

Fracture Analysis of Laminated composite plates using Extended Finite Element Method: A Review

Atul. A. Joshi^{1*}, Shailesh P. Palekar²

Abstract

Laminated composite plates are used in aerospace, automotive, and marine industries. They feature great durability against fatigue, a high strength-to-weight ratio, and mechanical attributes that may be altered. However, they are prone to fracture and delamination under complex loading, requiring accurate fracture analysis for structural integrity. Traditional finite element methods (FEM) need extensive mesh refinement for modelling crack propagation which increases the computational costs. The Extended Finite Element Method (XFEM) offers a more efficient alternative by incorporating discontinuous functions into the finite element framework, allowing accurate crack modelling without mesh dependency. The review identifies research gaps and future directions, emphasizing the need for improved XFEM formulations to model matrix cracking, fibre breakage, and delamination. XFEM can be integrated with computational tools like artificial neural network (ANN) for enhanced fracture predictions, aiding researchers in developing efficient predictive models for real-world applications.

Keywords: Laminated composite plates, fracture mechanics, extended finite element method, crack propagation

INTRODUCTION

Composite structures with technology-tailored mechanical properties are used in engineering applications such as aerospace, shipbuilding, automotive, medicine. Orthotropic structures like laminated composite plates goes under complex manufacturing process resulting in existence of various discontinuities such as cracks, holes, voids, inclusions, and flaws. These discontinuities significantly degrade a material's strength by formation of pre- cracks which grows during the service life of component and lead to the failure of engineering components. To prevent catastrophic failures of engineering components in service life, it becomes important to study the stress intensity factors at the crack tip and growth of crack for components made of laminated composites.

*Author for Correspondence

Atul. A. Joshi
E-mail: Joshiatulmech@sanjivani.org.in

¹Research Scholar, Department of Mechanical Engineering, Sanjivani College of Engineering, Kopergaon., Maharashtra, India

²Associate Professor, Department of Mechanical Engineering, Sanjivani College of Engineering, Kopergaon., Maharashtra, India

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In case of orthotropic material like laminated composite plate, to analyse material properties, four elastic moduli, five strength parameters, and two coefficients of thermal expansion are among the thirteen parameters that require experimental determination and these parameters depend on a number of variables like fibre volume fraction, packing geometry, lamina stacking sequence. It makes experimental evaluation of these parameters more expensive and time-consuming than in isotropic materials. Therefore, there is necessity and incentive for creating analytical models to determine these parameters for composite materials. The main findings in the fields of fracture mechanics, finite element methods, and composite

materials since 1966 are highlighted in this review, along with the different analytical techniques created to forecast composite material failure using XFEM.

DISCUSSION

Tsai S.W.-(1966) Set out the failure theory for unidirectional laminae which is based on the distortion-energy failure theory of Von Mises [1]. The distortion energy is part of the total strain energy in the body and is made up of two parts: one due to volume change and the other due to shape change where material properties are assumed to be similar in all direction but Tsai S W, Wu, E M. (1971) proposed theory based on total strain energy failure theory which differentiates the compressive strength from the tensile strength of the lamina with greater precision [2]. G. C Sih (1973) showed that modes of failure for unidirectional composites are governed by the properties of the constituent materials, as well as the conditions under which they were fabricated [3]. The presence of these initial flaws or cracks in the composite may appreciably affect the load at failure so it will be extremely difficult, if not completely impossible, to develop a single analytical model predicting all forms of fracture. As it was deduced that due to complexity of the problem make any one experimental method insufficient to predict the exact material properties, efforts to build analytical method started. In similar attempt, Owen, D. R. J., & Fawkes, A. J. (1983) developed few Singular finite Elements for simulating crack tip singular fields [4]. see Figure 1. The use of these elements has considerably upgraded the level of accuracy obtained by finite element method for simulation of crack tip fields but still these Singular elements lacked the capability of modelling discontinuity across a crack path.

Early numerical approaches for modeling discontinuities within finite element frameworks were developed in the late 1980s, laying the groundwork for subsequent advancements in discontinuity modeling techniques. A strong discontinuity can be modeled by modifying the principle of virtual work. In this approach, the displacement field is divided into a standard (continuous) component and an additional enhanced component, where the enhanced part accounts for the displacement jump across the discontinuity surface. Another approach for simulating fracture behavior is based on a cohesive surface traction–displacement relationship, which introduces an intrinsic length scale into the model. As a result, there is no need for a separate fracture criterion, and both crack initiation, growth, and propagation path emerge naturally from the analysis.. In conventional Finite Element Method (FEM), the non-smooth displacement at the crack tip is essentially modelled by local mesh refinement. Degree of Freedom considerably rises and incremental calculation of crack growth requires repeated remeshing leading to computationally intensive work so conventional FEM has reached its boundary for solving fracture mechanics problem.

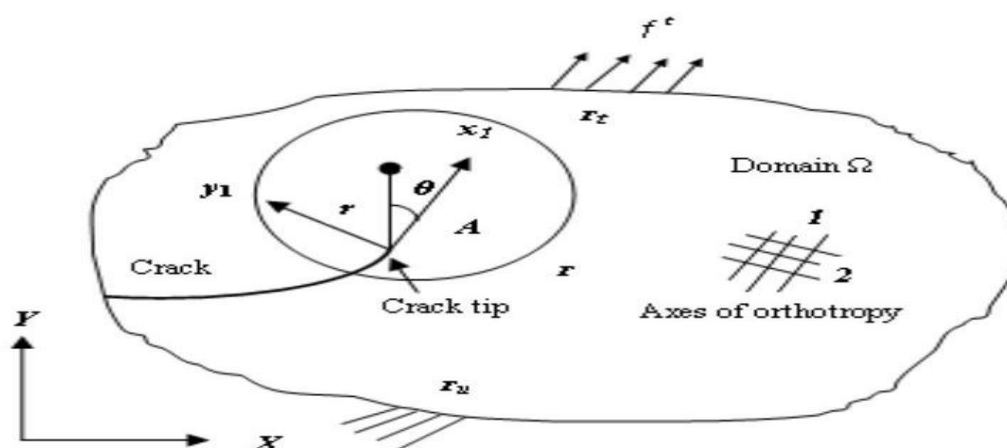


Figure 1. Shows An arbitrary orthotropic body with crack, subjected to traction f^t , having global Cartesian coordinate (X, Y) , local polar co-ordinate (r, θ) defined at the crack-tip surrounded by contour r and its interior area A with arbitrary boundary conditions.

In order to address this, Melenk and Babuska(1996) [5] and Belytschko (1999) [6] presented elementary mathematical background of the partition of unity finite element method (PUFEM) which later referred as extended finite element method. (X-FEM) which has new minimal remeshing technique of FEM to predict crack growth by including the discontinuous enrichment functions to the approximation space that contains a discontinuous displacement field This enabled a crack to be arbitrarily aligned during the analysis. This new method is called as Extended Finite Element method (XFEM). see Figure 2. In extended finite element method, first normal finite element mesh is generated then some degrees of freedom are added to the traditional finite element model in chosen nodes close to the discontinuity for providing increased level of accuracy [7, 8].

Moes, Dolbow and Belytschko [9] proposed new interaction energy integral method for computation of mixed mode stress intensity factor at the tips of arbitrary oriented cracks in functionally graded materials using minimal remeshing finite element method in which discontinuous enrichment functions are used with finite element approximation to account for presence of a crack Jirasek and Zimmermann (2001) [7] Combined X-FEM with damage theory and advocate new concept of a model with transition from a smeared to an embedded discrete crack.

Sukumar N, Chopp D.L, Moes N and Moran B (2003) presented numerical technique for planar three-dimensional fatigue crack growth simulations [8]. The new technique couples the extended finite element method (X-FEM) to the fast-marching method (FMM). In the X-FEM, a discontinuous function and the two-dimensional asymptotic crack-tip displacement fields are added to the finite element approximation to account for the crack using the notion of partition of unity. Zi, Ted Belytschko and Hao Chen-carried out numerical simulation of linear elastodynamic problems by using XFEM and concluded that XFEM can be used in stationary and propagating cracks and numerical examples [9].

Liu et al (2004) enhanced XFEM by direct computation of mixed mode stress Intensity factors [10]. Zienkiewicz et al (2005) illustrates how FEM is applicable for nonlinear materials and problems of large deformations. Nistor, O. Pantale, S. Capera (2005) have evolved methodology to simulate the dynamic discontinuities through numerical solution of the Extended Finite Element Method (XFEM) [11]. Nobile, L. and Carloni (2005)- Researched elastostatic fracture response of an orthotropic cracked plate loaded in a biaxial uniform load at infinity [12]. The influence of orthotropy and biaxial loading on the near crack tip elastic fields is noted. The strain energy density theory and maximum circumferential stress theory are addressed and applied to orthotropic materials. Numerical analysis is carried out for a broad range of orthotropic material properties and loads applied.

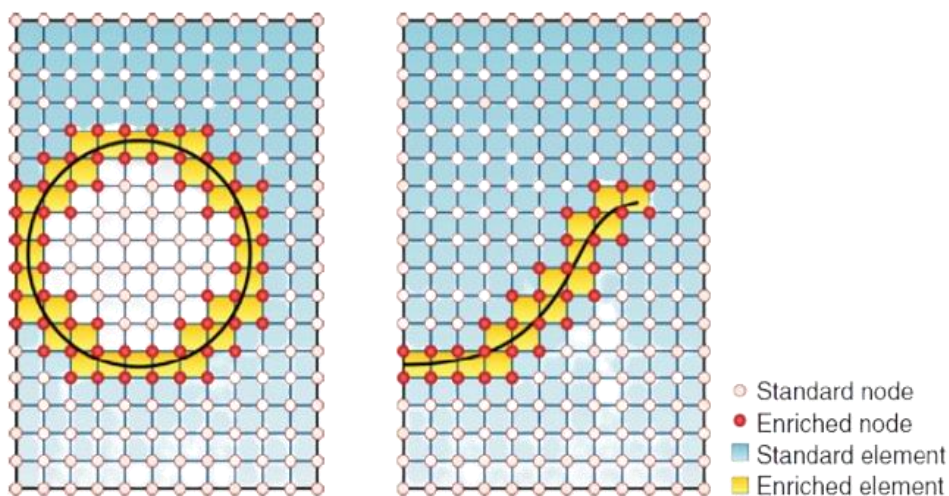


Figure 2. Modelling the discontinuity with extended FEM weak discontinuity as biomaterial interface and strong discontinuity as crack interface [52].

J. Rethore, A. Gravouil, A. Combescure (2005)-introduces a generalization of the Extended finite element method (X-FEM) for dynamic fracture and time-dependent problems from a more general perspective, and presents a proof of stability of the numerical scheme in the linear case. It demonstrates that Lagrangian conservation-based technique for dynamic stress intensity factor estimation for arbitrary 2D cracks [13]. The findings given for a number of applications are valid for stationary or moving cracks. Alireza Asadpoure, Soheil Mohammadi, Abolhasan Vafa (2006) suggested for an extended finite element method for the modelling of crack in orthotropic media. A piecewise discontinuous function and two-dimensional asymptotic crack-tip displacement fields are utilized in a traditional finite element approximation enriched with partition of unity framework [14]. Here, fracture characteristics of the models are specified through mixed-mode stress intensity factors (SIFs) that are derived through the domain expression of the interaction integral (M-integral). Numerical simulations are conducted to validate the method, and the reliability of the results is presented through comparison with other numerical or (semi-) analytical solutions N. Sukumar, J. E. Dolbow, N. Moës (2015)-provided retrospective analysis with a historic overview of the computational fracture mechanics, and emphasize the most significant advancements and recommended practices as they pertain to the development and numerical implementation of the X-FEM for fracture problems [15].

B. Prabel, A. Combescure, A. Gravouil (2007) created two improvements enhancing crack propagation modelling using the X-FEM approach [16]. It describes how one can employ simultaneously a structured mesh for an accurate and efficient level set update and an irregular unstructured one for the mechanics model. It also provides a new numerical formulation following the X-FEM approach for elastic-plastic dynamic situations.

T.P. Fries (2007) modified the enrichment functions such that they are zero in the standard elements, unchanged in the elements with all their nodes being enriched, and varying continuously in the blending elements. All nodes in the blending elements are enriched. The modified enrichment function can be reproduced exactly everywhere in the domain and no problems arise in the blending elements [17]. Thomas Hettich, Andrea Hund (2007)-The global mechanical response of composites in the linear and the nonlinear regime is not only determined by the material properties of the constituents and the bond but also by the material arrangement. The material structure is discretized and described on a small scale enabling to account for these effects. In the X-FEM the finite-element approximation is enriched with relevant functions using the partition of unity concept [18]. The material interface and crack geometry are modelled by the LSM (level set method) The combination of the two, X-FEM and LSM, proves to be quite natural because the enrichment can be explained and even formulated in terms of level set functions. S. H. Ebrahimi, S. Mohammadi, A. Asadpoure: (2008) characterized fracture properties of the models through the mixed-mode stress intensity factors (SIFs), which are calculated through the domain integral (M-integral) [19]. J. Réthoré, Combescure, A. Gravouil, D. Gregoire, (2008)- emphasizes cases where the crack is not along a straight line path and experiences stop-and-restart processes. It demonstrates that the X-FEM is a potential algorithm for the simulation of complex dynamic crack propagation [20]. A validated two-dimensional implementation of the proposed X-FEM method on dynamic experiments of a brittle isotropic plate. E. Giner (2009) et al – presented an implementation of the extended finite element method for fracture problems using the finite element software ABAQUS [21].

Y. Xu & H. Yuan (2009) introduced and implemented an extended finite element method (XFEM) containing strong discontinuity within elements in the commercial general-purpose software ABAQUS [22]. Matthew J. Pais (2009) combined XFEM method and level set method to simulate weak and strong discontinuities using element-based enrichment function implementation [23]. Thomas Menouillard and Ted Belytschko (2010)-suggested new technique for releasing the crack tip element through the utilization of correction force, it illustrates the relationship between the correction force and the location of the crack tip within the element and the smooth release when the crack propagates [24]. D. Motamedi, S. Mohammadi (2010) Examined dynamic crack growth of composites from the novel progress and

development of orthotropic enrichment functions in the partition of unity framework and the extended finite element method (XFEM) [25]. Several benchmark and test examples are simulated and the solutions are compared to available reference solutions. T. Menouillard, J. Song, Q. Duan, T. Belytschko (2010)- investigated various enrichment techniques for dynamic crack growth within the framework of the extended finite element method and the influence of various direction criteria on the direction of the crack [26]. A new enrichment technique with a time dependent enrichment function is introduced. Stress intensity factor results and crack paths for various enrichments and direction criteria are presented. T. Menouillard, J. Song, Q. Duan, T. Belytschko (b) (2010)-recommended enhancements in numerical features of dynamic crack growth routines by the extended finite element technique are explained and investigated [27]. Liu, Z.L., Menouillard, T. & Belytschko (2011) presented XFEM/Spectral element method for dynamic crack propagation [28]

Ghorashi, S. S., Mohammadi, S. and Yazdi (2011) new method for discrete crack modelling in two-dimensional orthotropic media using the element free Galerkin method is presented. To enhance the solution precision, newly presented orthotropic enrichment functions utilized within the extended finite element method are implemented together with a sub-triangle method to improve the accuracy of the Gauss quadrature at the crack [29]

Burlayenko V. N., Sadowski T. (2012) developed a finite element model for analysing the dynamic response of sandwich plates with partially damaged face sheet-to-core interface. Here, the three-dimensional finite element model of the sandwich plate with penny-shaped de bonded zone located at the plate center has been developed [30].

Hattori, et al (2012) The performance of the newly derived enrichment functions is investigated, and comparisons are made with the widely known classical crack-tip functions for isotropic materials [31]. R. Pourmodheji, M. Mashayekhi (2012) -The continuum damage mechanics (CDM) model is incorporated into the extended finite element method (XFEM) to achieve a model of ductile crack growth [32]. In this model, the critical damage parameter from continuum damage mechanics is used as the criterion for crack growth in the analysis of ductile materials. Crack propagation has been simulated using the proposed model in two examples; centre inclined crack and single edged notched beam for A533B1 steel. Tran Van-Xuan, Geniaut Samuel (2013)-presented model analysis for the calculation of the stress intensity factors under dynamic load at low frequency using XFEM. In this case, the modal stress intensity factors of a body with immovable crack are calculated from the displacement fields of the deformed model shapes [33].

R. Tiebreak, M. Bachene, B.K. Hachi, S. Rechak and M. Haboussi (2014) suggested a model considering both the transverse shear deformation and rotatory inertia has been numerically implemented by using X-FEM [34]. It is parametric research where crack length and position have been analyzed. In the numerical solution, standard finite element with no discontinuity is initially performed, followed by introduction of enriched functions into nodal displacement field for cracked element nodes. As a result, no remeshing of the domain is needed, resulting in a significant reduction in time computing. Wang, Y., & Waisman, H. (2014) progressive delamination by way of a discrete damage zone model in the extended finite element method is examined [35].

The model permits both bulk and interface damages to be easily tracked, irrespective of the mesh alignment underpinning. A number of benchmark delamination analyses and failure analyses for a fiber/epoxy unit cell are shown and elaborated on. The model proposed is compared with existing analytical/experimental data and is shown to be robust and mesh insensitive Meek, C and Ainsworth, R.A. (2015) conducted Finite element analysis for a Centre-cracked plate under a broad range of biaxial loading in plane strain for a variety of crack sizes and for a variety of plate lengths [36]. Jian-Ying Wu, Feng-Bo Li (2015) introduced A simple and efficient improved stable XFEM (Is-XFEM) with an innovative enrichment function [37].

The innovative enrichment is the Heaviside function stabilized by its linear interpolant. The presented Is-XFEM is of both high accuracy and well-conditioned system matrix.

Afshar, A. Daneshyar, S. Mohammadi (2015) extended the extended finite element method (XFEM) to investigate the fiber bridging phenomenon in fracture mechanics of unidirectional composites [38]. By coupling the ability of XFEM in simulating arbitrary crack path with the traction-separation behaviour of the fracture process zone in the model of crack-bridged zone, which enables simulation of fibre bridging in a crack growth process without prior knowledge of the crack path.

Longfei Wen & Rong Tian (2016)- suggested Improved XFEM to use for dynamic problems in order to determine dynamic SIF in the benchmark problems using an accelerated iterative solver [39]. Baran I. Cinar K., Ersoy N., Akkerman R., Hattel J.H. (2016)- suggested a numerical process model for virtual design and optimization of the composite manufacturing process which can be free from the costly trial-and-error based methods [40]. Arregui-Mena, J.D., Margetts L., Mummery P.M. (2016) presented brief review of the Stochastic Finite Element Method (SFEM) [41]. The review article presents the most widely applied approaches: direct Monte Carlo simulation, the perturbation method and the spectral stochastic finite element method. Subsequently, this article examines present software available on the SFEM and gives illustrations from the material science, and engineering sciences to demonstrate a variety of steps by which SFEM is utilised in real practice Kumar, S, Singh I.V., Mishra, B.K. Sharma, K., Khan, I. A. (2016) suggested A homogenized multigrid XFEM method to simulate the stable crack growth in ductile material based on finite strain plasticity [42]. The influence of different kinds of flaws has been seen on the load carrying capacity of the structures. Sh. Akhondzadeh, A.R. Khoei, P. Broumand (2016) proposed an effective enrichment strategy to simulate stress singularities in multilateral problems and crack-tips ending at a bi-material interface in the XFEM framework [43]. The eigen-function expansion method is used to compute eigenvalues and asymptotic fields near singular points. The generalized stress intensity factors are formulated based on direct and interaction integral techniques.

S. Dey, T. Mukhopadhyay, S. Adhikari (2017) suggested that as the laminated composite plate is dependent upon a large number of parameters involved in complex fabric action and production process of laminated composite plate, then the laminated composite system property can be of random type which leads to uncertainty in laminated composite plate response [44]. Hence, to properly define the initial problems and facilitate a better comprehension and description of the true behaviour of the laminated composite structures, it is of utmost significance that the intrinsic randomness in system parameters is integrated into the analysis.

Rossana Dimitri, Nicholas Fantuzzi, Yong Li, Francesco Tornabene (2017) introduced application of the level set method in combination with the numerical extended finite element method (XFEM) to simulate the fracture propagation direction in a specimen, and to calculate the stress intensity factor for cracked composite plates with varied loads [45]. R. Higuchi, T. Okabe, T. Nagashima (2017) suggested a high-fidelity mesoscale simulation approach able to predict carbon fibre reinforced plastic laminates' (CFRP's) progressive damage and corresponding failure [46]. The onset of matrix cracking and delamination, which causes post-peak softening of the local stress-strain curve, is modelled using cohesive zone models (CZM). The CZM for delamination is implemented with an interface element, but CZM for matrix cracking is implemented with an extended finite element method (XFEM). Yu, T., Bui, T. Q. (2018) proposed a 2-D crack numerical simulation for laminated composite plates at material interfaces [47]. Cracks or material interfaces' discontinuity and singularity are realized by local enrichments in the context of partition of unity. Bansal, M., Singh, I.V., Mishra, B.K. (2018)- Developed a parallel and computationally fast multi-split XFEM method for 3-D analysis of composite materials where the mesh size does not depend on the relative distance between the heterogeneities/discontinuities [48]. Patil, R.U., Mishra, B.K., Singh, I.V. (2019) introduced the phase field approach (PFM) combined with multiscale extended finite element method (MsXFEM) for the simulation of crack propagation in

strongly heterogeneous materials Numerical computations are carried out by taking account of three heterogeneous material types [49]: voids, particles with an ideal interface, and particles with a finite interface The performance of the suggested approach is confirmed by different numerical experiments. P. Shailesh and A. Lal (2021) analysed the fracture response of symmetric angle ply laminated composite plates having a central crack under biaxially applied tensile, shear and tensile and shear combined stresses is carried out by using well established extended finite element method (XFEM). (2022)-suggested a new extended finite element method (XFEM)-artificial neural network (ANN) strategy is introduced to forecast the performance of the aluminium panels repaired single-sided with a composite patch [50]. The deep neural network model optimized through data sets generated from XFEM—validated with experimental outcomes. The constructed ANN model is founded on XFEM data sets which will not capture damage and delamination of the composite patch, as well as non-linear material behaviour [51, 52].

CONCLUDING REMARKS ON REVIEWED LITERATURE

1. Numerous empirical and phenomenological models have been introduced in the last decade of fracture studies for composite materials by using XFEM.
2. Many researchers do not represent real world crack problems; they are only used as efficient simplified models for other highly complex dynamic phenomena.
3. In most of the cases, authors had limited experimental data available, and variable data they are modelling often fit to the any experimental data available
4. Advanced computational commercial packages like ABAQUS and MATLAB are used to simulate the standard cases of the crack propagation in isotropic and orthotropic materials. Many listed authors have used customized computational packages as standard package cannot solve all the practical cases documented in literature of crack propagation
5. The XFEM has lot of potential to investigate the of complex problems of fracture mechanics considering interactions of various discontinuities with possible variations like change in Shape of plate geometry and change in shape of discontinuities
6. Behaviour of Laminated composite plates under hygrothermal or dynamic loading can be studied computationally to predict the life of structure made of laminated composite plates. Computational approach is more practical in case of laminated composite plates because of orthotropic and stochastic nature of its material properties

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