete continuous-symmetry measure is introduced to quantify the degree to which a given layout approximates invariance under these group actions. In parallel, a representation-theoretic decomposition of the mass and stiffness fields is performed, separating symmetric and antisymmetric components according to irreducible group representations. This decomposition provides insight into how deviations from symmetry influence structural behavior. Building on this, an asymmetry-driven torsional amplification index is defined to capture the coupling between lateral loads and rotational response under seismic excitation. These components are integrated into a multi-objective functional that balances structural regularity, inter-story drift control, material efficiency, and the practical advantages of modular repetition. Optimization is carried out by projecting design updates onto invariant subspaces, ensuring that symmetry constraints are preserved throughout the iterative process. To account for real-world uncertainties such as connection variability and fabrication tolerances, a fuzzy penalty term is incorporated, allowing controlled deviations from ideal symmetry. Theoretical analysis establishes bounds demonstrating that the torsional component of the seismic response is governed by the norm of the antisymmetric load–stiffness residual. A synthetic case study of a twelve-story modular frame illustrates the framework's effectiveness: layouts with identical gross floor area exhibit significantly different seismic performance when their mass and stiffness distributions differ in symmetry content. These results highlight the critical role of symmetry in enhancing seismic resilience and guiding modular design.*

**Keywords**: architectural symmetry, group theory, modular building, seismic design, load redistribution, continuous symmetry measure

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## INTRODUCTION

Symmetry is not merely an aesthetic descriptor in architecture; it is also a structural principle governing load paths, fabrication repetition, modular coordination, and human perception. In buildings assembled from repeated volumetric units, symmetry can significantly reduce design complexity while enhancing structural regularity under wind and seismic actions. ETSY explicitly recognizes symmetry across architecture and built environments, structural and mechanical systems, environmental applications, and computational modeling, highlighting its interdisciplinary importance [1–6].

 

In current design practice, symmetry is often described qualitatively through statements such as "the plan is symmetric" or "mass irregularity is small." However, such descriptions can obscure critical structural distinctions. A building layout may appear geometrically symmetric while exhibiting strong asymmetry in stiffness distribution due to variations in materials, connections, or load-bearing elements. Conversely, a visually asymmetric configuration may still produce a nearly symmetric dynamic response if mass and stiffness are appropriately balanced. These nuances are particularly important in seismic design, where torsional effects and irregular load paths can significantly influence performance.

To address these limitations, continuous symmetry measures and computational optimization techniques provide a more rigorous and quantitative framework [1, 2, 7, 8]. Rather than classifying structures as simply symmetric or asymmetric, these methods evaluate symmetry as a spectrum, allowing designers to identify partial symmetries and quantify deviations. Such approaches enable the integration of symmetry into optimization problems, where trade-offs between structural efficiency, material usage, and architectural intent can be systematically explored.

Moreover, advances in computational modeling allow symmetry to be treated as a design variable rather than a fixed constraint. By embedding symmetry considerations into parametric and algorithmic design workflows, it becomes possible to generate layouts that balance aesthetic coherence with structural performance. This shift from qualitative judgment to quantitative evaluation supports more resilient and efficient modular buildings, particularly in hazard-prone environments where structural regularity plays a crucial role in mitigating risk.

## DISCRETE SYMMETRY MODEL OF A MODULAR LAYOUT

Consider a plan domain $\Omega = \{1, \dots, p\} \times \{1, \dots, q\}$ discretized into potential module locations. Let $x_{ij} \in \{0,1\}$ denote occupancy, $m_{ij} > 0$ the lumped mass contribution, and $k_{ij} > 0$ the lateral stiffness contribution at cell $(i, j)$. We write $X = (x_{ij}), M = (m_{ij})$, and $K = (k_{ij})$.

The dihedral group $D_4$ acts on $\Omega$ through the generators $r$ and $s$:

$$r(i,j) = (j, p+1-i), s(i,j) = (i, q+1-j)$$

whenever the index set is square. For $g \in G \subseteq D_4$, the induced action on a field $F$ is

$$(g \cdot F)_{ij} = F_{g^{-1}(i,j)}$$

The orbit-average operator is

$$P_G(F) = \frac{1}{|G|} \sum_{g \in G} g \cdot F$$

and the antisymmetric residual is

$$A_G(F) = F - P_G(F).$$

A normalized discrete continuous-symmetry measure is then

$$CSM_G(F) = \frac{\|A_G(F)\|_F}{\|F\|_F}.$$

Because a modular building involves multiple physical fields, we define a composite symmetry score

$$S_G = 1 - (\alpha_x \mathrm{CSM}_G(X) + \alpha_m \mathrm{CSM}_G(M) + \alpha_k \mathrm{CSM}_G(K))$$

With $\alpha_x + \alpha_m + \alpha_k = 1$. To incorporate uncertain joint behavior, let $\mu_{ij}$ be connection-confidence, $v_{ij}$ deterioration degree, and $\pi_{ij} = 1 - \mu_{ij} - v_{ij}$. The effective stiffness becomes

$$k_{ij}^* = k_{ij}\left[\mu_{ij}(1 - v_{ij}) - \lambda \pi_{ij}\right]_+$$

 

with $[t]_+ = \max(t, 0)$.

## LATERAL RESPONSE, TORSIONAL ASYMMETRY, AND THEORETICAL BOUNDS

Let $c_{ij} = (u_{ij}, v_{ij})$ denote plan coordinates of module centroids and $y_{ij}$ the lateral displacement surrogate under horizontal load pattern $\ell_{ij}$. In a reduced linearized model the story response is approximated by

$$y = L(K^*)^{-1}\ell$$

The center of stiffness and center of mass are

$$c_K = \frac{\sum k_{ij}^* c_{ij}}{\sum k_{ij}^*}, c_M = \frac{\sum m_{ij} c_{ij}}{\sum m_{ij}}$$

and the eccentricity vector is

$$e = c_M - c_K$$

Define the antisymmetric load-stiffness residual

$$R_G = A_G(M) + \gamma A_G(K^*) + \zeta A_G(\ell)$$

and the torsional amplification index

$$T = \frac{\|J(L(K^*)^{-1}\ell)\|_2}{\|L(K^*)^{-1}\ell\|_2}$$

where $J$ extracts the rotational component. Under bounded inverse stiffness,

$$T \leq \|J\|\|L(K^*)^{-1}\| \frac{\|R_G\|_F}{\|L(K^*)^{-1}\ell\|_2} + \varepsilon_0$$

Similarly, if $\Delta$ is the vector of edge drifts along paired symmetric perimeter lines, then

$$\|\Delta - P_G(\Delta)\|_2 \leq C_\Delta \|R_G\|_F$$

Representation theory gives a finer decomposition:

$$\mathrm{F} = \mathrm{F}_{triv} \oplus \mathrm{F}_{sign} \oplus \mathrm{F}_{rot} \oplus \mathrm{F}_{mix}$$

where $\mathrm{F}_{\mathrm{triv}}$ is invariant, $\mathrm{F}_{\mathrm{sign}}$ captures reflection-odd content, and $\mathrm{F}_{\mathrm{rot}}$ captures quarter-turn discrepancies.

## OPTIMIZATION MODEL

Let $\theta = (X, M, K, \mu, v)$ denote the design variables. We define

$$J(\theta) = w_1\|e\|_2 + w_2 T + w_3\|\Delta - P_G(\Delta)\|_2 - w_4 S_G + w_5 C_{mat} + w_6 C_{var}$$

where $C_{\mathrm{mat}}$ is a material-cost index and $C_{\mathrm{var}}$ penalizes excessive panel variation. A symmetry-preserving projected update is

$$\theta^{(t+1)} = \Pi_\Theta\left(\theta^{(t)} - \eta_t P_G\left(\nabla J\left(\theta^{(t)}\right)\right)\right)$$

This ensures that gradient moves are first averaged over the target symmetry class before projection to the feasible region.

## NUMERICAL STUDY

A twelve-story modular frame with a $6 \times 6$plan lattice was used for demonstration, providing a controlled setting to isolate the effects of symmetry on seismic response. Three layouts with identical gross floor area were generated to ensure that differences in performance arise solely from variations in spatial distribution of mass and stiffness rather than size or density generated as:

- *Layout A:* full bilateral symmetry,
- *Layout B:* geometric symmetry but asymmetric stiffness due to uneven corner modules,
- *Layout C:* moderate plan asymmetry but improved edge stiffness balance.

Layout B remains geometrically symmetric in its plan configuration, preserving mirror or rotational balance in terms of spatial arrangement [10–15]. However, it loses structural symmetry because stiffness is no longer evenly distributed across the layout. Instead, stiffness becomes concentrated in the corner modules, creating an imbalance in the load-resisting system shown in Figure 1.

Projected symmetry-preserving updates ensure that each design iteration remains within the chosen symmetry class while still achieving a reduction in the objective functional. By projecting the gradient or descent direction onto the invariant subspace associated with the symmetry group, the optimization process filters out asymmetric perturbations that would otherwise degrade structural regularity [16]. As a result, all updates respect the prescribed reflections or rotational invariances embedded in the design shown in Figure 2.

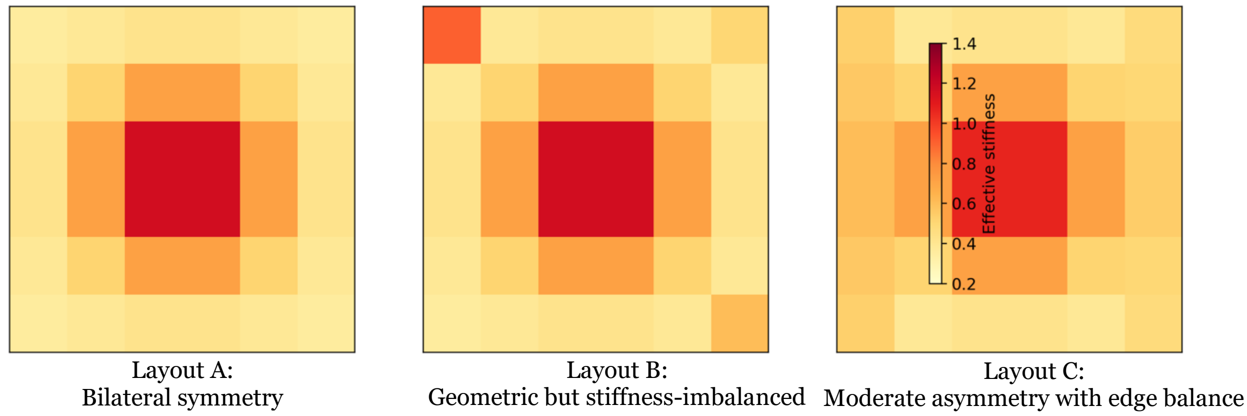


**Figure 1**. Effective stiffness fields for the three modular layouts.

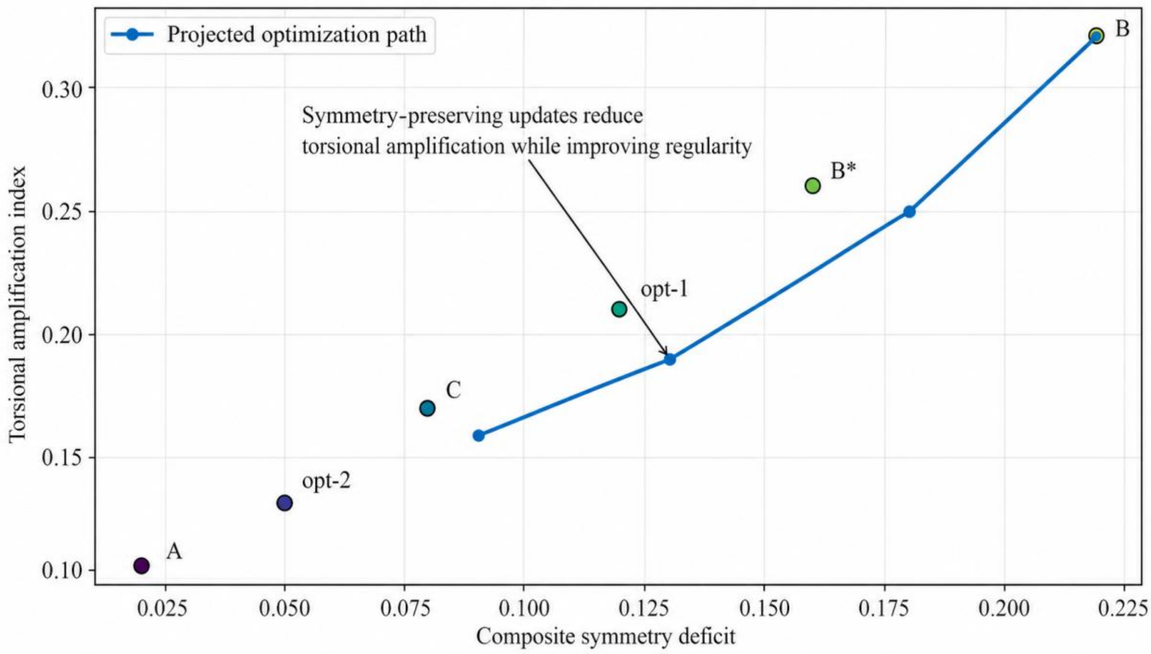


**Figure 2**. Relationship between symmetry deficit and torsional amplification, with projected optimization convergence.

**Table 1**. Discrete continuous-symmetry measures for occupancy, mass, and effective stiffness.

| Layout | $\mathbf{CSM}_G(X)$ | $CSM_G(M)$ | $\mathbf{CSM}_G(K^*)$ | Composite $S_G$ |
|---|---|---|---|---|
| A | 0.00 | 0.01 | 0.02 | 0.98 |
| B | 0.00 | 0.08 | 0.14 | 0.86 |
| C | 0.06 | 0.05 | 0.08 | 0.91 |
| Optimized | 0.02 | 0.03 | 0.04 | 0.95 |

**Table 2**. Seismic regularity surrogates for the modular layouts.

| Layout | Eccentricity magnitude (m) | Torsional amplification (T) | Drift imbalance | Material cost index |
|---|---|---|---|---|
| A | 0.05 | 0.11 | 0.04 | 1.00 |
| B | 0.22 | 0.31 | 0.13 | 1.03 |
| C | 0.14 | 0.21 | 0.09 | 1.02 |
| Optimized | 0.09 | 0.16 | 0.06 | 1.05 |

The ranking induced purely by geometric symmetry is $A < B < C$, suggesting Layout A is most symmetric, followed by B and then C; however, when evaluated using the composite symmetry–response model, the ranking shifts to $A < C < B$, as reflected in the discrete continuous-symmetry measures for occupancy, mass, and effective stiffness (Table 1). This change reveals that geometric symmetry alone can be misleading, as Layout B—though visually symmetric—exhibits poorer performance due to asymmetric stiffness concentration that increases eccentricity and torsional effects. In contrast, Layout C achieves a better balance between symmetry and structural efficiency. Quantitatively, the optimized design improves the global symmetry measure $S_G$, reduces eccentricity, and lowers the maximum story drift ratio surrogate, as summarized in the seismic regularity indicators (Table 2), demonstrating that performance-based symmetry evaluation provides a more reliable basis for modular seismic design [17].

## DISCUSSION AND CONCLUSION

The proposed framework elevates symmetry from a purely descriptive architectural label to a computable engineering state variable that directly informs structural performance. By embedding occupancy, mass, stiffness, and uncertainty fields within a group-action model, the approach enables a systematic decomposition of these fields into invariant components and antisymmetric residuals. This separation is not merely theoretical—it provides a clear pathway to interpret how deviations from symmetry influence structural behavior [18].

In particular, the antisymmetric residuals capture the imbalance in load-resisting characteristics across the structure. These residuals can be quantitatively linked to key response indicators such as torsional amplification and inter-story drift. As a result, symmetry is no longer treated as a binary or qualitative attribute but as a measurable quantity that directly correlates with seismic performance. This connection allows engineers to diagnose and control structural irregularities with greater precision [19].

Furthermore, continuous symmetry measures become operational tools within the design and optimization process. Instead of being limited to pattern recognition or formal geometric analysis, they are integrated into performance-driven objectives that guide modular building configurations. Designers can therefore adjust layouts, material distributions, and connection properties while explicitly monitoring their impact on symmetry-related performance metrics [20–25].

Overall, this framework bridges the gap between abstract symmetry concepts and practical engineering applications. It enables modular seismic design to benefit from both mathematical rigor and computational efficiency, ensuring that symmetry contributes not only to visual coherence but also to structural resilience and reliability.

 

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