Journal of
Polymer & Composites
ISSN: 2321-2810 (Online)
ISSN: 2321-8525 (Print)
Volume 13, Special Issue 3, 2025
DOI (Journal): 10.37591/JoPC
https://journals.stmjournals.com/jopc
Review JoPC

Free Vibration of Curved Sandwich Beams with Laminated Composite Facings Using Finite Element Method

Joydeep De[1], Priyankar Datta[2,*]

Abstract

The present work is focussed on the free vibration analyses of singly curved sandwich beams using three-dimensional finite element method in ABAQUS software. Three different layers are considered for soft core and stiff face sheets in ABAQUS solid modelling to accurately represent the effects of transverse shear deformation of sandwich structure. To validate the model, obtained natural frequencies from present model are first compared with the results available in the literature for sandwich plates and are found to be in very good agreement. Several parametric studies are carried out to investigate the effect of aspect ratio, core to face thickness ratio and different layup of composite face on the free vibration of the curved sandwich beams with different boundary conditions. It is seen that the natural frequency decreases with the increase in aspect ratio, while, different orientations of composite face lamina have little impact on the same. Core to face thickness ratio and different boundary conditions affect the first few natural frequencies of the curved sandwich beams. Technique has been described for utilizing readily available commercial finite element software ABAQUS to enquire the vibrational characteristics of curved sandwich beam, one, which is less investigated till date but have structural importance.

Keywords: Free vibrations, curved beam, sandwich beams, finite element method, shear deformation

INTRODUCTION

Curvatures in structural member are often commonly found in bridges, aerospace, tunnels, roof structures. Sandwich beam with soft core and curved geometry are increasingly getting popular due to their applicability to different industries like aerospace, marine, structure etc. Due to high strength to weight ratio, excellent damping capacity, thermal and acoustic insulation these materials are now matter of interest. As these materials are often used as structural members and subjected to variable excitations, proper investigations are required to predict the dynamic behaviour especially to avoid resonance. Although several damping techniques are developed for structural vibration control [1-4], it is essential to first identify the inherent dynamic properties of the base structure. Different investigations have been carried out in this aspect focussing on straight sandwich beam with isotropic or composite face and soft isotropic core. Very few available literatures have investigated the curved sandwich beam with composite face.

Chalak et al. [5] presents the study of free vibration of laminated sandwich beam having soft core using C^0 finite element beam model based on higher order zigzag theory considering in-plane cubic displacement for entire thickness. Transverse

*Author for Correspondence
Priyankar Datta

[1]Research Scholar, Department of Mechanical Engineering, Jadavpur University, Kolkata, West Bengal, India
[1]Sr. Lecturer, Department of Mechanical Engineering, Asansol Polytechnic, Asansol, West Bengal, India
[2]Assistant Professor, Department of Mechanical Engineering, Jadavpur University, Kolkata, West Bengal, India

Received Date: November 18, 2024
Accepted Date: February 10, 2025
Published Date: May 01, 2025

Citation: Joydeep De, Priyankar Datta. Free Vibration of Curved Sandwich Beams with Laminated Composite Facings Using Finite Element Method. Journal of Polymer & Composites. 2025; 13(Special Issue 3): S549–S561p.

displacements are considered constant for the faces and quadratic for the core and results are presented for different types of laminated composite sandwich beam. Baba BO [6] analysed the free vibration of curved sandwich beam with debonded core and face both experimentally and numerically. Their investigation shows that debond, which is a common phenomenon of curved sandwich beam, drastically reduce the natural frequency of the beam as a result of reduced stiffness. Effect of curvature is also reported. Sadeghpour et al. [7] also investigated natural frequencies of debonded curved sandwich beam implementing nonlinear contact condition of the debonded region. Here the displacement along the thickness is assumed as quadratic polynomial taking into account the radial and the circumferential rigidity of the core. Results reveal that curvature and boundary conditions affects the natural frequency. For circular curved beam with free-free boundary condition the natural frequency slowly decreases with increase in the curvature angle and while in fixed-fixed boundary condition, the natural frequency of symmetric modes increases with curvature angle. It is also concluded that effect of debond on natural frequency does not vary much between curved and flat beams.

Loja et al. [8] incorporated different shear deformation theories to formulate layerwise models to overcome the incapability of a single deformation theory applied to the entire thickness of the sandwich beam. The core is considered visco-elastic in nature and modelled with complex modulus approach. All the layerwise models are combined using kriging based finite element approach considering quadratic and cubic polynomials as displacement fields. Mishra et al. [9] investigated the free vibration of sandwich plate with cutout. As coutouts facilitate different required services including fuel and electric supply, accessibility to other parts etc., it becomes unavoidable in structures and results in changes in the dynamic behaviour. ANSYS 15.0 software is used to develop the finite element model for the same considering shell element and Block Lanczose method towards numerical prediction of mode shapes and natural frequencies. Results are reported for different parametric variations of sandwich plate with cutout. As the cotout size increases the natural frequency increases for the plate for both clamped and simply supported boundary conditions. However the change of the natural frequency is more prominent in clamped boundary condition.

Khdeir and Aldraihem [10] analysed free vibration of sandwich beam using zigzag beam theory and analytical solutions for mode shapes and natural frequencies for sandwich beams with soft core are developed based on state space approach. Results are compared with available literatures and it is concluded that zigzag beam theory is capable to predict the dynamic behaviour of sandwich structure. Pingulkar and Suresha [11] investigated free vibration of laminated composite plates using finite element software ANSYS. A number of commercially available composite plates are considered with cantilever boundary conditions and different parametric studies are carried out after validation of the model with experimental and numerical results of other available literatures. They conclude that hybridization and orientation of the outermost layer has major contributions on the natural frequencies of the laminated composite plates compared to other parameters like fiber volume fraction and matrix material. Yang et al. [12] reviews the state of the art free-in-plane vibration of curved beams. Different governing differential equations of motions of the curved beam based on different hypothesis are illustrated and compared. Analytical and numerical techniques are also discussed for solutions of the governing equations. It is observed that the effects of axial extensibility of curved beams are quite significant to their dynamic properties.

Sayyad AS and Ghugal [13] presented the static behaviour of functionally graded sandwich curved beams using the sinusoidal beam theory where isotropic core and functionally graded skin are considered for the analysis. Governing equilibrium equations are developed based on Hamilton's principle and analytical solution is carried out using Navier's technique. Numerical results are obtained for static analysis with different curvatures.

Wu et al. [14] presented the experimental procedures and analysis on free vibration of sandwich plates based on sinusoidal global-local theory. Experiments are carried out for sandwich plates having Aluminium face and PMI core and the natural frequencies are compared with numerical results using the developed numerical model. The proposed model is developed with the continuity conditions of the transverse shear stress at the interfaces of different layers and is able to predict the natural frequencies of composite sandwich plate quite accurately. It is also concluded that boundary conditions and length to thickness ratio influence the natural frequency. Arefi and Najafitabar [15] studied the buckling and free vibration of sandwich beams composed of soft core and functionally graded graphene nanoplatelets reinforced composite face sheets. Kinematic relations are assumed based on the extended higher order sandwich beam theory while the governing equations are derived using the Ritz-Lagrange formulations. Natural frequencies and buckling loads for the beams with assumed theory are presented for different boundary conditions and other parametric variations.

From the ongoing review of the available literature, it is evident that analyses on the free vibration characteristics of curved sandwich beams with composite face are limited and more investigations are required for different configurations. In the present work, the authors aim is to investigate the effect of different ply orientations of the composite face sheets for moderately thick and thin singly curved beams supported with different boundary conditions. Effect of core to face thickness ratio is also reported for all the cases. Several shear deformation theories [16-18] have been developed and refined over the past years for static and dynamic analyses of composite and sandwich structures. However, a customized finite element model needs to be prepared based on the used shear deformation theory. On the contrary, commercial finite element packages can be directly used based on the inbuilt modules for modelling and analysing structures for their static and dynamic behaviour. While choosing the face composite sheets this study has generalised using specific values of direction dependent material properties without specifying any fiber or matrix material. Both synthetic and natural fiber with known material properties can be used to manufacture the face sheets with customised stiffness as per requirement of the application. Though synthetic fibers are most common in curved or straight sandwich structure, recent studies have shown different benefits of using natural fibers in bio-composite laminates which are one of the most environment friendly and financially viable options as structural materials. Ramasubbu et al. [19] investigated the mechanical properties of bio-composite reinforced with natural Areca catechu fiber (ACF) with silicon carbide filler in epoxy matrix. Palaniappan et al. [20] introduced the use of novel aerial root of Ficus retusa L as an alternative to synthetic fibers as reinforcing elements in polymer matrix. Studies were carried out to find physical, thermo-chemical, mechanical and morphological properties of this bio-composite. Ramakrishnan et al. [21] focussed on finding mechanical, morphological and wear resistance properties of hybrid composite using different weight percentage mixture of both natural and glass fiber reinforcement in polymer matrix. Karthik et al. [22] reviewed the surface modification techniques of plant fibers for enhancing the properties of bio-composites. Here, the ABAQUS finite element software is used to develop the model for numerical study of mode shapes and corresponding natural frequencies. At first the modelling procedure is validated with the available results of Wu et al. [14] for straight sandwich beam with isotropic aluminium face and soft PMI core. After satisfactory validation the same modelling procedure is applied for singly curved beams both with isotropic and composite face sheets. Several parametric studies are carried out and the results are presented in tabular and graphical forms.

FINITE ELEMENT MODEL

A schematic diagram of the present problem is described in Figure 1. The singly curved beam is composed of soft isotropic core and stiffer face sheets. The curvature is considered as circular with the curvature angle 90^0. The thickness of each face sheet is denoted by t_f and that of the core is denoted by t_c such that the total thickness of the beam is $t = t_c + 2t_f$. The radius (R) of the mid plane of the curved beam is varied to maintain the aspect ratio of the curved beam during respective analyses. Width of the beam is denoted as w and considered uniform. Cylindrical co-ordinate (r, θ, z) system is considered with origin at the left bottom most point of the mid-plane. Layers of the sandwich curved beam are stacked along the radial thickness direction r. Curvature is in the r-θ plane and width of the beam is in z direction. For composite face sheets, the fiber orientations of each ply are considered as 'θ^k' in the z-θ plane with respect to curvature direction.

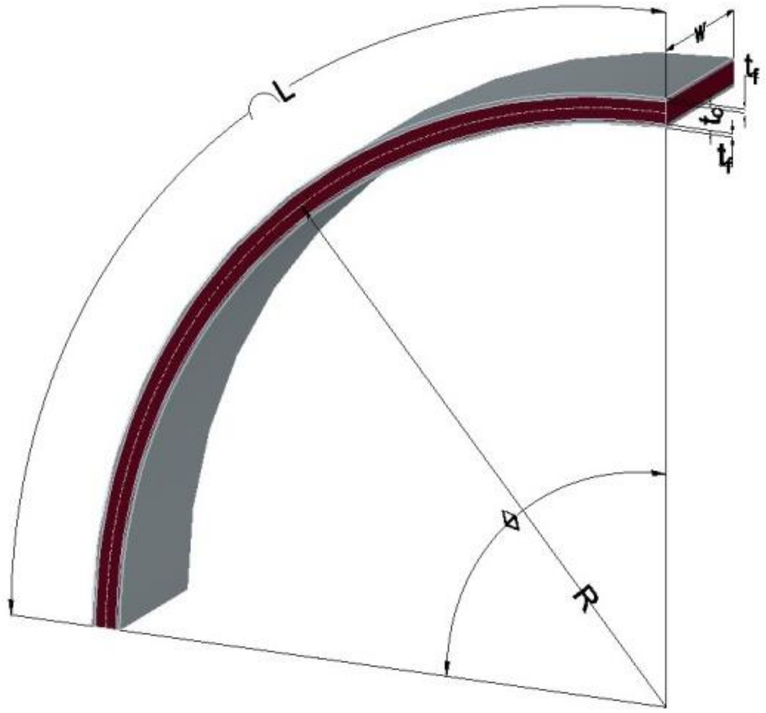


Figure 1. Schematic diagram of singly curved sandwich beam.

Finite element analysers are very powerful tools towards solutions of complex and variety of engineering problems. Applicability of these tools are widely accepted due to ease of model design and customised integration scheme for solving highly nonlinear systems with various load and boundary conditions. The commercially available softwares in this field are user-friendly, well-developed and continuously updated. ABAQUS is one of the most powerful and popular multi-purpose Finite-Element analysers that employs both implicit and explicit integration scheme. In the present work, the Finite Element analyses of the curved sandwich beams are carried out in ABAQUS 6.14 using standard implicit model.

Modelling Procedure of 3D Solid Finite Element

A 3D deformable model of curved sandwich beam is constructed for the present analysis. The curvature is maintained circular with varying the mid-plane lengths while 90^0 curvature angle is maintained as shown in Figure 1. Partitions in the thickness direction parallel to curved face are created to implement different properties of the layers. The thicknesses of the partitions are considered based on the top and bottom face sheet thickness t_f and core layer thickness t_c. For isotropic core and face sheets, solid homogeneous sections are created and assigned to different layers. In case of composite face sheets solid composite layup module is used with user defined cylindrical coordinate system (r, θ, z) as datum with curvature in r-θ plane and width of the beam is in z direction. As the ratio of elastic modulus between the stiffer face sheets and soft core is very high, the transverse shear effects are predominant and solid composite sections are used to analyse the dynamic behaviours of the beams most accurately.

C3D20R (3 dimensional 20 node quadratic brick element, reduced integration) with 6 degrees of freedom at each node is used to create the mesh. Number of elements in the thickness direction of the soft core is considered as 6 and for the top and bottom face sheets are considered in different manner for isotropic and composite materials as described below. For isotropic face sheet a single element in thickness direction is taken as it is enough to converge the numerical results and creating more elements unnecessarily increase the computational effort. For composite face sheet, the number of elements in the thickness direction is considered same as the number of ply on each face. Lanczos eigensolver is used in linear perturbation to find the first 5 bending modes of the curved sandwich beam. Figure 2 depicts the meshed structure of the curved sandwich beam.

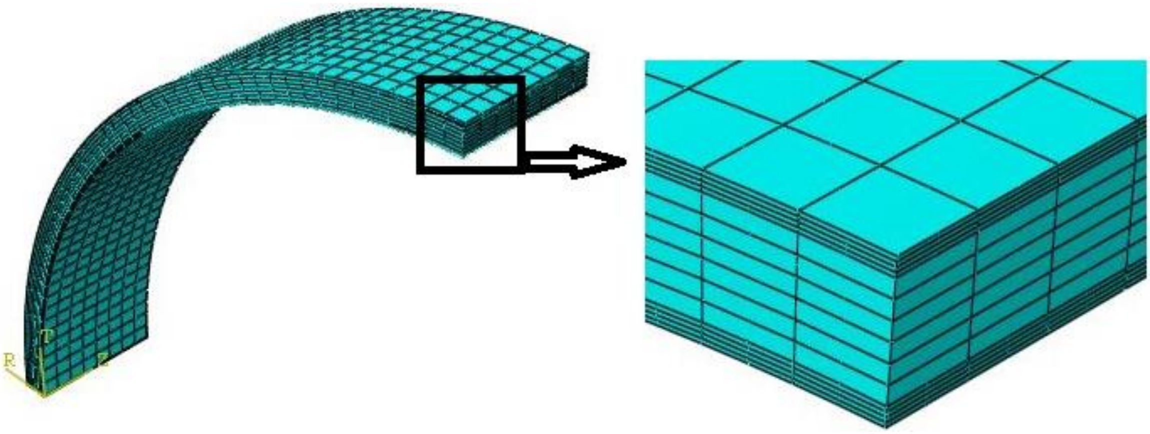


Figure 2. Finite element meshing of curved sandwich beam in ABAQUS.

Constitutive Relations for Fiber Reinforced Composites

As the sandwich curved beam under analysis is composed of composite face sheet in which the substrate has several ply orientations, the basic constitutive relation for unidirectional fiber reinforced composites lamina is illustrated. The constitutive relations for the k^{th} orthotropic lamina of the substrate in which its principal material axes are aligned with the laminate coordinate axes (r, θ, z) are given by [16]

$$\{\sigma^k\} = [C_{ij}^k]\{\varepsilon^k\}, \text{ i.e.}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \tag{1}$$

The elastic constants C_{ij} with respect to the principal material coordinate axes of the layer are given by

$$C_{11} = \frac{E_1(1-\nu_{23}\nu_{32})}{\Delta}, C_{12} = \frac{E_1(\nu_{21}+\nu_{31}\nu_{23})}{\Delta} = \frac{E_2(\nu_{12}+\nu_{32}\nu_{13})}{\Delta};$$
$$C_{13} = \frac{E_1(\nu_{31}+\nu_{21}\nu_{32})}{\Delta} = \frac{E_3(\nu_{13}+\nu_{12}\nu_{23})}{\Delta};$$
$$C_{23} = \frac{E_2(1-\nu_{13}\nu_{31})}{\Delta}, C_{23} = \frac{E_2(\nu_{32}+\nu_{12}\nu_{31})}{\Delta} = \frac{E_3(\nu_{23}+\nu_{13}\nu_{21})}{\Delta}; C_{33} = \frac{E_3(1-\nu_{12}\nu_{21})}{\Delta};$$
$$C_{66} = G_{12}; C_{55} = G_{23}; C_{44} = G_{13}; and$$
$$\Delta = 1 - \nu_{12}\nu_{21} - \nu_{13}\nu_{31} - \nu_{23}\nu_{32} - 2\nu_{32}\nu_{21}\nu_{13}. \tag{2}$$

In Eq. (2), for any layer, E_1, E_2 and E_3 represent the axial Young's moduli along the principal material coordinate axes, $\nu_{12}, \nu_{13}, \nu_{23}$ are the major Poisson's ratios and $\nu_{21}, \nu_{31}, \nu_{32}$ are the minor Poisson's ratios, respectively. G_{12} is the in-plane shear modulus, G_{13} and G_{23} are transverse shear moduli. As the laminated composite plate is made of several orthotropic layers with their principal material axes arbitrarily oriented with respect to the laminate coordinate system, the constitutive equations of each layer needs to be transformed to the laminate coordinate system (r, θ, z). Thus the stress–strain relations for the k^{th} off-axis lamina with respect to the laminate coordinate system are given by

$$\{\sigma^k\}=[\bar{C}^k]\{\varepsilon^k\} \tag{3}$$

Where

$$[\bar{C}^k]=[T]^T[C^k][T] \tag{4}$$

In Eq. (4) $[T]$ is the transformation matrix with the following form

$$[T]=\begin{bmatrix} m^2 & l^2 & 0 & 0 & 0 & -2lm \\ l^2 & m^2 & 0 & 0 & 0 & 2lm \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & l & 0 \\ 0 & 0 & 0 & -l & m & 0 \\ lm & -lm & 0 & 0 & 0 & l^2-m^2 \end{bmatrix} \tag{5}$$

Where $l=\sin\theta^k$ and $m=\cos\theta^k$; θ^k is the global orientation of k^{th} layer.

Constitutive Relation for Soft Isotropic Core and Isotropic Face Sheet:

The soft core and aluminium face sheet under this analysis are isotropic in nature having infinite number of planes of material symmetry. The number of independent elastic constant here reduces to only 2 and can be represented as:

$$E_1=E_2=E_3=E,\ G_{12}=G_{13}=G_{23}=G,\ \nu_{12}=\nu_{23}=\nu_{13}=\nu \tag{6}$$

And the constitutive relation can be represented as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}=\Lambda\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) \end{bmatrix}\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \tag{7}$$

Where $\Lambda=\dfrac{E}{(1+\nu)(1-2\nu)}$;

RESULT AND DISCUSSION

Validation of the Finite Element Model

It is mentioned in the previous section that the elastic modulus of the top and bottom composite sheets are much higher compared to that of the isotropic core. Consequently, there is a sharp change in transverse shear deformation in these layers. To accurately represent the effect of layer wise transverse shear stress the beam is modelled using partitions for each layer. To verify the accuracy of the proposed 3D modelling a flat sandwich beam is modelled in ABAQUS using 3D solid element and the obtained natural frequencies are compared with that presented by Wu et al., 2021 [14]. Two flat sandwich beams are considered having top and bottom aluminium face sheets (each are 1 mm thick) and soft PMI core (6 mm thick). Lengths of the beams are taken as 184 mm and 284 mm and categorised as Group I and Group II respectively. Width for both the sandwich beams are considered as 40 mm. Geometry of the flat sandwich beam is shown in Figure 3. The material properties are given in Table 1 [14]. Based on the present model the beams are analysed and first five natural frequencies are compared in Table 2. It is observed that the results obtained by the present model excellently matches with those reported by Wu et al., 2021 [14]. Similar procedure is followed for modelling the curved sandwich beam and the corresponding results are presented in following section.

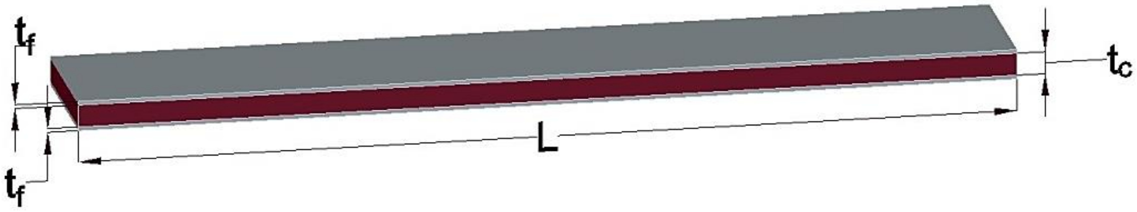


Figure 3. Schematic diagram of flat sandwich beam.

Table 1. Material properties for sandwich beam [14].

	E **(MPa)**	***v***	***ρ*** **(kg/m^3)**
Aluminium (Face Sheet)	71 x 10^3	0.3	2700
PMI Foam (Core)	92	0.375	75

Table 2. Comparison of natural frequencies (Hz) of flat sandwich beams.

Configuration	**Mode**	**3D FEM(Wu et al., 2021)**	**Present FEM ABAQUS**
Group I	1	604.2	608.5
	2	1236.5	1249.7
	3	1954.9	1979.2
	4	2727.4	2768.1
	5	3578.8	3640.4
Group II	1	356.8	358.3
	2	733.9	738.3
	3	1163.7	1172.0
	4	1608.6	1622.3
	5	2076.1	2096.6

Numerical Results

In this section numerical results obtained for singly curved sandwich beams are presented. For this purpose quarter circular beams are considered with uniform cross section with thickness of t = 8 mm and width of w = 40 mm unless specified otherwise. Also the thickness of the soft core and each faces are taken as t_c = 6 mm and t_f = 1 mm, respectively. First the effect of aspect ratio is investigated (without changing the curvature angle). The obtained natural frequencies are reported in Table 3 for various singly curved sandwich beams considering both Fixed-Fixed and Fixed-Free boundary conditions.

Only the bending modes are presented here as it is represented as a beam. The face and core materials are chosen as isotropic aluminium and PMI foam respectively and the properties are mentioned in Table 1. The obtained results show that the natural frequency for first five modes reduces rapidly from thick to thin beams. This is due to the reduction in stiffness due to the overall increase in length, while the cross section area remains unaltered. This trend is similar for both flat and curved beams.

Next, curved and sandwich beams with composite face and soft core are analysed. Similar thicknesses are considered as before, i.e. t_f = 1 mm and t_c = 6 mm. The material properties of the composite face is given by [23],

$$E_1 = 181 \text{ GPa}, E_2 = E_3 = 10.3 \text{ GPa}, G_{12} = G_{13} = 7.17 \text{ GPa}, G_{23} = 2.87 \text{ GPa}, \nu_{12} = \nu_{13} = 0.25, \nu_{23} = 0.33 \text{ and } \rho = 1578 \text{ kg/m}^3 \quad (8)$$

whereas the core material is PMI Foam with properties given in Table 1. These material properties will be considered for all subsequent cases, unless mentioned otherwise. Here also the results are compared for both Fixed-Fixed and Fixed-Free boundary conditions. For both top and bottom face sheets ($0^0/90^0/0^0$) symmetric cross ply and ($0^0/90^0/0^0/90^0$) anti-symmetric cross ply is considered. Similarly, ($+45^0/-45^0/+45^0$) symmetric angle ply and ($+45^0/-45^0$) anti-symmetric angle ply is also taken to study the effect of composite layup on the natural frequency of curved sandwich beams.

Table 3. Natural frequencies (Hz) of curved sandwich beams with isotropic face and soft core.

Boundary Condition	Mode	Aspect Ratio (*L*/*t*)				
		20	30	40	50	100
Fixed-Fixed	1	1309.5	800.7	565.3	428.9	164.2
	2	2105.8	1279.5	903.9	687.6	269.9
	3	3208.8	1918.2	1357.5	1041.0	427.6
	4	4194.2	2466.4	1737.0	1332.6	557.1
	5	5537.9	3164.6	2204.6	1686.0	713.7
Cantilever (Fixed-Free)	1	269.0	142.9	87.3	58.4	15.5
	2	694.7	408.8	276.6	201.2	65.6
	3	1610.9	977.8	679.3	506.0	181.2
	4	2498.3	1526.1	1080.4	822.1	319.5
	5	3487.1	2100.9	1489.2	1141.4	466.1

Table 4. Natural frequencies (Hz) of curved sandwich beams with composite face sheets and soft core for L/t = 30.

Boundary Condition	Mode	$0^0/90^0/0^0/90^0$ /Core/ $0^0/90^0/0^0/90^0$	$+45^0/-45^0/+45^0$ /Core/ $+45^0/-45^0/+45^0$	$45^0/-45^0$ /Core/ $45^0/-45^0$	$0^0/90^0/0^0$ /Core/ $0^0/90^0/0^0$
Fixed-Fixed	1	947.1	943.7	942.9	947.1
	2	1513.8	1508.7	1507.7	1513.9
	3	2289.2	2283.8	2282.3	2289.2
	4	2903.7	2893.8	2892.4	2903.8
	5	3728.9	3720.9	3718.9	3728.9
Cantilever (Fixed-Free)	1	159.9	157.87	157.81	159.89
	2	466.8	463.48	463.18	466.67
	3	1118.1	1110.8	1110.2	1117.7
	4	1806.5	1798.1	1797.3	1805.8
	5	2505.9	2496.6	2495.8	2505.0

Analyses are performed for moderately thick (Aspect Ratio 30) and thin (Aspect Ratio 100) curved beams. The natural frequencies are reported in Table 4 and Table 5 considering only the first five bending modes. Mode shapes as obtained from ABAQUS are also presented in Figure 4 for both the boundary conditions. It may be noted that the use of cross ply or angle ply marginally affects the change

ISSN: 2321-2810 (Online), ISSN: 2321-8525 (Print)

in natural frequency. Thus the orientation or layup of composite face sheets does not have major influence on their natural frequency, rather it mainly depends on the orientation of the outermost layer. The same phenomenon is observed by [11] for flat sandwich beams.

However, for the present analyses the ratio of the longitudinal modulus (E_1) to the transverse modulus (E_2) of the composite material is around 18. This value is quite low resulting insignificant changes in the natural frequency of the overall curved beam, especially for the first few modes.

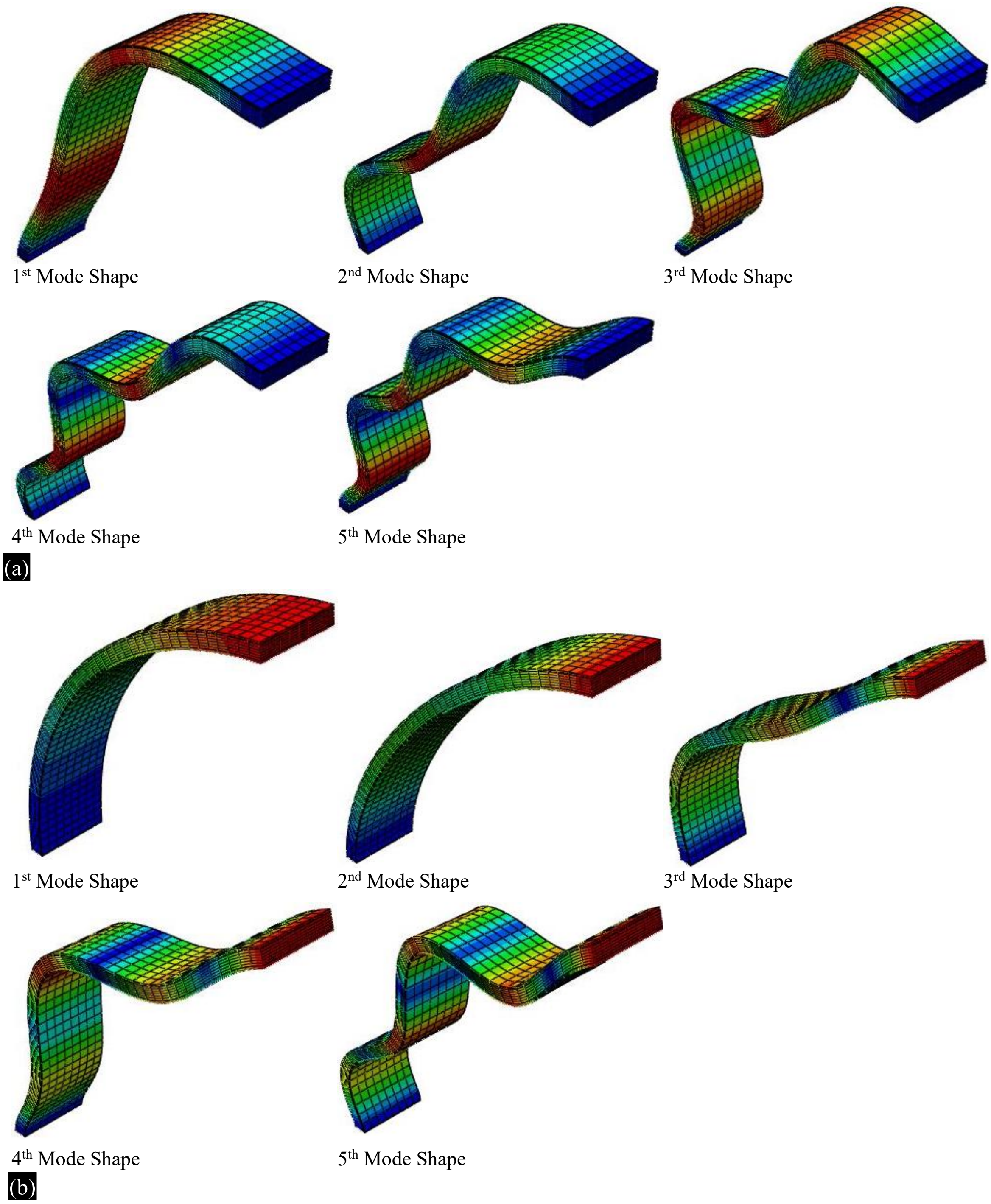


Figure 4. Mode shapes of curved sandwich beam with composite face (ABAQUS), (a) Fixed-fixed beams, (b) Fixed-free beams.

Further analyses have been carried out to investigate the effect of core to face thickness ratio (t_c / $2t_f$) to the natural frequencies of the curved sandwich beam with composite face. As the natural frequency does not vary much with lamina sequence, here results are presented for ($0^0/90^0/0^0/90^0$) anti-symmetric cross ply and ($+45^0/-45^0/+45^0$) anti-symmetric angle ply face sheets for both Fixed-Fixed and Fixed-Free boundary conditions.

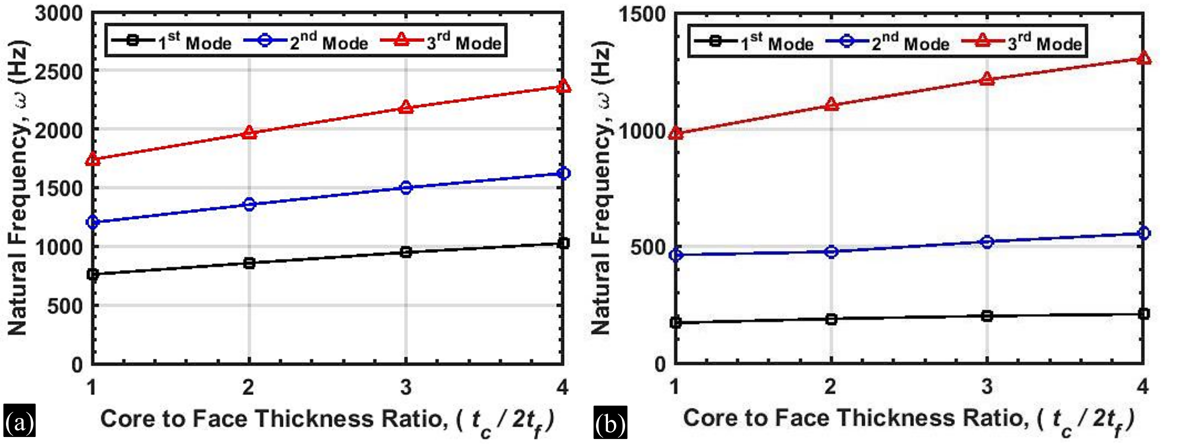
Various curved beams with different core to face thickness ratios are considered keeping the overall thickness of the beams same, i.e. $t = 8$ mm. Comparison of natural frequencies for the first three bending modes are plotted in Figures 5 and 6 for moderately thick (Aspect Ratio 30) and thin (Aspect Ratio 100) beams, respectively, for both Fixed-Fixed and Fixed-Free boundary conditions.

The results show that as the core to face thickness ratio increases, the natural frequency gradually increases. Plot of third mode is steeper than first two modes in almost all configurations, so it can be concluded that the rate of increase in natural frequency with respect to core-face thickness ratio is higher from third mode onwards.

The percentage increase of natural frequency is presented in Tables 6 and 7. Both the moderately thick and thin configurations show similar amount of increase in natural frequency. First 3 bending modes of vibration are considered and reported here for Fixed-Fixed and Fixed-Free boundary conditions.

Table 5. Natural frequencies (Hz) of curved sandwich beams with composite face and soft core for L/t = 100.

Boundary Condition	Mode	$0^0/90^0/0^0/90^0$ /Core/ $0^0/90^0/0^0/90^0$	$+45^0/-45^0/+45^0$ /Core/ $+45^0/-45^0/+45^0$	$45^0/-45^0$ /Core/ $45^0/-45^0$	$0^0/90^0/0^0$ /Core/ $0^0/90^0/0^0$
Fixed-Fixed	1	177.4	175.1	175.1	177.4
	2	307.2	304.3	304.3	307.2
	3	501.8	497.9	497.9	501.8
	4	666.1	661.6	661.6	666.0
	5	872.9	868.1	868.2	872.8
Cantilever (Fixed-Free)	1	20.1	19.9	19.9	20.1
	2	69.4	68.5	68.5	69.4
	3	189.2	186.7	186.6	189.2
	4	348.7	344.7	344.7	348.6
	5	528.6	523.5	523.5	528.5



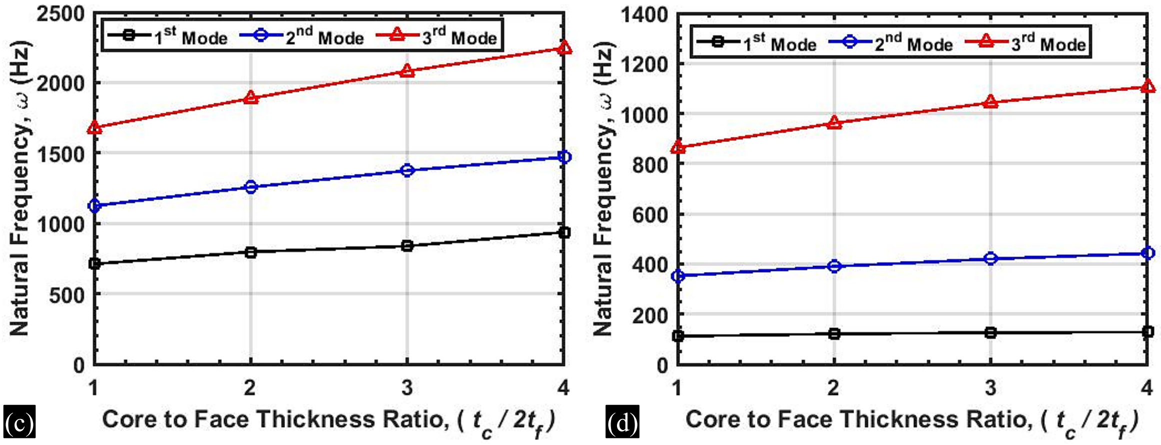


Figure 5. Natural frequency v/s core to face thickness ratio of curved sandwich beams for L/t = 30, (a) ($0^0/90^0/0^0/90^0$/Core/$0^0/90^0/0^0/90^0$) beam with fixed-fixed boundary condition, (b) ($0^0/90^0/0^0/90^0$/Core/$0^0/90^0/0^0/90^0$) beam with fixed-free boundary condition, (c) ($45^0/-45^0/45^0$/Core/$45^0/-45^0/45^0$) beam with fixed-fixed boundary condition, (d) ($45^0/-45^0/45^0$/Core/$45^0/-45^0/45^0$) beam with fixed-free boundary condition.

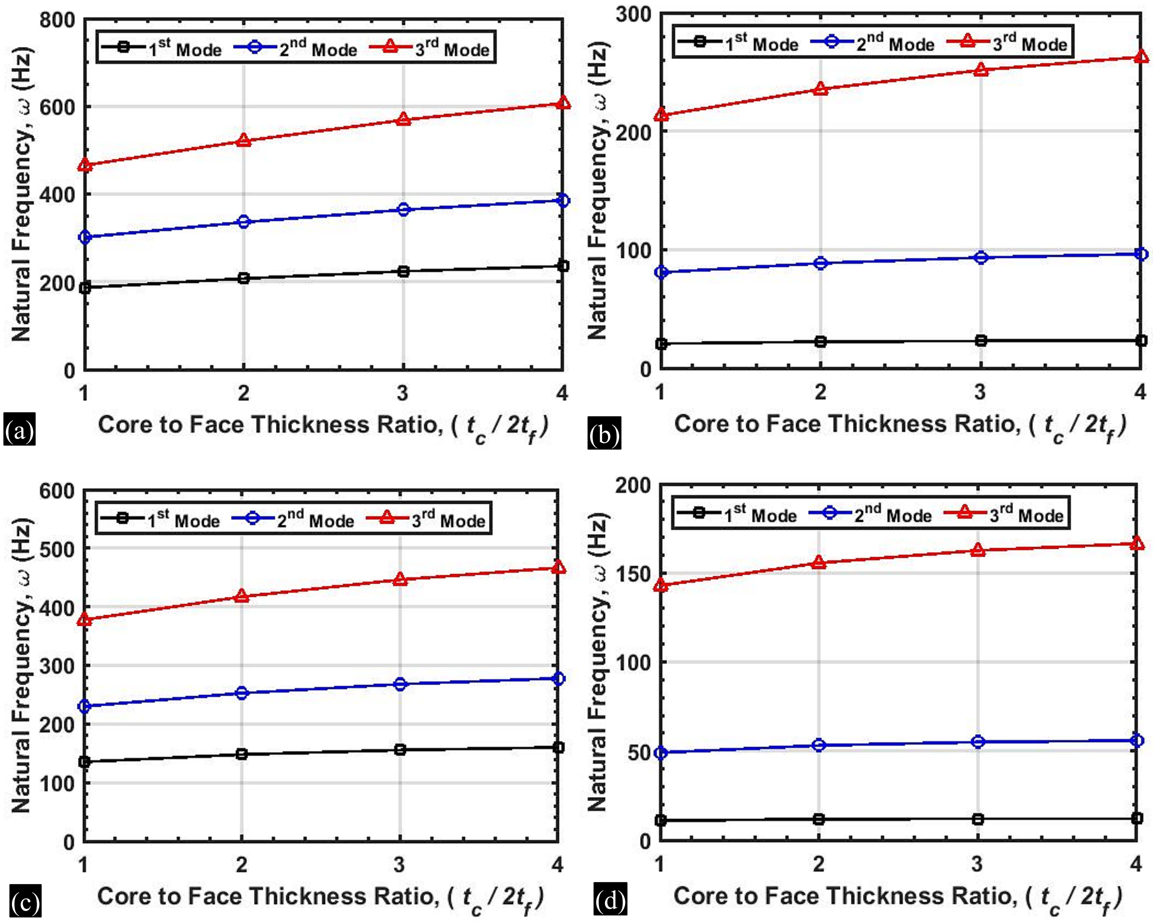


Figure 6. Natural frequency v/s core to face thickness ratio of curved sandwich beams for L/t = 100, (a) ($0^0/90^0/0^0/90^0$/Core/$0^0/90^0/0^0/90^0$) beam with fixed-fixed boundary condition, (b) ($0^0/90^0/0^0/90^0$/Core/$0^0/90^0/0^0/90^0$) beam with fixed-free boundary condition, (c) ($45^0/-45^0/45^0$/Core/$45^0/-45^0/45^0$) Beam with fixed-fixed boundary condition, (d) ($45^0/-45^0/45^0$/Core/$45^0/-45^0/45^0$) beam with fixed-free boundary condition.

Table 6. Percentage (%) change in natural frequencies with core to face thickness ratio for sandwich beam with L/t = 30.

Lamina Orientation of beam	Boundary Condition	Mode	Core to Face Thickness Ratio		
			2	*3*	*4*
($0^0/90^0/0^0/90^0$/core/$0^0/90^0/0^0/90^0$)	Fixed-Fixed	1	12.7	24.7	34.8
		2	12.7	24.7	34.9
		3	12.9	25.3	36.0
	Cantilever (Fixed-Free)	1	10.2	17.2	21.8
		2	3.0	12.4	19.9
		3	12.4	23.7	33.0
($+45^0/-45^0/+45^0$/core/$+45^0/-45^0/+45^0$)	Fixed-Fixed	1	12.1	17.9	31.8
		2	11.8	22.3	30.8
		3	12.4	24.0	33.7
	Cantilever (Fixed-Free)	1	8.2	12.6	14.8
		2	10.8	19.3	25.5
		3	11.3	20.8	28.2

Note: Natural frequency at core to face thickness ratio 1 is taken as reference.

Table 7. Percentage (%) change in natural frequency with core to face thickness ratio for sandwich beam with L/t = 100.

Lamina Orientation of beam	Boundary Condition	Mode	Core to Face Thickness Ratio		
			2	*3*	*4*
($0^0/90^0/0^0/90^0$/core/$0^0/90^0/0^0/90^0$)	Fixed-Fixed	1	11.2	19.9	26.4
		2	11.4	20.7	27.9
		3	11.9	22.2	30.4
	Cantilever (Fixed-Free)	1	8.0	11.3	12.5
		2	9.6	15.6	19.2
		3	10.5	18.0	23.2
($+45^0/-45^0/+45^0$/core/$+45^0/-45^0/+45^0$)	Fixed-Fixed	1	9.3	15.0	18.3
		2	9.8	16.5	20.8
		3	10.5	18.2	23.6
	Cantilever (Fixed-Free)	1	8.0	9.9	11.0
		2	8.3	12.1	13.9
		3	8.9	13.8	16.5

Note: Natural frequency at core to face thickness ratio 1 is taken as reference.

CONCLUSIONS

In this paper free vibration of singly curved sandwich beams are carried out with composite and isotropic face sheets and soft isotropic core. A finite element model is developed in ABAQUS software and natural frequencies are obtained for different beam aspect ratio, face composite layup, and core to face thickness ratio with both cantilever and Fixed-Fixed boundary conditions. Results presented here can be used as a ready reference for any future study related to curved sandwich beam with soft core. Some of the key findings of the present analyses are summarized as follows. As the aspect ratio increases, the natural frequency of the curved sandwich beams reduces rapidly from thick to thin beams for first five bending modes. Natural frequency of curved sandwich beam with composite face sheets does not vary much with respect to different ply layup. It depends mainly on the fiber orientation of the outermost ply. However, here results are presented with $E_1 / E_2 = 18$ for composite faces and effect of higher values of E_1 / E_2 needs to be investigated in future. Natural frequency of singly curved sandwich beam with composite face sheet increases as core to face thickness ratio increases. Change in frequency is gradual irrespective of face composite layup and boundary conditions. Also, the rate of increase in the natural frequency with respect to the core to face thickness ratio is higher from third mode onwards.

REFERENCES

1. Mead DJ. Passive Vibration Control. Chichester, Sussex: John Wiley; 1998.
2. Trindade MA, Benjeddou A. Hybrid active-passive damping treatments using viscoelastic and piezoelectric materials: review and assessment. *JVC*. 2002; 8(6): 699-745.
3. Datta P, Ray MC. Effect of carbon nanotube waviness on smart damping of geometrically nonlinear vibrations of fuzzy-fiber reinforced composite plates. *JIMSS.*2019; 30(7): 977-997.
4. Datta P. (2021). Active vibration control of axially functionally graded cantilever beams by finite element method. *Materials Today: Proceedings*, *44*, 2543-2550.
5. Chalak HD, Chakrabarti A, Iqbal MA, et al. Vibration of laminated sandwich beams having soft core. *JVC.*2021; 18(10): 1422-1435.
6. Baba BO. Free vibration analysis of curved sandwich beams with face/core debond using theory and experiment. *MAMS.*2012; 19(5): 350-359.
7. Sadeghpour E, Sadighi M, Ohadi A. Free vibration analysis of a debonded curved sandwich beam. *EJM-A/Solids.*2016; 57: 71-84.
8. Loja MAR, Barbosa JI, Soares CM. Dynamic behaviour of soft core sandwich beam structures using kriging-based layerwise models. *CS.*2015; 134:883-894.
9. Mishra N, Basa B, Sarangi SK. (2016, September). Free vibration Analysis of Sandwich Plates with cutout. In *IOP Conference Series: Materials Science and Engineering* (Vol. 149, No. 1, p. 012149). IOP Publishing.
10. Khdeir AA, Aldraihem OJ. Free vibration of sandwich beams with soft core. *CS.*2016: 154: 179-189.
11. Pingulkar P, Suresha B. Free vibration analysis of laminated composite plates using finite element method. *PPC.*2016; 24(7): 529-538.
12. Yang F, Sedaghati R, Esmailzadeh E. Free in-plane vibration of curved beam structures: a tutorial and the state of the art. *JVC.*2018; 24(12): 2400-2417.
13. Sayyad AS, Ghugal YM. A sinusoidal beam theory for functionally graded sandwich curved beams. *CS.*2019; 226; 111246.
14. Wu Z, Mei J, Xiao YS, et al. Experiment and analysis on free vibration of sandwich plates based on an alternative sinusoidal global-local theory. *CS.*2021; 257: 113145.
15. Arefi M, Najafitabar F. Buckling and free vibration analyses of a sandwich beam made of a soft core with FG-GNPs reinforced composite face-sheets using Ritz Method. *TWS.*2021; 158: 107200.
16. Reddy JN. Mechanics of laminated composite plates and shells: theory and analysis. CRC press; 2004.
17. Datta P, Ray MC. Finite element analysis of laminated composite plates using zeroth-order shear deformation theory. *IJM&MD.*2016; 12: 387-400.
18. Biswas S, Datta P. (2021, November). Finite element model for free vibration analyses of FG-CNT reinforced composite beams using refined shear deformation theories. In *IOP Conference Series: Materials Science and Engineering* (Vol. 1206, No. 1, p. 012019). IOP Publishing.
19. Ramasubbu R, Kayambu A, Palanisamy S, et al. Mechanical properties of epoxy composites reinforced with Areca catechu fibers containing silicon carbide. *BioResources*.2024; 19(2):2353-2370.
20. Palaniappan M, Palanisamy S, Mani T, et al. Novel Ficus retusa L. aerial root fiber: a sustainable alternative for synthetic fibres in polymer composites reinforcement. *Biomass Conversion and Biorefinery*.2024.
21. Ramakrishnan S K, Palanisamy S, Khan T, et al. Mechanical, morphological and wear resistance of natural fiber / glass fiber-based polymer composites. *BioResources*.2024; 19(2):3271-3289.
22. Karthik A, Bhuvaneshwaran M, Kumar MSS, et al. A Review on Surface Modification of Plant Fibers for Enhancing Properties of Biocomposites. *ChemistrySelect*.2024; 9(21).
23. Kapuria S, Dumir PC, Jain NK (2004). Assessment of zigzag theory for static loading, buckling, free and forced response of composite and sandwich beams. *CS.*2004; 64(3-4): 317-327.

