

Research to Improve the Performance of Position Control System Using the SPMSM Based on the Time-varying High Order Sliding Mode Control Method

Jun-Bom Kim*

Abstract

This paper proposes the method of position control system design using permanent magnet synchronous motor (PMSM) based on the time-varying high order sliding mode control (TVHOSMC) strategy. In the conventional sliding mode control (SMC) system the chattering phenomenon appears in the vicinity of the sliding plane and while the error state of system reaches the sliding plane, the system is sensitive to the external disturbance and the parameter change. Thus, in this case it is difficult to guarantee the robustness of the system. For this reason, in this pexternalhod of position control system design using the PMSM based on the TVHOSMC strategy is proposed to eliminate the chattering in sliding mode control, perform the fast and accurate position control and guarantee global stability and robustness of the system in the presence of the external disturbances and parameter uncertainties. Simulation and experimental results are then carried out to demonstrate its tracking performance and robustness against torque disturbance and chatter.

Keywords: Time-varying sliding control, high order sliding mode control (HOSMC), PMSM, Lyapunov stability

INTRODUCTION

In recent years PMSM is widely used because it has advantages such as high driving speed, high power density and high efficiency. PMSM is widely used in many applications requiring high control performances such as the robotic field, cosmic field and wind generator system. But the control system using PMSM is the multi-variable nonlinear system and we must consider the problems such as parameter uncertainties and external disturbances during the control process. In the practical application controlling is closely related to nonlinear. Therefore, various nonlinear control methods have been studied and developed, and research to apply them into practice is being done [1-3].

In general, the sliding control method is known as a good one to improve the robustness of the system and reduce the transient time compared to other control methods [5], SMC has been focused on by engineers and researchers for it's simple in structure, good in performance - robustness to external disturbances and low sensitivity to system parameter variations [8, 10].

But the disadvantage of sliding control method is the chattering phenomenon in a vicinity of the sliding plane. When the error state of system reaches the sliding plane, it is difficult to guarantee robustness and stabilization perfectly [13, 16-18]. A lot of research achievements have been made to

*Author for Correspondence

Jun-Bom Kim

E-mail: JB.KIM1024@star-co.net.kp

Faculty of Mathematics, Kim Il Sung University, Pyongyang,
Democratic People's Republic of Korea

Receiving Date: May 08, 2024

Accepted Date: May 16, 2024

Published Date: May 25, 2024

Citation: Jun-Bom Kim. Research to Improve the Performance of Position Control System Using the SPMSM Based on the Time-varying High Order Sliding Mode Control Method. Journal of Control & Instrumentation. 2024; 15(1): 29–44p.

improve the control performances of the position control system because the speed or position control system operates with parameter variation and external disturbances [20-22]. SMC is being studied in many ways. The first thing is to research the sliding plane. Various sliding planes have been investigated since the common linear sliding plane was studied.

R. Ghazali, Y. M. Sam, M. F. Rahmat, A. W. I. M. Hashim and Zulfatman (2010) applied the PID sliding surface to improve the performances of SMC [34]. The focus to be followed in the study of the sliding plane is the time-varying sliding mode control (TVSMC) method. Generally in the SMC there are two stages, that is, one is for the error state of system to reach the sliding plane and the other one is for the state obtained from the above to be kept in the sliding plane [25]. Thus, it is important to reduce or eliminate the stage to let the state of the system reach the sliding plane. As a result, the TVSMC method has been researched and used. The following literatures deal with the TVSMC, and other control methods combined with each other [27-29].

Yi-biao Sun, Lan Yang and Cheng-yuan Wang (2012) proposed to guarantee the global stabilization of the system in the presence of the parameter variation and external disturbances by introducing the TVSMC to the linear servo motor [6]. Ferhun Yorgancioglu and Soydan Redif (2019) applied the terminal decoupling sliding mode control method using the time-varying sliding plane to guarantee the satisfied transient response characteristics of the system and to reduce the steady state error [12]. Grzegorz Tarchala (2017) composed the speed control system of the induction motor (IM) by the SMC using the time-varying switching plane [15]. Lin Zhao and Yingmin Jia (2014) proposed the finite-time attitude tracking control for a rigid spacecraft using time-varying terminal sliding mode techniques [24]. Lin Li, Qi-zhi Zhang and Nurzat Rasol (2011) introduced the time-varying sliding mode adaptive control method to the rotary drilling system [45].

Husnu Bayramoglu and Hasan Komurcugil (2014) proposed the method to combine the TVSMC and the TSMC and introduced it to the fourth order system [46]. In the TVSMC the sliding plane is designed to cross the error state point according to the initial error states of the system and then it moves gradually with a constant velocity to cross the origin of the error state space [31, 33]. That is, the reaching stage to let the state of system reach the sliding plane is removed to meet the needs of the system stabilization by moving the sliding plane with the time.

The next thing to be focused on is to reduce the chattering phenomenon to apply the SMC. Several control strategies are proposed to avoid the chattering phenomenon [35-36]. The literatures to overcome the chattering phenomenon are as follows;

Huangfu Yigeng, Liu Weiguo, S. Laghrouche and A. Miraoui (2009) [47], Dailin ZHANG, Tom C.KONG and Ruxu DU (2012) [9], Faa-Jeng Lin, Ying-Chih Hung and Kai-Chun Ruan (2014) [23], Xiangjie Liu and Yaozhen Han(2014) applied the HOSMC to overcome the chattering phenomenon of the SMC.[48] and P. Li and Z.Q. Zheng(2012) [30], Mohamed Assaad Humida, Alain Glumineau, Jesus de Leon and Luc Loron (2014) proposed new strategies by combining the HOSMC and robust adaptive control method to improve the control performance. [26] In addition, control strategies were created by combining the sliding control method and intelligent control method including fuzzy control and neural network control, which has brought about great successes to improve the control performance. REN Li-tong, XIE Shou-sheng, MIAO Zhuo-guang, TIAN Hu-sen and PENG Jing-bo (2016) [32], Bo Wang, Peng Shi and Hamid Reza Karimi (2014) [4], Bhavani. Rayala, D. Prabhavathi and K. Prakasam(2014) proposed the strategies to combine the fuzzy control and sliding control [7]; Ghania Debbache and Nouredine Golea (2012) proposed the strategies to combine the neural network control and adaptive sliding control and adaptive SMC based on the model reference adaptive control method and recurrent radial basis function network [14], adaptive Wavelet neural network back stepping sliding mode control method were proposed.(I. Vesely, L. Vesely & Z. Bradac, 2018 [19]; Fayez F.M. El-Sousy, 2013 [11]; Xiaoguang Zhang, Lizhi Sun, Ke Zhao & Li Sun, 2013 [37]

In this paper HOSMC method is used to eliminate the chattering phenomenon. In the HOSMC the new sliding plane, exactly high order sliding plane is composed of sliding plane of standard SMC and its derivation [38-43]. At this moment the new sliding plane is related to the first order or high order derivative of manipulated variable and the discontinuous term (sign function) includes the first order or high order derivative of manipulated variable. Comparing the HOSMC with the standard SMC, the HOSMC is regarded as an effective way to eliminate the chattering phenomenon in the vicinity of the sliding plane because the higher order derivative of errors is included in the switching function [44]. In the above consideration, this paper proposes the method to improve the control characters of the PMSM position servo system using the TVHOSMC strategy and verifies the effectiveness of the proposed control strategy through the simulation and the result analysis [49].

This paper gives the following solutions.

1. A new sliding control system design method by combining the TVSMC and HOSMC to eliminate the chattering phenomenon and perform fast and accurate position control, guarantee the global stabilization and robustness of the system in the presence of the parameter variation and external disturbances is proposed.
2. Using the strategy proposed in the paper, the control system is designed in detail, and it is applied to the PMSM position control system and the advantage of proposed method is demonstrated by comparing the control performances with the HOSMC and conventional SMC.

This paper consists of five sections.

1. Section 2 describes the mathematical model of SPMSM.
2. Section 3 describes the design method of the PMSM position control system based on the TVHOSMC.
3. Section 4 represents the stabilization analysis of the designed system.
4. Section 5 describes the determination, the simulation and the analysis of the plant and the parameters of controller using the detail values.

MATHEMATICAL MODEL OF PMSM

Transformation of coordinates to the d-q axes

To establish the mathematical model of the PMSM, it is assumed as follows.

- The magnetic circuit of the motor is not saturated.
- The winding of the stator is three phases symmetrical winding.
- Magnetic hysteresis loss and eddy current loss are neglected.
- The variations of the flux induced in the stator phases by the rotor magnets are sinusoidal, which implies that the induced electromotive forces are sinusoidal.

It is more suitable to analyse the control characteristics of PMSM in the two-phase synchronous rotating coordinate than in the three-standstill coordinate. In this paper, the control law was established based on the d-q axes model.

But the three phase voltages are inputted to the PMSM and three phase currents are output. Thus, in the output three phase – two phase standstill transform is performed, and it is represented as the d-q axes values. The d-q axes are rotating as the synchronous speed (electrical angular velocity) with the rotator. If the angle between the d-axis in this space coordinate and a-axis in the fixed coordinate system, that is, the angular position of rotator in the fixed coordinate system is represented as θ , the coordinate transformation matrix is given as follows.

$$C = \begin{bmatrix} \cos \theta & \cos(\theta - 120^\circ) & \cos(\theta + 120^\circ) \\ -\sin \theta & -\sin(\theta - 120^\circ) & \sin(\theta - 120^\circ) \end{bmatrix} \quad (1)$$

The currents in the d-q axes coordinate are represented as follows.

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos \theta & \cos(\theta - 120^\circ) & \cos(\theta + 120^\circ) \\ -\sin \theta & -\sin(\theta - 120^\circ) & \sin(\theta - 120^\circ) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (2)$$

If two electric current sensors are used, the transformation relation of coordinates is as follows.

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \sqrt{2} \begin{bmatrix} \sin(\theta + 120^\circ) & \sin \theta \\ \cos(\theta + 120^\circ) & \cos \theta \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} \quad (3)$$

In the d-q axes the voltage equations of the PMSM are as follows.

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = \begin{bmatrix} R_s + sL_d & -\omega_e L_q \\ \omega_e L_d & R_s + sL_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega_e \begin{bmatrix} 0 \\ \psi_f \end{bmatrix} \quad (4)$$

where u_d , u_q are stator voltages in the d, q axes, i_d , i_q are stator currents in the d, q axes, ω_e is the electrical angular velocity, ψ_f is flux created by the rotor magnets, R_s is the resistance of stator, L_d , L_q are stator inductances in the d, q axes and s is the differential operator.

The mechanical rotation angle is as follows.

$$\theta = \int \omega_e dt \quad (5)$$

The torque balance equation of PMSM is as follows.

$$T_e - T_L = J \frac{d\omega_m}{dt} + f_v \omega_m \quad (6)$$

Where T_e is the electromagnetic torque, T_L -load torque, f_v -viscous friction coefficient, ω_m -mechanical angular velocity, $\omega_e = p\omega_m$. So it is clear that the d-q axes currents can be controlled by the d-q axes voltages u_d , u_q . If the position of magnetic pole is measured, the d-q axes armature voltages u_d , u_q can be controlled.

Current Feedback Compensation Decoupling Control

From the voltage equation (4), it should be noted that the d-q axes voltages are related each other by the d-q axes inductances, thus there is coupling speed electromotive force. They affect the i_d , i_q , but they cannot control directly. The block diagram of dynamic decoupling of PMSM is showed in Figure 1.

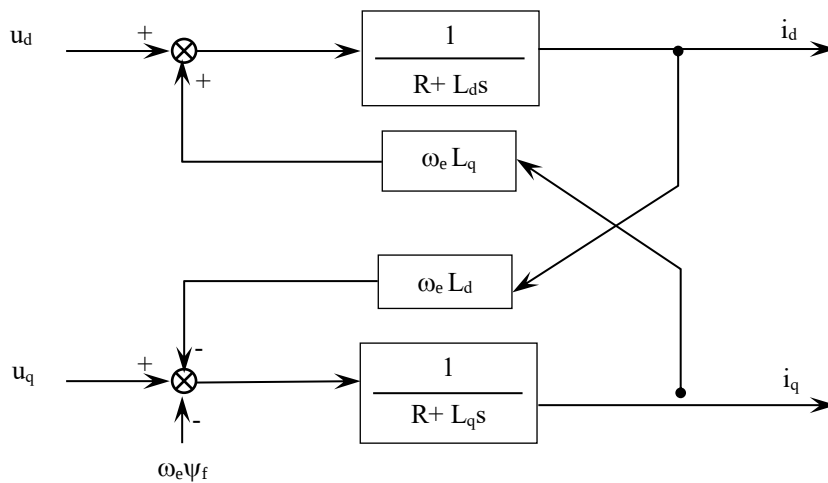


Figure 1. Dynamic decoupling relation block diagram of PMSM.

The voltage coupling term can be offset by the current feedback compensation. The compensation term which is the same in size, but different in signal with the coupling term of d-q axes voltage equation is substituted to the output term of d-q axes current controller as shown in Figure 2.

Using the feedback compensation terms u_{d0}, u_{q0} , the voltage equation (4) is represented as follows;

$$\begin{cases} u_d^* = G_1(s)(i_d^* - i_d) + u_{d0} \\ u_q^* = G_2(s)(i_q^* - i_q) + u_{q0} \end{cases} \quad (7)$$

Where $u_{d0} = -\omega_e \hat{L}_q i_q, u_{q0} = \omega_e \hat{L}_d i_d + \omega_e \psi_f$.

Mathematical model of PMSM

As a result, the control to maintain the d-axis current as zero and the control to change the moment by q-axis current i_q become independent and easy to control by the decoupling control. The electromagnetic torque equation is as follows;

$$T_e = p[\psi_f i_q + (L_d - L_q)i_d i_q] \quad (8)$$

Where p is the number of pole pair.

To perform the vector control of PMSM, $i_d = 0$ is used.

In the surface mounting PMSM (SPMSM), $L_d = L_q$, so the electromagnetic torque equation (8) is transformed as follows;

$$T_e = p\psi_f i_q \quad (9)$$

Flux ψ_f is affected by the temperature and magnetic saturation, but it hardly changes, so the torque can be controlled by the i_q .

If $i_d = 0$, flux ψ_f is determined by the permanent magnet, then there is linear relation between the i_q and T_e , so the PMSM has the control characteristics same as the DC motor and the current of q-axis could be maximum value when the d-axis current is 0, so that maximum-torque-per-ampere (MTPA) strategy can be performed and copper loss can be reduced.

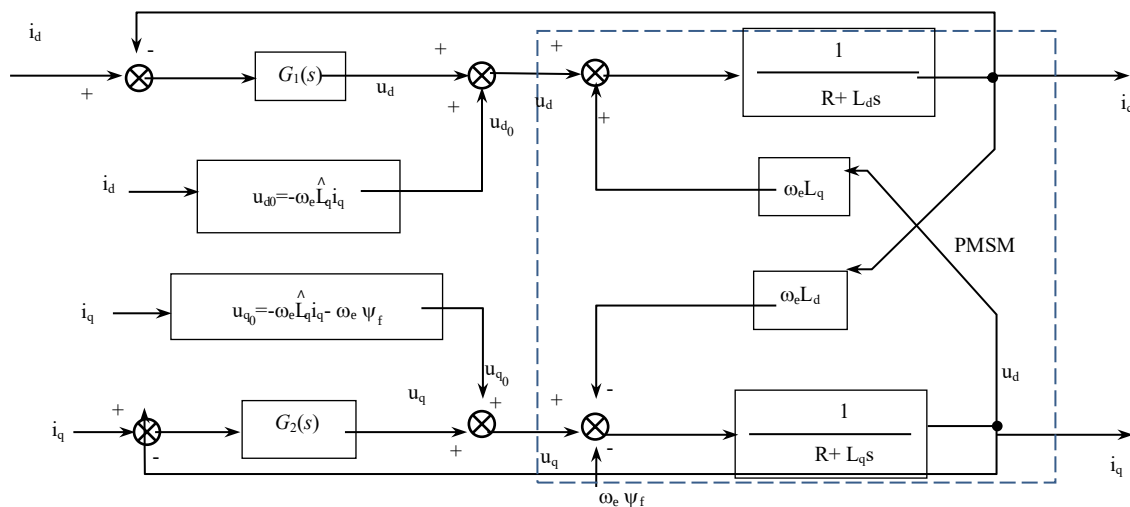


Figure 2. Current feedback decoupling control diagram.

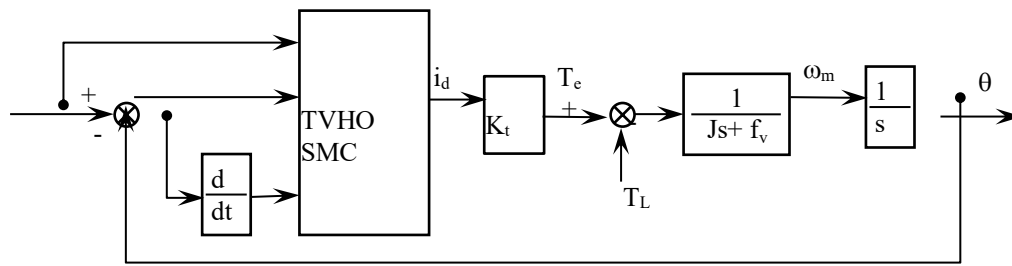


Figure 3. Block diagram of PMSM position control system based on the TVHOSMC method.

Thus, in the vector control of SPMSM $i_d = 0$, control method is also MTPA strategy. This paper expresses the method performing position control by controlling the current of q-axis i_q . That is, the position and speed controllers of position control system are designed by the time-varying high order sliding mode control (TVHOSMC) method.

Therefore, the block diagram of position control system using the PMSM based on the TVHOSMC is shown in Figure 3.

Let us set the state variables of the system as follows.

$$\begin{cases} x_1 = \theta^* - \theta \\ x_2 = \dot{\theta}^* - \omega_m \end{cases} \quad (10)$$

Where θ^* is the reference value of position signal, θ is measured value of position signal, ω_m is the measured value of mechanical speed.

The torque balance equation (6) is substituted as follows.

$$\dot{x}_2 = \ddot{\theta}^* - \ddot{\theta} = \ddot{\theta}^* - \dot{\omega}_m = \ddot{\theta}^* - \frac{1}{J}[T_e - T_L - f_v \omega_m] = \ddot{\theta}^* - \frac{1}{J}[p\psi_f i_q - T_L - f_v \omega_m] \quad (11)$$

But in practical condition there must be disturbances. Hence, considering the external disturbances, the mathematical model is as follows.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{1.5p\psi_f}{J} i_q + \ddot{\theta}^* + \frac{T_L + T_f}{J} + d(t) \end{cases} \quad (12)$$

where $d(t)$ is the random disturbance acting on the plant.

From the above, the control system must be designed to guarantee robustness in the presence of these disturbances. It is desirable to use the SMC for the control system. However, to avoid the chattering phenomenon surrounding the sliding plane and guarantee global stability, the TVHOSMC is used for the system.

DESIGN METHOD OF PMSM POSITION CONTROL SYSTEM BASED ON THE TVHOSMC

The first order sliding plane is designed as follows.

$$S = cx_1 + x_2 = 0 \quad (13)$$

By using it the second order sliding plane is given as follows.

$$S_{SO SMC} = kS + \dot{S} = ckx_1 + (c+k)x_2 + \dot{x}_2 = 0 \quad (14)$$

In the HOSMC the sliding controller is designed by combining the switching function S and its derivation. Currently the new switching function is related to the control signal and its high order derivation, so the control signal and its integration is a continuous signal. Thus, the chattering could be overcome.

Time-varying second order sliding mode control (TVSOSMC) plane is designed as follows.

$$S_{TVSOSMC} = ckx_1 + (c + k)x_2 + \dot{x}_2 + p(t) \quad (15)$$

Where A, B are constant, $t_f > 0$ and $p(t)$ are $\begin{cases} A + Bt, & t \leq t_f \\ 0, & t > t_f \end{cases}$.

The sliding plane is designed according to the initial state of the system.

We set the time-varying sliding plane as the initial error state of system can be placed on the sliding plane and gradually move the sliding plane to the stability state to cross the origin. The initial value of time-varying sliding control is $t = t_0 = 0$, thus the time-varying second order sliding plane is represented as follows;

$$S_{TVSOSMC}(t_0) = ckx_1(t_0) + (c + k)x_2(t_0) + \dot{x}_2(t_0) + p(t_0) \quad (16)$$

In the initial time $x_2(0) = 0, \dot{x}_2(0) = 0$, it is given as follows.

$$t_f = -\frac{A}{B} = \frac{e_0}{B}$$

Assuming that e_0 is the distance between the switching plane and present point, so that it is represented as follows.

$$e_0 = ckx_1(t_0) + (c + k)x_2(t_0) + \dot{x}_2(t_0)$$

And in the initial moment $x_2(t_0) = 0, \dot{x}_2(t_0) = 0$, so $e_0 = ckx_1(t_0)$

As a result of this, the derivation of time-varying second order sliding plane is as follows.

$$\begin{aligned} S_{TVSOSMC} &= ckx_1 + (c + k)x_2 + \dot{x}_2 + p(t) \\ &= ckx_1 + (c + k)x_2 + [\Delta f(x, t) - \frac{1.5p\psi_f}{J}i_q + \ddot{\theta}^* + \frac{T_L + T_f}{J} + d(t)] + p(t) \end{aligned} \quad (17)$$

In the SMC the following should be satisfied from the arrival and stabilization condition.

$$S_{TVSOSMC} \cdot \dot{S}_{TVSOSMC} < 0$$

From the above, the following expression is given below.

$$\dot{S}_{TVSOSMC} = -\rho \operatorname{sgn}(S_{TVSOSMC})$$

Where $\rho > 0$.

Then, the following expression is.

$$\dot{S}_{TVSOSMC} = ck\dot{x}_1 + (c + k)\dot{x}_2 + \ddot{x}_2 + \dot{p}(t) = -\rho \operatorname{sgn}(S_{TVSOSMC}) \quad (18)$$

The integration result of the above is as follows.

$$S_{TVSOSMC} = ckx_1 + (c + k)x_2 + \dot{x}_2 + p(t) = -\rho \int_{t_0}^t \operatorname{sgn}(S_{TVSOSMC}) d\tau \quad (19)$$

The control signal is given as follows.

$$\begin{aligned}
u = i_q &= \frac{J}{1.5p\psi_f} \times [ckx_1 + (c+k)x_2 + \ddot{\theta}^* + p(t)] + \frac{J}{1.5p\psi_f} \rho \int_{t_0}^t \text{sgn}(S_{TVSOSMC}) d\tau \\
&= \frac{J}{1.5p\psi_f} \times [ck(\theta^* - \theta) + (c+k)(\dot{\theta}^* - \omega_m) + \ddot{\theta}^* + p(t) \\
&\quad + \frac{J}{1.5p\psi_f} \rho \int_{t_0}^t \text{sgn}(S_{TVSOSMC}) d\tau
\end{aligned} \tag{20}$$

STABILITY ANALYSIS OF TIME-VARYING SECOND ORDER SLIDING CONTROLLER

Let us analyse the stability of time-varying second order sliding mode control system by the Lyapunov method.

Defining the Lyapunov function V as follows.

$$V_{TVSOSMC} = \frac{1}{2} S_{TVSOSMC}^2 \tag{21}$$

Thus, the derivation of Lyapunov function is given as follows.

$$\dot{V}_{TVSOSMC} = S_{TVSOSMC} \dot{S}_{TVSOSMC} = S_{TVSOSMC} \times \left[\frac{\dot{T}_L + \dot{T}_f}{J} + \dot{d}(t) - \rho \text{sgn}(S_{TVSOSMC}) \right] \leq 0 \tag{22}$$

As a result, if $\rho \geq \left| \frac{\dot{T}_L + \dot{T}_f}{J} + \dot{d}(t) \right|$, the requirement of stabilization is satisfied.

That is, in this case it is satisfied to the Lyapunov stabilization requirement.

$$\dot{V}_{TVSOSMC} = S_{TVSOSMC} \dot{S}_{TVSOSMC} \leq 0$$

Where, the load torque and friction torque are given and the random torques are added here.

Assume that the inequality is satisfied as follows;

$$\left| \frac{\dot{T}_L + \dot{T}_f}{J} + \dot{d}(t) \right| \leq D_{max} \tag{23}$$

Where D_{max} is the given value.

If it does not satisfy the stabilization due to a certain accident disturbance torque acting on the plant, the system moves to the time-varying condition and guarantees the global stabilization of the system.

The controller parameters of PMSM position control system are optimally determined to guarantee the contraction to the origin.

In the case of $0 \leq t \leq t_f$ the time-varying high order sliding plane is designed as follows.

$$S_{TVSOSMC} = ckx_1 + (c+k)x_2 + \dot{x}_2 + A + Bt = 0 \tag{24}$$

To analyze the function, the differential equation is given as follows.

$$ckx + (c+k)\dot{x} + \ddot{x} + A + Bt = 0 \tag{25}$$

To solve the second order linear nonhomogeneous differential equation, characteristic equation of associated homogeneous equation is given as follows.

$$\lambda^2 + (c + k)\lambda + c = 0 \quad (26)$$

The following are characteristic values.

$$\begin{aligned} (\lambda + c)(\lambda + 1) &= 0 \\ \lambda_1 &= -k, \quad \lambda_2 = -c \end{aligned}$$

Then, the characteristic equation needs to be solved in a double way to contract to the origin with high speed without vibration. That is, $k = c$

On the next stage let us obtain the special solution of nonhomogeneous differential equation by the method of undetermined coefficient.

Let us define the special solution of nonhomogeneous differential equation as follows.

$$\begin{aligned} \bar{x} &= A_0 t + B_0 \\ \bar{x}' &= A_0 \\ \bar{x}'' &= 0 \end{aligned} \quad (27)$$

Substituting (27) into (25), the following expression is given as follows;

$$(B_0 + A_0 t)c^2 + 2cA_0 = -At - B \quad (28)$$

Calculating coefficients by the method of undetermined coefficients,

$$A_0 = \frac{-B}{c^2}, \quad B_0 = \frac{\frac{2B}{c} - A}{c^2} \quad (29)$$

Therefore, the solution of differential equation is expressed as the sum of general solution of associated homogeneous equation and special solution of nonhomogeneous differential equation as follows.

$$x_1(t) = c_1 e^{-ct} - \frac{B}{c^2} t + \frac{-A + \frac{2B}{c}}{c^2} \quad (30)$$

$$x_2(t) = -c_1 c e^{-ct} - \frac{B}{c^2} \quad (31)$$

Where c_1, c_2 are constants and they could be determined from the initial condition.

That is, considering $x_1(t_0) = \frac{e_0}{c}, x_2(t_0) = 0$ in the initial time of control,

$$x_1(t) = \left[\left(\frac{e_0}{c} + \frac{A}{c^2} - \frac{2B}{c^3} \right) + \left(\frac{A-B}{c} + e_0 \right) t \right] e^{-ct} - \frac{B}{c^2} t + \frac{-A + \frac{2B}{c}}{c^2} \quad (32)$$

$$x_2(t) = - \left[\left(e_0 + \frac{A}{c} - \frac{2B}{c^2} \right) + (A - B + e_0 c) t - \left(\frac{A-B}{c} + e_0 \right) \right] e^{-ct} - \frac{B}{c^2} \quad (33)$$

In the same way, in the case of $t > t_i$, the sliding plane is given as follows.

$$c^2 x + 2c\dot{x} + \ddot{x} = 0 \quad (34)$$

The solution is given as follows.

$$x_1(t) = (c_3 + c_4 t) e^{-ct} \quad (35)$$

$$x_2(t) = (-cc_3 + c_4 - cc_4t)e^{-ct} \quad (36)$$

Where c_3, c_4 are constants and they could be determined from the initial condition.

At the moment of $t = t_f$ from the (32) and (33) we can obtain as follows.

$$x_1(t_f) = \left[\left(\frac{e_0}{c} + \frac{A}{c^2} - \frac{2B}{c^3} \right) + \left(\frac{A-B}{c} + e_0 \right) t_f \right] e^{-ct_f} - \frac{B}{c^2} t_f + \frac{-A + \frac{2B}{c}}{c^2} \quad (37)$$

$$x_2(t_f) = - \left[\left(e_0 + \frac{A}{c} - \frac{2B}{c^2} \right) + (A - B + e_0c) t_f - \left(\frac{A-B}{c} + e_0 \right) \right] e^{-ct_f} - \frac{B}{c^2} \quad (38)$$

Substituting $t_f = \frac{e_0}{B}$,

$$x_1(t_f) = \left[\left(\frac{e_0}{c} + \frac{A}{c^2} - \frac{2B}{c^3} \right) + \left(\frac{A-B}{c} + e_0 \right) \frac{e_0}{B} \right] e^{-\frac{e_0c}{B}} - \frac{e_0}{c} + \frac{-A + \frac{2B}{c}}{c^2} \quad (39)$$

$$x_2(t_f) = - \left[\left(e_0 + \frac{A}{c} - \frac{2B}{c^2} \right) + (A - B + e_0c) \frac{e_0}{B} - \left(\frac{A-B}{c} + e_0 \right) \right] e^{-\frac{e_0c}{B}} - \frac{B}{c^2} \quad (40)$$

Then (39) can be used as the initial condition in (35).

$$\begin{aligned} x_1(t_f) &= (c_3 + c_4 t) e^{-\frac{e_0c}{B}} = \\ &= \left[\left(\frac{e_0}{c} + \frac{A}{c^2} - \frac{2B}{c^3} \right) + \left(\frac{A-B}{c} + e_0 \right) \frac{e_0}{B} \right] e^{-\frac{e_0c}{B}} - \frac{e_0}{c} + \frac{-A + \frac{2B}{c}}{c^2} \end{aligned} \quad (41)$$

$$\begin{aligned} c_3 &= \left(\frac{e_0}{c} + \frac{A}{c^2} - \frac{2B}{c^3} \right) - \left(\frac{e_0}{c^2} - \frac{-A + \frac{2B}{c}}{c^2} \right) e^{\frac{e_0c}{B}} \\ c_4 &= \frac{A-B}{c} + e_0 \end{aligned} \quad (42)$$

Therefore,

$$x_1(t) = \left[\left(\frac{e_0}{c} + \frac{A}{c^2} - \frac{2B}{c^3} \right) + \left(\frac{A-B}{c} + e_0 \right) - \left(\frac{e_0}{c^2} - \frac{-A + \frac{2B}{c}}{c^2} \right) e^{\frac{e_0c}{B}} \right] e^{-ct} \quad (43)$$

$$x_2(t) = - \left[\left(e_0 + \frac{A}{c} - \frac{2B}{c^2} \right) + (A - B + e_0c) - \left(\frac{A-B}{c} + e_0 \right) - \left(\frac{e_0}{c} - \frac{-A + \frac{2B}{c}}{c} \right) e^{\frac{e_0c}{B}} \right] e^{-ct} \quad (44)$$

From the above, the action of the state values contracting to the origin can be shown. As can be seen from the above equation, the speed of contracting is related to the parameter c . That is, the transient time could be ensured by the parameter c .

ANALYSIS OF SIMULATION AND RESULT

Determining Parameters of Plant

The parameters of the plant are described in Table 1.

Simulation Results and Analysis

Simulation of stabilization characteristics for the position reference signals. The stabilization characteristics for the position reference signals are simulated as follows in Figure 4.

As can be seen from the simulation result, the transient characteristic of the position control system by the proposed TVHOSMC is better than other control methods (HOSMC, SMC).

Table 1. Parameters of the plant.

?????	?????
Rated power of motor	$P_n = 0.82\text{kW}$
Inertial moment of motor	$J_m = 3.01 \times 10^{-4}\text{kg} \cdot \text{m}^2$
Resistance value of stator winding	$R = 0.447\Omega$
Polar pairs	$2p = 8$
Rated voltage	115 V
Rated current	$I_n = 1.95\text{A}$
Rated number of rotations	$n_n = 3000\text{rpm} \pm 10\%$
Equivalent inductance of stator winding	$L = 4.8\text{mH}$
Standard value of frictional coefficient	$f_v = 5 \cdot 10^{-4}\text{Nm} \cdot \text{s/rad}$
Maximum allowable current	$I_{A_{max}}$
Mechanical time constant	$t = 2.37\text{ms}$
Nominal torque	2.6 Nm

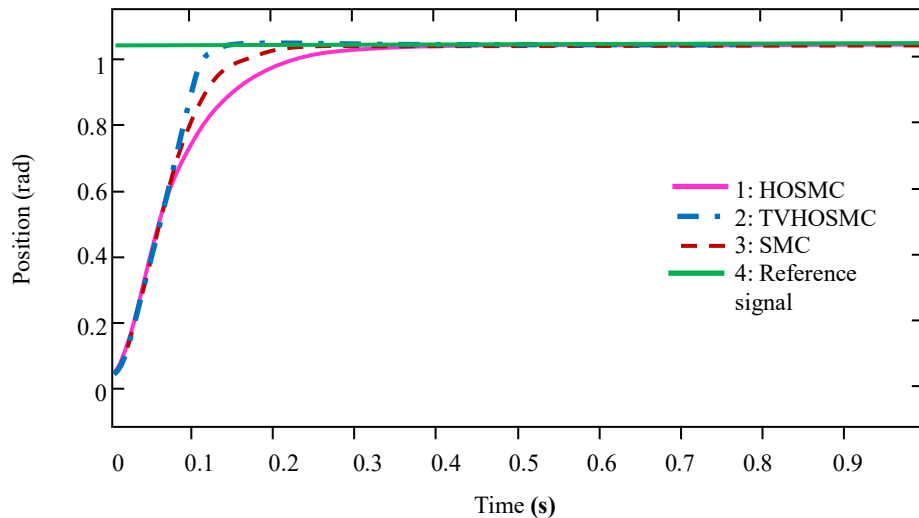


Figure 4. Transient response characteristic of position reference signal.

To evaluate the accuracy, the error characteristics are shown according to every control methods. To compare the accuracy, the error characteristics are zoomed out to verify that suggested control method is better than others in control performance.

Simulation of stabilization characteristics for the position reference signals (in the presence of the external disturbance) in Figure 5.

Robustness of system to overcome the external disturbances was proved under the condition that the system was given by the impulse torque for 0.05s now of 0.5s.

The error characteristics are zoomed out as follows in Figure 6.

As can be seen from the simulation results, the rejection characteristics on the external disturbances vary with the control methods. That is, SMC returns to the stabilization state in 0.15s against the external disturbance, in HOSMC it takes 0.15s and in TVHOSMC it takes 0.08s.

As for accuracy, the error value of SMC is 0.18° , in HOSMC it is 0.12° and in TVHOSMC it is about 0.015° .

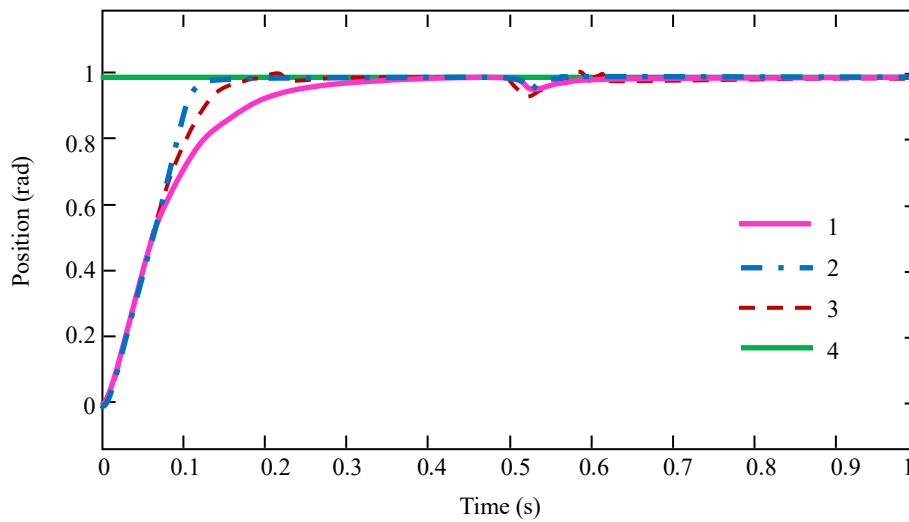


Figure 5. Transient response characteristic of position reference signal (in the presence of external disturbance)

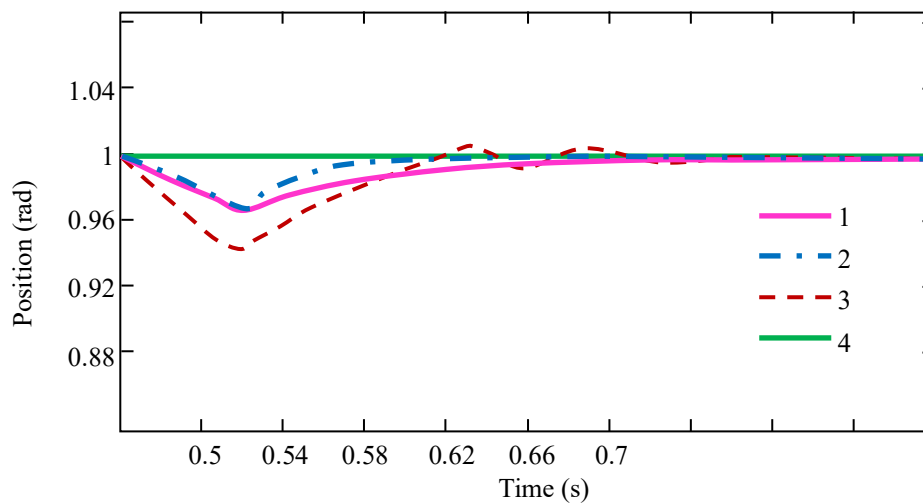


Figure 6. Rejection characteristics of external disturbance

Simulation of tracking characteristics for sine reference signals as shown in Figure 7. To compare the states of system in the presence of the external disturbance, the state curve was zoomed out for consideration.

As can be seen from Figure 8, the external disturbance rejection characteristics and tracking characteristics are better than any others.

That is, in SMC it returns to the tracking state in 0.4 s against the external disturbance, in HOSMC it takes 0.3s and in TVHOSMC it takes 0.1 s. For accuracy, the error value of SMC is 0.3° , 0.25° in the HOSMC and about 0.2° in the TVHOSMC.

Simulation of tracking characteristics for the speed reference signal. And the tracking characteristics for the speed reference signal are considered as follows.

As can be seen from the simulation result, the system control characteristics was improved by the proposed method. The fast response property and accuracy was improved by the proposed control method TVHOSMC.

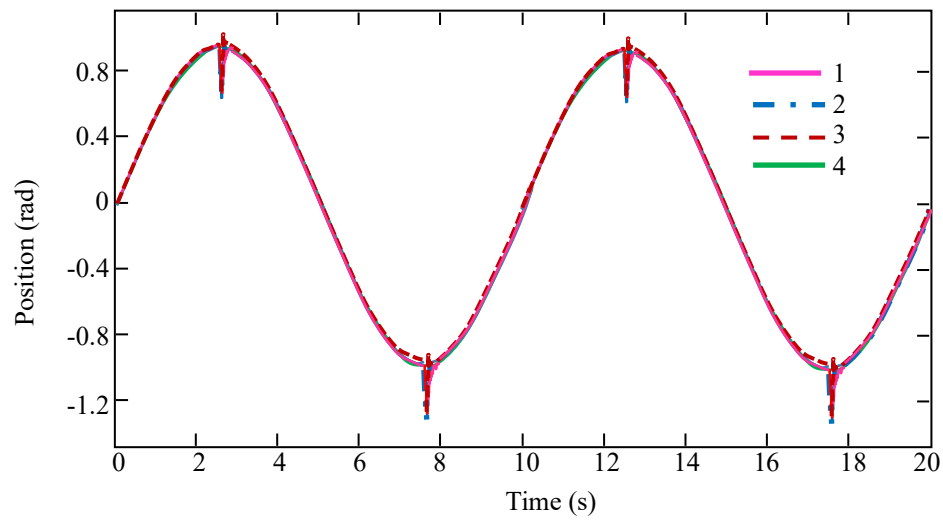


Figure 7. Following characteristics for sine reference signals (in the presence of the external disturbance)

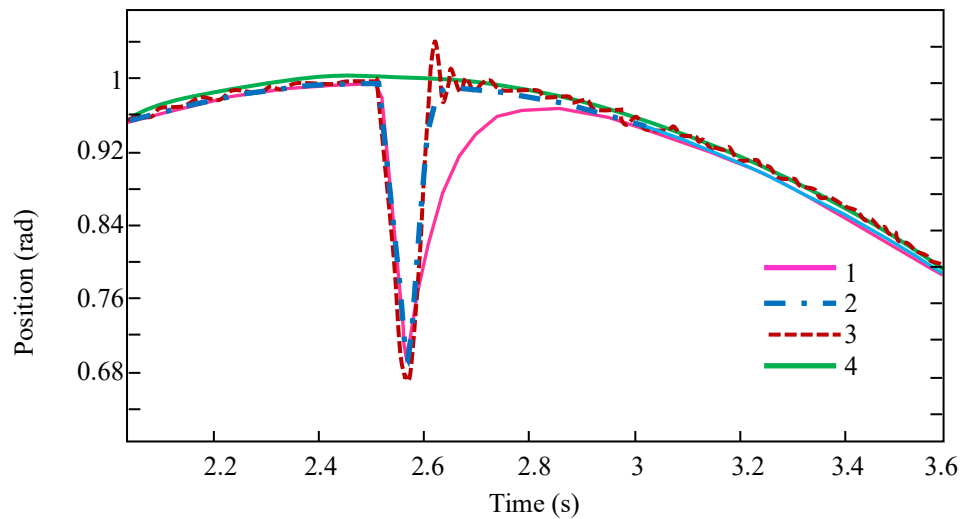


Figure 8. Tracking characteristics for sine reference signals.

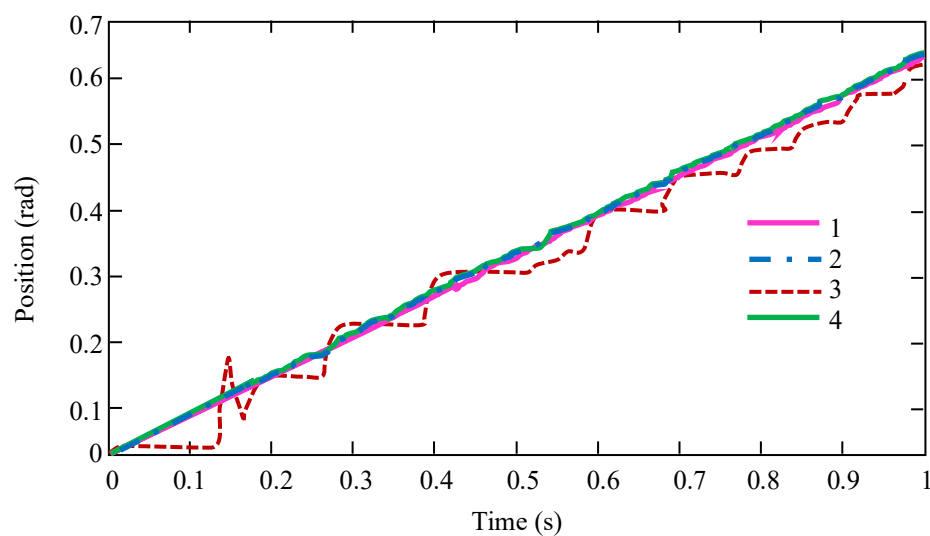


Figure 9. Comparison of tracking characteristics.

CONCLUSION

This paper proposed the method of position control system design using the PMSM based on the TVHOSMC strategy to eliminate the chattering in sliding mode control, perform the fast and accurate position control and guarantee global stability and robustness of the system in the presence of the external disturbances and parameter uncertainties. In addition, in the presence of external disturbances and parameter uncertainties, the proposed control method was verified through the analysis of the performance of the PMSM position control system.

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