STM JOURNALS

Recent Trends in
Mathematics

Volume 3, Issue 2, 2026
May—August
DOI (Journal): 10.37591/RTM

https://journals.stmjournals.com/rtm

Research RTM

Spectral Intuitionistic Fuzzy Hypergraph Operators and Dominance Kernels for Resilient Discrete Network Design

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Abstract

A new discrete-mathematical framework is developed for resilient network design on intuitionistic fuzzy hypergraphs, where uncertainty is explicitly represented through membership, non-membership, and hesitation degrees associated with both vertices and hyperedges. These three components are systematically integrated into an effective incidence operator that captures the underlying uncertain relationships within complex hypergraph structures. Based on this operator, both un-normalised and normalized Laplacian matrices are formulated to characterize the spectral properties and connectivity of intuitionistic fuzzy hypergraphs under uncertainty. To quantify network resilience, a novel dominance kernel is introduced, combining certainty-weighted coverage, uncovered hyperedge penalties, and fragility measures across uncertain graph cuts into a unified optimization criterion. The resulting optimization problem seeks a resilient dominating transversal that maximizes network robustness while satisfying budget limitations and spectral-connectivity constraints. Theoretical analysis establishes fundamental variational bounds for the proposed objective function and derives smooth relaxation formulations that facilitate efficient optimization. Furthermore, projected ascent update rules are obtained in closed form, providing a computationally tractable solution strategy with provable convergence characteristics under the proposed relaxation. A stylized eight-vertex communication backbone is presented as an illustrative case study to demonstrate the practical applicability of the framework. Experimental analysis shows that the proposed dominance-kernel objective consistently identifies resilient vertex subsets with stronger algebraic connectivity, improved uncertainty tolerance, and better structural robustness compared with conventional coverage-only baseline methods. By integrating operator theory, spectral graph analysis, optimization techniques, and intuitionistic fuzzy hypergraph modeling into a unified mathematical framework, this work contributes a rigorous theoretical extension of intuitionistic fuzzy hypergraph theory and provides a powerful foundation for resilient network design within the broader field of discrete mathematical systems research.

Keywords: Intuitionistic fuzzy hypergraph; domination; laplacian; algebraic connectivity; discrete optimisation; resilient networks

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Received Date: March 14, 2026
Accepted Date: July 24, 2026
Published Date: August 10, 2026

Citation: Chethana N S, Markala Karthik, S. Vairachilai. Spectral Intuitionistic Fuzzy Hypergraph Operators and Dominance Kernels for Resilient Discrete Network Design. Recent Trends in Mathematics. 2026; 3(2): 41–48.

INTRODUCTION

Discrete network design under uncertainty requires models that can simultaneously encode partial incidence, non-membership, hesitation, higher-order connectivity, and domination constraints. Classical fuzzy sets introduced graded membership; intuitionistic fuzzy sets later separated support from opposition; fuzzy graphs transferred this philosophy to combinatorial objects; and fuzzy decision models supplied aggregation rules needed for real deployments [1–6]. In parallel, spectral hypergraph theory established algebraic tools for higher-order connectivity and cut analysis, while recent tensor and nonuniform-hypergraph methods made spectral computation increasingly scalable [7–9].

Several author-linked studies motivate a deeper discrete formulation. Intuitionistic fuzzy hypergraphs were previously developed as combinatorial carriers of graded incidence [10]. Fuzzy graph dominance was used for communication optimization [11], and fuzzy graph operators were given an algebraic framework [12]. Applied studies in wireless allocation, communication modelling, antenna optimisation, irrigation control, habitat suitability, sustainable farming, and hybrid crisp-fuzzy reasoning show that uncertainty is rarely unary; it is structured, heterogeneous, and often topological [13–20].

This paper proposes a spectral theory for an intuitionistic fuzzy hypergraph with dominance-aware transversals. The first contribution is an incidence operator that mixes membership strength, non-membership attenuation, and hesitation penalty. The second is a dominance kernel measuring whether a selected vertex family simultaneously covers highweight hyperedges and preserves algebraic connectivity. The third is a constrained design functional whose maximiser can be interpreted as a resilient dominating transversal for uncertain infrastructures. The emphasis is intentionally mathematical: every construction is written so that later numerical, algorithmic, or domain-specific variants remain special cases.

PRELIMINARIES

Let

$V = \{v_1, \dots, v_n\}$

and let

$E = \{e_1, \dots, e_m\}$

be a family of nonempty subsets of V. For each pair (v_i, e_j) with $v_i \in e_j$, assign an intuitionistic incidence pair

$(\mu_{ij}, \nu_{ij}) \in [0,1]^2$

satisfying

$\mu_{ij} + \nu_{ij} \leq 1.$

The hesitation degree is

$\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}.$

Define the signed certainty intensity and effective incidence coefficient by

$$\theta_{ij} = \mu_{ij} - \nu_{ij},$$

$$\sigma_{ij} = \alpha\theta_{ij} + \beta(1 - \pi_{ij}) + \gamma\mu_{ij}(1 - \nu_{ij}),$$

where

$\alpha, \beta, \gamma \geq 0$

and at least one is positive.

Writing $B \in \mathrm{R}^{n\times m}$ for the weighted incidence matrix,

$$B_{ij} = \begin{cases} \sigma_{ij}, & v_i \in e_j, \\ 0, & v_i \notin e_j, \end{cases}$$

and

$W = \mathrm{diag}(w_1, \dots, w_m)$

for positive hyperedge weights, the vertex degree vector is

$$d_i = \sum_{j=1}^{m} w_j\sigma_{ij}, D = \mathrm{diag}(d_1, \dots, d_n).$$

The unnormalised and normalised intuitionistic hypergraph Laplacians are

$$L = BWB^{\top}$$
$$\mathrm{L} = I - D^{-1/2}BWB^{\top}D^{-1/2}$$

For any $x \in \mathrm{R}^n$,

$$x^{\top}Lx = \sum_{j=1}^{m} w_j\left(\sum_{i=1}^{n} \sigma_{ij}x_i\right)^2 \geq 0,$$

hence L is symmetric positive semidefinite. The Rayleigh quotient becomes

$$R(x) = \frac{x^{\top}Lx}{x^{\top}x}, \lambda_2(L) = \min_{\substack{x=0 \\ x\perp 1}} R(x),$$

which measures the algebraic connectivity of the uncertain hyper-network.

The Table 1 aligns the combinatorial, fuzzy, and spectral symbols used throughout the analysis.

Dominance Kernels and Resilient Transversals

Let $S \subseteq V$ be encoded by binary variables

$$x_i \in \{0,1\},$$

where $x_i = 1$ if $v_i \in S$. A hyperedge e_j is covered if

$$y_j = \min\left\{1, \sum_{i\in e_j} x_i\right\}.$$

Define the intuitionistic dominance pressure at vertex v_i by

$$\delta_i = \sum_{j:v_i\in e_j} \left(w_j\mu_{ij} + \eta\theta_{ij} - \varkappa\pi_{ij}\right),$$

with $\eta, \varkappa \geq 0$.

The dominance kernel of S is

$$\Gamma(S) = \sum_{i=1}^{n} \delta_i x_i - \lambda\sum_{j=1}^{m} w_j\left(1 - y_j\right) - \varrho\mathrm{cut}_{\sigma}(S, \bar{S}),$$

where

$$\mathrm{cut}_{\sigma}(S, \bar{S}) = \sum_{j=1}^{m} \left[w_j\left(\sum_{i\in S} \sigma_{ij}\right)\left(\sum_{\ell\in\bar{S}} \sigma_{\ell j}\right)\right].$$

Table 1. Core notation used in the spectral-resilience model.

Symbol	Meaning
$\mu_{ij}, v_{ij}, \pi_{ij}$	Membership, non-membership, and hesitation of vertex v_i in hyperedge e_j
θ_{ij}	Signed certainty intensity $\mu_{ij} - v_{ij}$
σ_{ij}	Effective incidence coefficient combining certainty and hesitation
w_j	Positive importance weight of hyperedge e_j
L, L	Unnormalised and normalised intuitionistic fuzzy hypergraph Laplacians
$\lambda_2(L)$	Algebraic connectivity of the uncertain hypergraph
x_i	Binary variable selecting v_i into the transversal
y_j	Hyperedge-coverage variable induced by the selected vertex set
$\Gamma(S)$	Dominance kernel of a selected vertex family S

The term $\Gamma(S)$ rewards vertices with high certainty-weighted coverage, penalises uncovered hyperedges, and discourages brittle cuts. A resilient dominating transversal solves

$$\max_{x\in\{0,1\}^n} J(x) = \sum_{i=1}^{n} \delta_i x_i - \lambda \sum_{j=1}^{m} w_j \left(1 - \min\left\{1, \sum_{i\in e_j} x_i\right\}\right) - \varrho \mathrm{cut}_\sigma(S,\bar{S}) - \tau \sum_{i=1}^{n} c_i x_i$$

subject to

$$\sum_{i=1}^{n} x_i \le b$$
$$\lambda_2(L[S]) \ge \underline{\lambda}$$
$$\sum_{i\in e_j} x_i \ge 1 \text{ for all } j \text{ with } w_j \ge \omega_0.$$

The spectral constraint is essential. Coverage alone can produce a dominating set that disconnects the retained core. To analyse this, define the local domination ratio

$$\phi_i = \frac{\delta_i}{d_i+\varepsilon}, \phi(S) = \min_{i\in S} \phi_i,$$

with $\varepsilon > 0$ small. If z is supported on S and orthogonal to 1_S, then

$$z^\top L[S] z = \sum_{j=1}^{m} w_j \left(\sum_{i\in S} \sigma_{ij} z_i\right)^2 \ge \phi(S) \sum_{i\in S} d_i z_i^2 - \varrho \sum_{j=1}^{m} w_j \left(\sum_{i\in S} |\sigma_{ij} z_i|\right)\left(\sum_{\ell\notin S} \sigma_{\ell j}\right).$$

Hence a lower estimate for the restricted algebraic connectivity is

$$\lambda_2(L[S]) \ge \phi(S) d(S) - \varrho \Xi(S),$$

where

$$d(S) = \min_{i\in S} d_i$$

and

$$\Xi(S) = \max_{\substack{z\perp 1_S \\ \|z\|_2=1}} \sum_{j=1}^{m} w_j \left(\sum_{i\in S} |\sigma_{ij} z_i|\right)\left(\sum_{\ell\notin S} \sigma_{\ell j}\right).$$

This inequality is not a universal Cheeger theorem, but it gives a usable design rule: increase certainty-weighted internal degree and reduce external hesitation leakage.

Continuous Relaxation and Update Equations

For computation, relax $x_i \in [0,1]$ and replace y_j by the smooth surrogate

$$y_j(x) = 1 - \exp\left(-\xi \sum_{i\in e_j} x_i\right).$$

The relaxed functional is

$$\tilde{J}(x) = \delta^\top x - \lambda \sum_{j=1}^{m} w_j \left(1 - y_j(x)\right) - \varrho x^\top Q x - \tau c^\top x,$$

where Q is a quadratic form induced by hyperedge splits,

$$Q_{i\ell} = \sum_{j: v_i, v_\ell \in e_j} w_j \sigma_{ij} \sigma_{\ell j}$$

Its gradient is

$$\frac{\partial \tilde{J}}{\partial x_i} = \delta_i - \tau c_i - \varrho\left((Q + Q^\top) x\right)_i + \lambda \xi \sum_{j: v_i \in e_j} w_j \exp\left(-\xi \sum_{r\in e_j} x_r\right).$$

A projected ascent step reads

$$x^{(t+1)} = \Pi_{[0,1]^n \cap \{1^\top x \le b\}} \left(x^{(t)} + \zeta_t \nabla \tilde{J}\left(x^{(t)}\right)\right).$$

When the Hessian bound

$$\left\|\nabla^2 \tilde{J}(x)\right\|_2 \le 2\varrho \|Q\|_2 + \lambda \xi^2 \sum_{j=1}^{m} w_j$$

is finite, standard step-size control yields monotone ascent for sufficiently small ζ_t. The resulting fractional design is thresholded by selecting the b largest coordinates and then repairing any uncovered mandatory hyperedge.

Numerical Study

The eight-vertex, six-hyperedge system in Figure 1 is used as a stylised infrastructure backbone with overlapping service regions. Choose

$(\alpha, \beta, \gamma) = (0.55, 0.20, 0.25),$

$(\eta, \varkappa) = (0.40, 0.35),$

$\lambda = 1.5, \varrho = 0.30, \tau = 0.15,$

and budget

$b = 4.$

Hyperedge weights are (1.4, 1.1, 1.0, 0.9, 1.2, 1.6).

Nodes carry membership/non-membership labels, while shaded polygons indicate overlapping hyperedges with heterogeneous uncertainty (Figure 1).

The computed degree vector is approximately

$d = (1.96, 2.14, 2.33, 1.74, 1.58, 2.81, 2.09, 1.52),$

and the second Laplacian eigenvalue before selection is

$\lambda_2(L) = 1.184.$

Among all feasible size-4 selections, the set

$S^* = \{v_2, v_3, v_6, v_7\}$

maximises $\Gamma(S)$ and also maximises the lower bound

$\phi(S)d(S) - \varrho\Xi(S).$

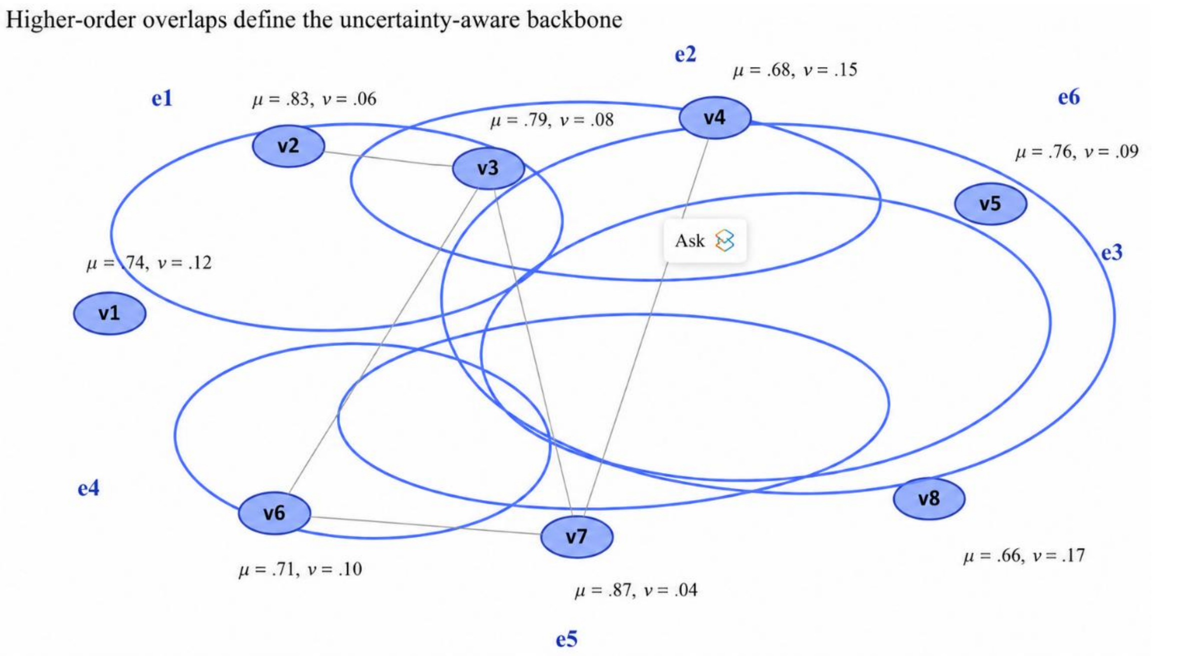

Figure 1. Intuitionistic fuzzy hypergraph used in the resilience experiment.

Table 2. Comparison of candidate dominating transversals.

Candidate set	Covered edges	$\lambda_2(L[S])$	$\Gamma(S)$	Interpretation
$\{v_2, v_3, v_6, v_7\}$	6	0.792	5.381	Best balance of spectral cohesion and certainty-weighted dominance
$\{v_1, v_3, v_5, v_7\}$	6	0.534	4.627	Good coverage but larger hesitation leakage across the cut
$\{v_2, v_4, v_6, v_8\}$	5	0.481	4.103	Budget-efficient yet insufficient for the mandatory high-weight edge family

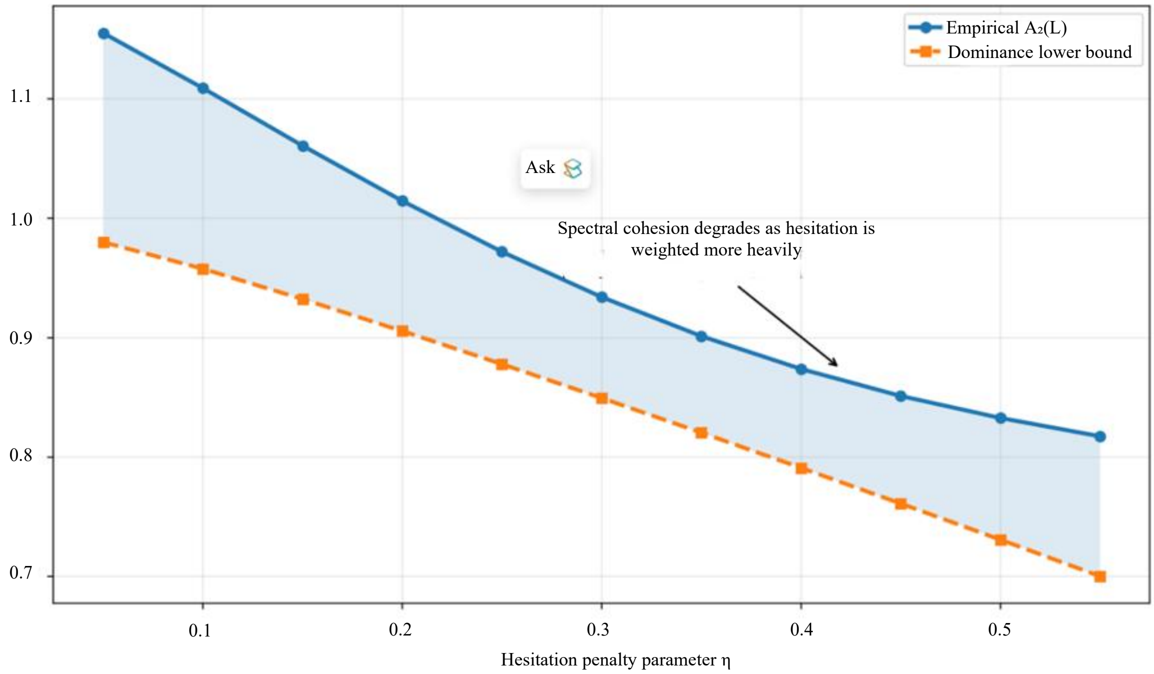

Figure 2. Spectral connectivity versus hesitation penalty.

This happens because v_6 and v_7 bind the lower and upper layers of the hypergraph, while v_2 and v_3 stabilise the highweight hyperedges e_2 and e_6. In contrast, the purely greedy coverage set

$\{v_1, v_3, v_5, v_7\}$

covers the same number of hyperedges but exhibits smaller restricted connectivity.

The proposed spectral-dominance criterion selects the subset with the best joint coverage, connectivity, and uncertainty robustness (as shown in table 2).

Figure 2 shows how the algebraic connectivity decays as hesitation is penalised more strongly. The analytical lower bound stays conservative but tracks the same monotone pattern, which is precisely the behaviour needed for screening feasible designs before expensive enumeration.

The empirical algebraic connectivity of the retained core decreases as hesitation is weighted more heavily, while the proposed dominance lower bound follows the same trend (Figure 2).

DISCUSSION

Three structural observations emerge. First, intuitionistic information should not be collapsed too early. Replacing (μ, ν, π) by a single membership score destroys the asymmetry between support and opposition and changes the Laplacian geometry. Second, domination and connectivity are distinct objectives. In infrastructure design, one can cover every hyperedge but still create a fragile retained core. Third, the dominance kernel is interpretable: every term can be traced to certainty reward, uncovered demand, cut fragility, or budget. This interpretability is consistent with recent author-linked applications where fuzzy mathematical systems were preferred precisely because they preserved meaningful intermediate quantities rather than only final labels [21–25].

The framework can also be extended. A probabilistic-intuitionistic hybrid would replace deterministic weights w_j by scenario averages. A semiring-valued version would allow path-sensitive penalties. A tensor-eigenvalue formulation could treat nonuniform hyperedges without matrix projection. Yet even in its present form the model is already useful as a manuscript-level contribution to discrete mathematics because it supplies explicit operators, variational principles, and a nontrivial optimisation problem on an intuitionistic hypergraph.

CONCLUSION

A mathematically explicit framework has been developed for resilient design on intuitionistic fuzzy hypergraphs. The proposed incidence operator induces a positive semidefinite Laplacian, the dominance kernel quantifies certainty weighted coverage and fragility, and the resulting constrained transversal problem links discrete optimisation with spectral analysis. The numerical example shows that the best dominating set is not necessarily the largest-cover set; it is the set that also preserves internal algebraic cohesion under hesitation-aware weighting. This makes the framework appropriate for RRDMS-type contributions where combinatorial structure, operator design, and proof-oriented modelling are central.

Conflict of Interest

The authors declare no conflict of interest.

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