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Unifrom Beam Dynamics Under Exponentially Moving Load on Bi-paeametric Foundation with Time-dependent Boundary Effects

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Abstract

This study investigates the problem of a uniform elastic beam subjected to exponentially varying moving load with time dependent boundary conditions. A complex structural dynamic problem that has very significant implications for various applications such as railway systems, bridge structures, industrial machinery will be addressed in this research. A damping term is built into the uniform beam model to give a more realistic picture of the structural behaviour, while the material and geometric properties, both of which strongly affect the response amplitude, are held constant throughout the work. The supporting medium is represented as a bi-parametric foundation combining Winkler and Pasternak parameters, which together capture the normal and shear interactions between the beam and its base. The governing equations are solved through a combination of methods: the Mindlin-Goodman technique converts the non-homogeneous boundary conditions into homogeneous ones, integral Fourier series transformation reduces the resulting fourth-order partial differential equation to a second-order ordinary differential equation, and this equation is then solved using Laplace transformation together with the convolution theorem. All results are presented graphically using Maple software. It was revealed from the graphs that, as the values of the structural parameters such as Winkler foundations parameter (K), Pasternak foundation parameter (G), Axial force (N), Damping coefficient (epsilon), and load speed (v) increases, the response amplitudes of the beam decreases. The results also reveals that, the effect of Pasternak foundation parameter is more noticeable compare to that of Winkler foundation parameter. Finally, higher values of damping coefficient and load speed significantly reduces the response amplitudes of the beam there by reduces the risk of resonance effect and the safety of the occupant of the structural member will be guaranteed

Keywords: Time Dependent Boundary Conditions, Bi-Parametric Foundation, Exponentially Moving Load, Damping Coefficient, Uniform Beam, Mindlin-Goodman method.

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INTRODUCTION

In any engineering fields of theoretical and practical significance, the dynamic analysis of elastic structures such as beam resting on elastic foundation is a fundamental problem and this area of research has been enriched in the last few decades by the development of high-speed railway networks. As a result of modern trends toward higher speed in the field of railway engineering so as to accurately predicted the behaviour of the railway track has further intensify the researchers in this areas of research. Most of the works in this area of research has been reported in the literatures. For example, the work of Ogunlusi *et al* [1] who used Galerkin's method, Laplace transformation along side with the theory of convolution to explored dynamic analysis of a non-uniform prismatic damped cantilever beam resting on exponential decaying foundation under constant and harmonic distributed load. Their findings revealed that load speed significantly influence the respond amplitudes of the beam subjected to both constant and harmonic moving load. Also from their findings, as the values of the axial force and damping coefficient increases, the response amplitude decreases. Ogunlusi and Okafor [2] studied the vibration of a non-prismatic damped beam whose thickness varies exponentially while resting on an exponentially decaying foundation under a distributed load. Anantheswar *et al* [3] considered treatment of inelastic material models within a dynamic ALE formulation for structures subjected to moving load. Dynamic behaviour of beam with an elastic foundation under the extended moving load for electromagnetic Launch rail-structure. Was investigated by Zhang *et al* [4]. Okafor *et al* [5] studied how an exponentially decaying foundation influences the dynamic response of a non-uniform damped Rayleigh beam under a harmonic moving load with general boundary conditions. Sunday *et al* [6] formulated a symmetric, compact three-step algorithm for solving the spatio-temporal generalized Fitzhugh-Nagumo equation. Using the finite element method, Arathy and Bennet [7] analysed a beam resting on an elastic foundation and found that raising the soil's safe bearing capacity, the beam's depth and width, and Young's modulus all act to reduce beam deflection. Usman *et al* [8] applied the Laplace differential transform method to examine the free vibration of a non-uniform double Euler-Bernoulli beam on a Winkler foundation. Chen *et al* [9] investigated analytical determination of dynamic response of layered saturated frozen foundation to moving loads under plane strain conditions. Quizizi *et al* [10] examined the nonlinear dynamics of a beam resting on a nonlinear fractional viscoelastic foundation under a moving load of variable speed.

All the aforementioned researchers considered Winkler foundation in their studies. The major short coming of the Winkler foundation model is its failure to capture the complex and continuous nature of the soil whereas in reality, soil is a continuous medium and a load applied at one point causes settlement in the surrounding area. In order to improve upon the Winkler foundation model so as to accounting for shear interaction between spring elements, two parameters foundation model called Pasternak foundation model was proposed by Pasternak [11]. The most significance of this model is that, it account for vertical shearing effects of the soil and also improved the prediction of structural deflection, bending moment and shear force in the foundation. Pasternak foundation model are reported in the literatures like the works of Olotu *et al* [12] who examined naturally vibration analysis of axially graded tapered Rayleigh beam on quadratic bi-parametric elastic foundation. Ermis *et al* [13] used mixed finite element method to investigate free vibration of axially FG curved beam on orthotropic Pasternak foundation. Lucos *et al* [14] examined the vibration of an axially loaded Euler-Bernoulli beam on a two-parameter foundation, combining the finite element method with the Rayleigh-Ritz method and presenting their findings as plotted curves; the results showed the axial load lowering the frequency parameter while the foundation parameter raised it. Awodola *et al* [15] studied the vibration of a non-uniform beam on a Pasternak elastic foundation under moving distributed masses of variable magnitude, obtaining a closed-form solution through Galerkin's method, a series representation of the Heaviside function, and Laplace transformation combined with the convolution theorem; their graphical results indicated that increasing the axial force, foundation modulus, and shear modulus all reduce the beam's response amplitude. Ajijola [16] used finite Fourier series together with Laplace transformation and the convolution theorem to investigate the dynamic response of a prestressed, damped shear beam resting on a bi-parametric elastic foundation. He obtains conditions in which the dynamical system experience resonance phenomenal and the critical speed at it occur. His results show that increases in the values of axial force, circular frequency, foundation modulus and shear modulus reduce the deflection profiles of the beam. He concluded that, increases in the values of those structural parameters will significantly enhances the stability of the beam subjected to moving load and hence reduces the risk of resonance

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effect and the safety of the occupant of the structural member will be guaranteed. Jimoh *et al* [17] combined Fourier series with integral transformation to compare the behaviour of clamped-clamped and clamped-free elastic beams resting on bi-parametric subgrades under a concentrated moving load. Some of the other researchers that considered two parameters foundation in their studies are Jimah *et al* [18], Ahmed *et al* [19], Saurabh [20], Quifeng [21], Giorgie [22] to mention but a few.

It is worthy to note at this point that, the above researchers considered only the general boundary conditions which includes simply supported, clamped-clamped, and clamped-free boundary conditions which are homogeneous in nature but in real situation the boundary conditions are not homogeneous. These boundary conditions are also referred to classical boundary conditions. In real life situation, problem that has to do with vibration deal with continuous system whose one or more boundaries are not stationeries but undergoes sort of displacement which varies with time. These boundary conditions are called non-classical boundary conditions. Example of such boundary conditions are elastically supported and time dependent boundary conditions. Research works where the non-classical boundary conditions are considered are very scanty in the literatures because of the difficulty involved in solving the governing differential equations describing such dynamical system. Some of the few researchers that considered non-classical boundary conditions in their studies are Mindlin and Goodman [23], Adedowole [24], Bayem *et al* [25], Jimoh *et al* [26], Jimoh *et al* [27], Oni and Awodola [28], Ajibola [29]. None of those researchers considered exponential load with non-classical boundary conditions and this remained outstanding in the literatures.

This study, therefore, focuses on a uniform elastic damped beam resting on a bi-parametric foundation and subjected to an exponentially varying moving load under time-dependent boundary conditions.

GOVERNING EQUATION

The dynamic response $\psi(x,t)$ of uniform elastic beam resting on bi-parametric foundations subjected to exponentially varying moving load with time dependent boundary conditions is governed by Frybal [30]

$$EJ\frac{\partial^4\psi(x,t)}{\partial x^4}+\mu\frac{\partial^2\psi(x,t)}{\partial t^2}-N\frac{\partial^2\psi(x,t)}{\partial x^2}+\varepsilon\frac{\partial\psi(x,t)}{\partial t}+K\psi(x,t)-G\frac{\partial^2\psi(x,t)}{\partial x^2}$$

$$=Mg\,e^{-\lambda t}\,\delta(x-vt)\;0<x<L,t>0 \tag{1}$$

Where

EJ= flexural rigidity of the structures, μ = mass per unit length of the beam

K = Winkler foundation parameter, G = Pasternak foundation parameter, N= axial force,

$Epsilon(\varepsilon)$ = damping coefficient, *E*= young's modulus, J = moment of inertia,

ψ *(x,t)*= transverse displacement, *x* = spatial coordinate and *t* = time coordinate

v = Load speed, *M*= mass of the moving load, g=acceleration due to gravity.

λ = rate of decay for the exponential function.

The boundary conditions corresponding to equation (1) are taken to be time dependent given as

$$H_i[\psi(0,t)]=r_i(t), i=1,2 \tag{2}$$

$$H_i[\psi(L,t)]=r_i(t), i=1,2 \tag{3}$$

Where H_i are differential operators of the order less than or equal to three.

For example, if the beam in question has simple support at both ends, then

$$H_1=1, H_2=\frac{\partial^2}{\partial x^2}\,, H_3=1\; H_4=\frac{\partial^2}{\partial x^2} \tag{4}$$

The initial conditions are given as

$$\psi(x,0) = 0 = \frac{\partial\psi(x,0)}{\partial t} \tag{5}$$

METHODS OF SOLUTION

In order to solve the above initial boundary valued problems, we introduce the auxiliary variable $\varphi(x,0)$ in the form

$$\psi(x,0) = \varphi(x,0) + \sum_{i=0}^{4} r_i(t)\, s_i(x) \tag{6}$$

The substitution of (6) into the boundary valued problems (1), (2) and (3) transform the latter to a boundary valued problem in terms of $\varphi(x,0)$. The functions $s_i(x)$ are to be chosen so as to render the boundary conditions for the boundary valued problem in $\varphi(x,0)$ homogeneous.

Substituting (6) into (1) after simplification yields

$$EJ\frac{\partial^4\psi(x,t)}{\partial x^4} + \mu\frac{\partial^2\psi(x,t)}{\partial t^2} - N\frac{\partial^2\psi(x,t)}{\partial x^2} + \varepsilon\frac{\partial\psi(x,t)}{\partial t} + K\psi(x,t) - G\frac{\partial^2\psi(x,t)}{\partial x^2} =$$

$$Mg\,e^{-\lambda t}\delta(x-vt) - \sum_{i=1}^{4}\begin{Bmatrix} r_i(t)EI\frac{\partial^4 s_i(x)}{\partial x^4} + \mu s_i(x)\ddot{r}_i(\mathrm{t}) + \sum s_i(x)\dot{r}_i(t) \\ r_i(t)N\frac{\partial^2 s_i(x)}{\partial x^2} + r_i(t)Ks_i(x) - r_i(t)\frac{\partial^2 g_i(x)}{\partial x^2} \end{Bmatrix} \tag{7}$$

Now, the assumed expression (6) must satisfy the boundary conditions (2) and (3).

Thus,

$$H_i[\varphi(0,t)] + \sum_{j=0}^{4} r_i(t)\, H_i[s_i(0)] = r_i(t),\ i = 1,2 \tag{8}$$

$$H_i[\varphi(L,t)] + \sum_{j=0}^{4} r_i(t)\, H_i[s_i(L)] = r_i(t),\ i = 3,4 \tag{9}$$

Substituting (6) into the initial conditions (5) to obtains

$$\varphi(x,0) = -\sum_{i=0}^{4} r_i(0)\, s_i(x) \tag{10}$$

$$\frac{\partial\varphi(x,0)}{\partial t} = -\sum_{i=0}^{4} r_i(0)\, s_i(x) \tag{11}$$

Using the Mindlin-Goodman method as applied in [26], the boundary conditions (8) and (9) in terms of $\varphi(x,t)$ can be made homogeneous if the function $s_i(x)$ are chosen so that the conditions given by

$$H_i\left[s_j(0)\right] = \delta_{ij}(i = 1,2\ j = 1,2,3,4) \tag{12}$$

$$H_i\left[s_j(L)\right] = \delta_{ij}(i = 1,2\ j = 1,2,3,4) \tag{13}$$

are satisfied.

Where

$$\delta_{ij} = \begin{cases} 0, i \neq j \\ 1, i = j \end{cases} \tag{14}$$

Using equations (12) and (13) in the non-homogeneous boundary conditions (8) and (9), we obtains the homogeneous boundary conditions as

$$H_i[\varphi(0,t)] = 0, i = 1,2 \tag{15}$$

$$H_i[\varphi(L,t)] = 0, i = 1,2 \tag{16}$$

The original problem now reduces to that of solving the non-homogeneous partial differential equation (7) subject to the homogeneous boundary conditions (15) and (16) with initial conditions (10) and (11).

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To solve the initial boundary valued problem (7), (15) and (16) above, one employed the method of generalized finite integral transform defined by

$$W(m,t) = \int_0^L W(x,t)\sin\frac{m\pi x}{L}\,dx, m = 1,2,3 \tag{17a}$$

With the inverse

$$W(x,t) = \frac{2}{L}\sum_{m=1}^{\infty} W(m,t)\sin\frac{m\pi x}{L} \tag{17b}$$

Now, employing (17a) into (7) after some simplification and rearrangement to obtains

$$W_{tt}(m,t) + a_2 W_t(m,t) + A_5\,W(x,t) = \frac{Mg}{\mu}e^{-\lambda t}\delta(x-vt)$$

$$[V_1(t) + V_2(t) + V_3(t) - V_4(t) + V_5(t)] \tag{18}$$

Where

$$a_1 = \frac{EJ}{\mu}, a_2 = \frac{\varepsilon}{\mu}, a_3 = \frac{N+G}{\mu}, a_4 = \frac{K}{\mu}, A_5 = R_1 + R_2 + a_4 \tag{19}$$

$$R_1 = \frac{m^4\pi^4 a_1}{L^4}, R_2 = \frac{m^2\pi^2\,a_3}{L^4} \tag{20}$$

$$V_1(t) = \sum_{i=1}^{4} a_1 r_i(t)\int_0^L \frac{\partial^4 s_i(x)}{\partial x^4}Sin\frac{m\pi x}{L}\,dx \tag{21}$$

$$V_2(t) = \sum_{i=0}^{4} \ddot{r}_i(t)\int_0^L s_j(x)Sin\frac{m\pi x}{L}\,dx \tag{22}$$

$$V_3(t) = \sum_{i=1}^{4} a_2\,\dot{r}_i(t)\int_0^L s_j(x)Sin\frac{m\pi x}{L}\,dx \tag{23}$$

$$V_4(t) = \sum_{i=1}^{4} a_2 r_i(t)\int_0^L \frac{\partial^2 s_i(x)}{\partial x^2}Sin\frac{m\pi x}{L}\,dx \tag{24}$$

$$V_5(t) = \sum_{i=1}^{4} a_4\,r_i(t)\int_0^L s_j(x)Sin\frac{m\pi x}{L}\,dx \tag{25}$$

Evaluating (21-25) and substituting the results into (18), after some simplification and rearrangement to obtains

$$W_{tt}(m,t) + a_2 W_t(m,t) + A_5 W(m,t) = \frac{Mg}{\mu}e^{-\lambda t}Sin\frac{m\pi ct}{L} - (U_1 + U_2 + U_3) \tag{26}$$

Where

$$U_1 = \ddot{r}_i(t)\,(J_1 - J_2) + \ddot{r}_3(t)J_2 \tag{27}$$

$$U_2 = a_2(r_i(t)(J_1 - J_2) + r_3(t)J_2) \tag{28}$$

$$U_3 = a_4(r_i(t)(J_1 - J_2) + r_3(t)J_2) \tag{29}$$

$$J_1 = \frac{L}{m\pi}(1 - cosm\pi) \tag{30}$$

$$J_2 = \frac{L^2}{m\pi}((-1)^{m+1}) \tag{31}$$

Equation (26) is the transformed equation of uniform elastic beam on bi-parametric foundation subjected to exponentially varying moving load with time dependent boundary conditions.

Analysis of the transformed equation

Recall that we can write

$$r_1(t) = Sin\Omega t \text{ and } r_3(t) = e^{-\beta t}Sin\Omega t \tag{32}$$

Where Ω and β are frequency and parameter respectively.

Differentiating (32) with respect to t to obtain

$$\left.\begin{aligned} \dot{r}_1(t) = \Omega Cos\Omega t, \ddot{r}_1(t) = -\Omega^2 Sin\Omega t \\ \dot{r}_3(t) = -\beta e^{-\beta t} Sin\Omega t + \Omega e^{-\beta t} Cos \\ \ddot{r}_3(t) = \beta^2 e^{-\beta t} Sin\Omega t - 2\beta\Omega e^{-\beta t} Cos\Omega t - \Omega^2 e^{-\beta t} Sin\Omega t \end{aligned}\right\} \quad (33)$$

Using equation (33) in equation (26), after simplification and rearrangement to obtain

$$W_{tt}(m,t) + a_2 W_t(m,t) + A_5 W(m,t) = \frac{Mg}{\mu} e^{-\lambda t} Sin\frac{m\pi ct}{L}$$
$$-\begin{Bmatrix} -\Omega^2 Sin\Omega t\,(J_1 - J_2) + J_2\left(\beta^2 e^{-\beta t} Sin\Omega t - 2\beta\Omega e^{-\beta t} Cos\Omega t - \Omega^2 e^{-\beta t} Sin\Omega t\right) \\ +a_2\left[Sin\Omega t(J_1 - J_2) + J_2 e^{-\beta t} Sin\Omega t\right] + a_4\left(Sin\Omega t\,(J_1 - J_2) + J_2 e^{-\beta t} Sin\Omega t\right) \end{Bmatrix} \quad (34)$$

Taking the Laplace transform of (34) with initial conditions (5), after some simplification and rearrangement to obtains

$$\bar{W}(m,s) = \frac{P_f}{(S^2 + a_2 S \mp A_5)}\left(\frac{\Phi}{(s+\lambda)^2 + \Phi^2}\right) - \frac{1}{(S^2 + a_2 S \mp A_5)}\Big[C_{f1}\left(\frac{\Omega}{S^2+\Omega^2}\right) + C_{f2}\left(\frac{1}{S+\beta}\right) - C_{f3}\left(\frac{s+\beta}{(s+\beta)^2+\Omega^2}\right) - C_{f4}\left(\frac{\Omega}{(s+\beta)^2+\Omega^2}\right) + C_{f5}\left(\frac{\Omega}{S^2+\Omega^2}\right) + C_{f6}\left(\frac{\Omega}{(s+\beta)^2+\Omega^2}\right) + C_{f7}\left(\frac{\Omega}{S^2+\Omega^2}\right) + C_{f4}\left(\frac{\Omega}{(s+\beta)^2+\Omega^2}\right)\Big] \quad (35)$$

Where

$$\left.\begin{aligned} P_f = \frac{\mathrm{Mg}}{\mu}, \mathrm{C_{f1}} = \Omega^2(J_1 - J_2), \mathrm{C_{f2}} = J_2\,\beta^2, \mathrm{C_{f3}} = 2\Omega\beta\,J_2 \\ \mathrm{C_{f4}} = \Omega^2 J_2, \mathrm{C_{f5}} = a_2\,(J_1 - J_2), \mathrm{C_{f6}} = a_2\,J_2, \\ \mathrm{C_{f7}} = a_4\,(J_1 - J_2), \mathrm{C_{f8}} = a_4\,J_2, \Phi = \frac{m\pi v}{L} \end{aligned}\right\} \quad (36)$$

By adopting the followings representation

$$\left.\begin{aligned} F_1(S) = \frac{\Phi P_f}{(s+\lambda)^2+\Phi^2}, F_2(S) = \frac{-\Omega C_A}{S^2+\Omega^2}, \\ F_4(S) = \left(\frac{-C_{f2}}{S+\beta}\right), F_3(S) = \frac{-C_B\Omega}{(S+\Omega)^2+\Omega^2} + \frac{B_7(S+\beta)}{(S+\beta)^2+\Omega^2} + \frac{B_8 S}{S^2+\Omega^2} \\ G_1(S) = \frac{1}{s+n_2},, G_2(S) = \frac{1}{s+n_1} \\ n_1 = -\frac{1}{2}\left(a_2 - \sqrt{a_2^2 - 4A_5}\right) n_2 = -\frac{1}{2}\left(a_2 + \sqrt{a_2^2 - 4A_5}\right) \\ Q_r = \frac{1}{n_1 - n_2}, C_A = C_{f1}C_{f5}C_{f7}, C_B = C_{f4}C_{f6}C_{f8} \end{aligned}\right\} \quad (37)$$

The Laplace inversion of (36) can be obtained as

$$W(m,t) = Q_r[(L_1 - L_6) + (L_2 - L_7) + (L_3 - L_8) + (L_4 - L_9) + (L_5 - L_{10})] \quad (38)$$

Where the integrals $(L_1 - L_{10})$ and their results are given as

$$L_1 = P_f e^{-n_2 t} \int_0^t e^{-(\lambda - n_2)u}\, Sin\Phi u\, du$$
$$= \frac{\Phi P_f e^{-n_2 t} - e^{-\left(\lambda + 2n_2\right)t}\left(\left(\lambda + n_2\right) Sin\Phi t + \Phi Cos\Phi t\right)}{(n_2 + \lambda)^2 + \Phi^2} \quad (39)$$

$$L_2 = -C_A e^{-n_2 t} \int_0^t e^{n_2 t} Sin\Omega u du$$

$$= \frac{-C_A n_2 Sin\Omega t - \Omega Cos\Omega t + \Omega e^{-n_2 t}}{n_2^2 + \Omega^2} \tag{40}$$

$$L_3 = -C_B e^{-n_2 t} \int_0^t e^{-(\beta - n_2)u} Sin\Omega u du$$

$$= \frac{-C_B e^{-\beta t} - ((n_2 - \beta) Sin\Omega t + \Omega Cos\Omega t) + \Omega e^{-n_2 t}}{(n_2 - \beta)^2 + \Omega^2} \tag{41}$$

$$L_4 = -C_{f2} e^{-n_2 t} \int_0^t e^{-(\beta - n_2)u} du$$

$$= \frac{-C_{f2} e^{-n_2 t} (1 - e^{-(\beta - n_2)t})}{\beta - n_2} \tag{42}$$

$$L_5 = C_{f3} e^{-n_2 t} \int_0^t e^{-(\beta - n_2)u} Cos\Omega u du$$

$$= \frac{C_{f3} e^{-\beta t} ((n_2 - \beta) Cos\Omega t + \Omega Sin\Omega t) + (\beta - n_2) e^{-n_2 t}}{(\beta - n_2)^2 + \Omega^2} \tag{43}$$

$$L_6 = P_f e^{-n_1 t} \int_0^t e^{-(\lambda - n_1)u} Sin\Phi u du$$

$$= \frac{\Phi P_f e^{-n_1 t} - e^{-\left(\lambda + 2n_1\right)t\left(\left(\lambda + n_1\right) Sin\Phi t + \Phi Cos\Phi t\right)}}{(n_1 + \lambda)^2 + \Phi^2} \tag{44}$$

$$L_7 = -C_A e^{-n_1 t} \int_0^t e^{n_1 t} Sin\Omega u du$$

$$= \frac{-C_A n_1 Sin\Omega t - \Omega Cos\Omega t + \Omega e^{-n_1 t}}{n_1^2 + \Omega^2} \tag{45}$$

$$L_8 = -C_B e^{-n_1 t} \int_0^t e^{-(\beta - n_1)u} Sin\Omega u du$$

$$= \frac{-C_B e^{-\beta t} - ((n_1 - \beta) Sin\Omega t + \Omega Cos\Omega t) + \Omega e^{-n_1 t}}{(n_1 - \beta)^2 + \Omega^2} \tag{46}$$

$$L_9 = -C_{f2} e^{-n_1 t} \int_0^t e^{-(\beta - n_1)u} du$$

$$= \frac{-C_{f2} e^{-n_1 t} (1 - e^{-(\beta - n_1)t})}{\beta - n_1} \tag{47}$$

$$L_{10} = C_{f3} e^{-n_1 t} \int_0^t e^{-(\beta - n_1)u} Cos\Omega u du$$

$$= \frac{C_{f3} e^{-\beta t} ((n_1 - \beta) Cos\Omega t + \Omega Sin\Omega t) + (\beta - n_1) e^{-n_1 t}}{(\beta - n_1)^2 + \Omega^2} \tag{48}$$

Putting (38) into (17b) to obtains

$$W(x,t) = \frac{2}{L} \sum \sum_{m=1}^{\infty} Q_r \begin{bmatrix} (L_1 - L_6) + (L_2 - L_7) + (L_3 - L_8) \\ +(L_4 - L_9) + (L_5 - L_{10}) \end{bmatrix} \sin \frac{m\pi x}{L} \tag{49}$$

Equation (49) describe the response amplitude of uniform beam resting on bi-parametric foundation under exponentially varying moving load with time dependent boundary conditions.

NUMERICAL ANALYSIS AND DISCUSSION OF RESULTS

This work explored dynamic responses of uniform time dependent elastic damped beam resting on bi-parametric foundation and subjected to exponentially varying moving load for various values of Winkler foundation parameter (K), Pasternak foundation parameter (G), axial force (N), damping coefficient (epsilon), and load speed (v) and the results are shown in figures (1-5).

Figures 1 depicts decreases in the response amplitude of the beam subjected to exponential moving load as the values of the axial force increases. As seen from the graph, the beam undergoes downward deflection toward the negative axis up till the middle before its experiencing upward direction towards the positive axis. The deflection profile of the beam subjected to exponential load is shown in figure 2 and it decreases as the values of the Pasternak foundation parameter increases. From the graph, there is slight deflection at the beginning while higher deflection was experience as the values of the parameter increases.

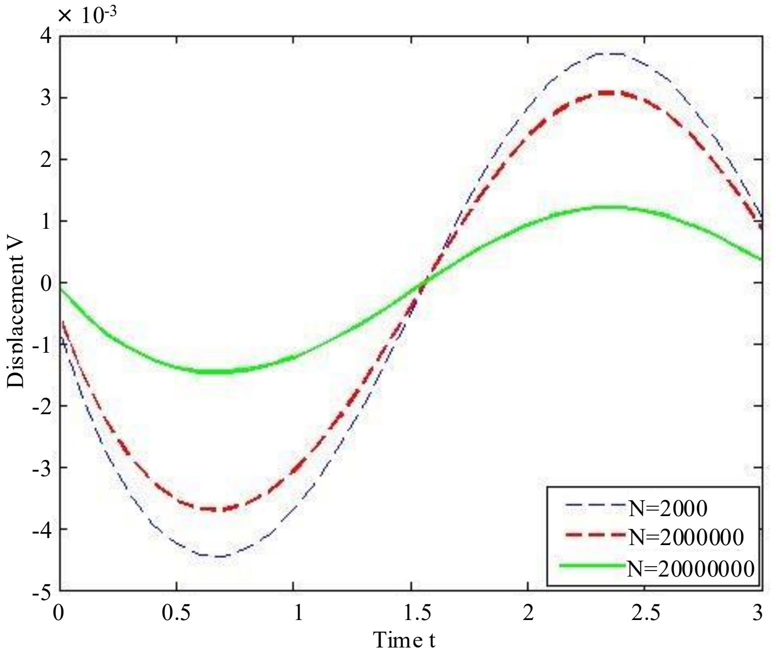

Figure 1. Response amplitude of uniform elastic beam resting on bi-parametric foundations subjected to exponentially moving load with varying axial force (N).

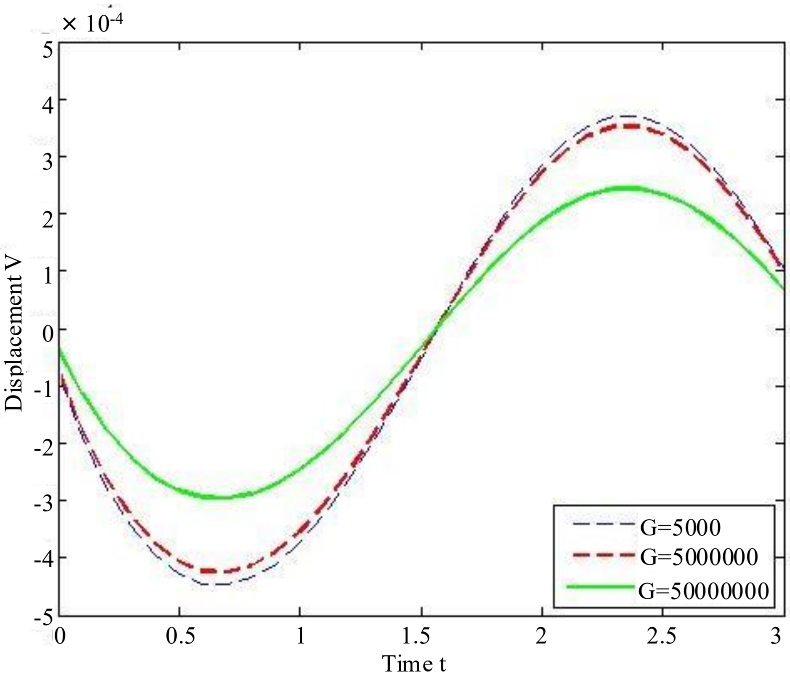

Figure 2. Response amplitude of uniform elastic beam resting on bi-parametric foundations subjected to exponentially varying moving load with varying shear modulus (G).

Figures 3, 4 and 5 shows the response amplitudes of the beam subjected to exponential moving load for various values of the Winkler foundation parameter, damping coefficient, and load speed respectively. From the figures, it is observed that increasing the values of each of the parameters lead to decreases in the response amplitudes of the beam.

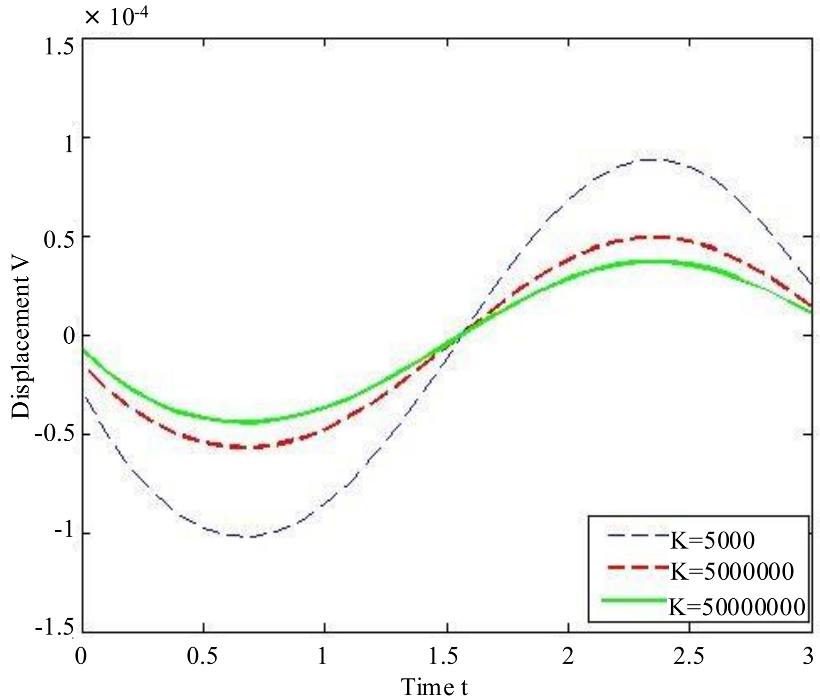

Figure 3. Response amplitude of uniform elastic beam resting on bi-parametric foundations subjected to exponentially varying moving load with varying foundation modulus (K).

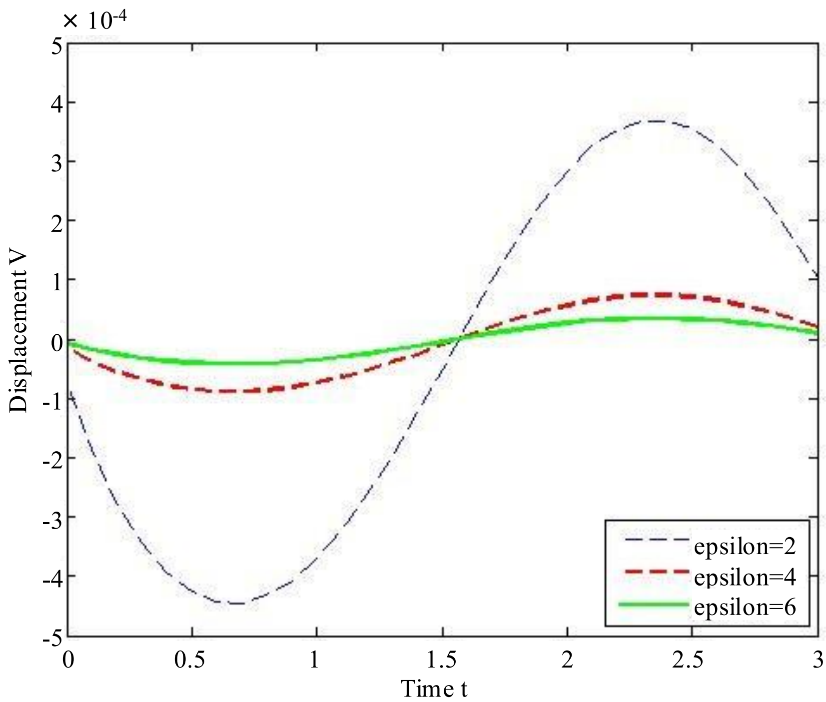

Figure 4. Response amplitude of uniform elastic beam resting on bi-parametric foundations subjected to exponentially varying moving load with varying damping coefficient (epsilon).

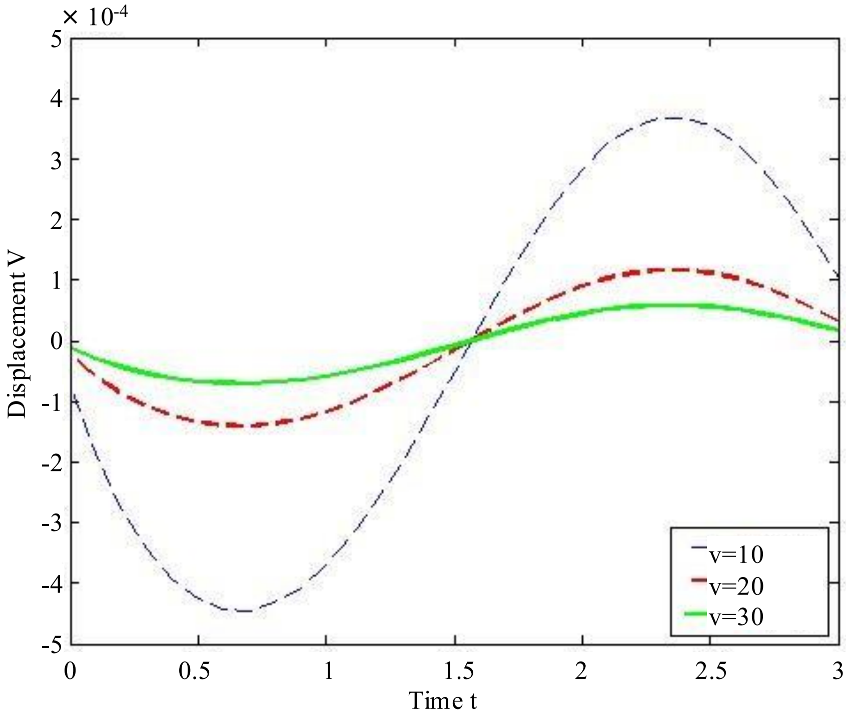

Figure 5. Response amplitude of uniform elastic beam resting on bi-parametric foundations subjected to exponentially varying moving load with varying load speed (v).

CONCLUSION

A theoretical investigation of elastic beam resting on bi-parametric foundation subjected to exponentially varying magnitude moving load has been carried out. The beam's properties such as moment of inertia and mass per unit length were considered to be constant with position x throughout the span L of the beam. The boundary conditions considered in this work is non-classical, time dependent in particular. Because of the difficulty in resolving the present mathematical problems, computation and graphical representation were done with the aid maple software.

The effects of the structural parameters such as Winkler foundation parameter, Pasternak foundation parameter, axial force, damping coefficient, and load speed are displayed in the graphs as shown below. The results revealed that, as the values of those parameters increases, the responds amplitudes of the beam under exponential load with time dependent boundary conditions decreases. The results indicate that the influence of the Pasternak foundation parameter is more pronounced than that of the Winkler foundation parameter. Furthermore, increasing the damping coefficient and load speed significantly reduces the response amplitudes of the beam.

Finally, higher values of damping coefficient and load speed significantly reduces the dynamical deflection and vibration intensity leading to a more stable structure and reduces the risk of resonance which could in effect guarantee the safety of life and properties.

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