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Presence of a Bulk Viscous Universe Within f (R, T) Gravity

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Abstract

This paper offers a comprehensive analysis of a bulk viscous universe in the context of f(R, T) gravity, where (R) signifies the Ricci scalar and (T) represents the trace of the energy-momentum tensor. The primary objective of our work is to get explicit solutions to the modified field equations by using a power-law scale factor representation. With this method, we have obtained functions of cosmic time and redshift for the Hubble parameter and the deceleration parameter. To validate our theoretical model, we have estimated the current values of these cosmological parameters using observational data. Specifically, we have utilized observational Hubble data and Supernova Type Ia (SN Ia) datasets to constrain the model. At the 1σ confidence level, we have determined that the present value of the deceleration parameter, q0 = −0.30 ± 0.05, and the model parameter, m = 0.70 ± 0.02. Additionally, we have examined the energy requirements in this cosmological framework and performed an Om(z) analysis within the context of the anisotropic Locally Rotationally Symmetric (LRS) Bianchi type I model. Our results show a strong agreement between our theoretical model and the observational data. This alignment offers valuable new insights into the characteristics and dynamics of a bulk viscous universe in modified gravity theories, highlighting the model's relevance and precision in explaining cosmological observations.

Keywords: Cosmological parameters; bulk viscosity; *f* (*R, T*) gravity; LRS Bianchi-I space-time, gravitational field theories

INTRODUCTION

Recent astrophysical observations such as the *H(z)* measurements from Type Ia supernovae [1-5], the Cosmic Microwave Background *(CMB)* [6, 7], baryon acoustic oscillations *(BAO)* [8-10], and PLANCK data [11]—have confirmed that our universe is accelerating. These findings indicate that nearly two-thirds of the universe's total energy density is comprised of a mysterious form of energy. This acceleration is attributed to dark energy *(DE)*, although its precise nature is still being explored. Numerous models with different DE candidates have been proposed in the literature. Among these models, the *ΛCDM* model is the most widely endorsed by theoretical physicists for explaining dark energy and the universe's late-time acceleration. Nevertheless, the *ΛCDM* model encounters two significant challenges: fine-tuning at the Planck scale and the cosmic coincidence problem [12]. The challenges associated with the *ΛCDM* model have driven the search for new gravitational field theories capable of explaining late-time cosmic acceleration without the need for a cosmological constant in Einstein's field equations. However, the idea of modifying general relativity did not arise solely from the discovery of the accelerating universe [13, 14]. Various modified theories, such as *f(R)* theory [15, 16], f(G) gravity [17], *f(T)* theory [18, 19], and *f(R, T)* theory [20], have existed for a considerable

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time, driven by the combined needs of astrophysics, high energy physics, and cosmology. Initially, efforts to modify general relativity focused on altering the geometrical part of the Einstein-Hilbert gravitational action. In [20], the authors investigated a non-minimal coupling between matter and geometry within the context of an effective gravitational Lagrangian that includes both the Ricci scalar *(R)* and the trace of the energy-momentum tensor *(T)*. This approach led to the development of the *f(R, T)* theory of gravitation.. In this theory, the inclusion of T accounts for the presence of certain imperfect fluids, potentially providing a comprehensive explanation for the universe's late-time acceleration without invoking dark energy.

The remarkable features of *f(R, T)* theory of gravitation have attracted researchers to study and reconstruct this theory within various contexts of astrophysics and cosmology [21, 22]. Recently, Nojiri et al. [23] have investigated inflation, bounce, and late-time evolution in the framework of modified gravity. It is important to note that *f(R, T)* theory is also applicable for examining the effects of expansion-free conditions in the formulation of structure scalars. Several useful applications and the existence of relativistic stellar objects within *f(R, T)* theory of gravity are discussed in references [24–26]. Following the discovery of the Wilkinson Microwave Anisotropy Probe [27, 28], homogeneous and anisotropic models have garnered increasing attention and momentum in observational cosmology, as researchers seek a relativistic picture of the early universe [29-31]. A spatially homogeneous Bianchi I (BI) space-time possesses three-dimensional groups that act transitively on space-like three-dimensional orbits. Consequently, the universe is expected to achieve. With the *f(R, T)* theory of gravity serving as the foundation, the primary objective of this research is to examine the bulk viscous anisotropic universe [32, 33].

METRIC AND FIELD EQUATIONS: FORMULATION

The spatially homogeneous and anisotropic LRS BI metric is given as

$$ds^2 = -dt^2 + A^2 + B^2(dy^2 + dz^2) \tag{1}$$

Here, A(t) and B(t) represent scale factors in the spatial directions. By simply subtracting the Einstein-Hilbert action from the action representing this version of General Relativity, the action characterising this modification of General Relativity is obtained. R is exchanged for a generic nonlinear function [34-36].

$$S = \frac{c^4}{16\pi G_n}\int d^4x\sqrt{-gf(R)}S_m \tag{2}$$

The energy momentum tensor for bulk-viscous fluid is read as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \tag{3}$$

where

$u_\mu = (0, 0, 0, 1)$ is the four velocity vector in co-moving co-ordinate system satisfying
$u_\mu u_\nu = 1$.

The Einstein field equations can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = (8\pi + 3\zeta)p - \zeta\rho \tag{5}$$

$$\frac{-\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} = (8\pi + 3\zeta)p - \zeta\rho \tag{6}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}=(8\pi+3\zeta)\rho-\zeta p \tag{7}$$

We can define the Hubble's parameter (H) in the term of scale factor

$$H=\frac{\dot{a}}{a} \tag{8}$$

Applying the hybrid expansion law

$$a=(nDt)^{\frac{1}{n}} \tag{9}$$

where *m* and *D* are constants.

The following formulas for the scale factors A and B in different directions.

$$A=(n_1D_1t)^{\frac{1}{n_1}} \tag{10}$$

$$B=(n_2D_2t)^{\frac{1}{n_2}} \tag{11}$$

Where, m_1, m_2, D_1 and D_2 are constants.

DYNAMICS OF THE MODEL'S PHYSICAL BEHAVIOR

The Hubble parameter (*H*) and deceleration parameter (*q*) can be expressed as

$$H=\frac{1}{nt}$$

$$q=-1+n$$

The energy density (ρ) and pressure (p) in model (1) are expressed as

$$\rho=\{8\pi+(-\gamma+3)\zeta\}^{-1}[\{m_1+2m_2(m_1m_2^2t^2)^{-1}\}-3\xi(mm_2^2t^3)^{-1}] \tag{12}$$

$$P=\gamma\{8\pi+(-\gamma+3)\zeta\}^{-1}[\{m_1+2m_2(m_1m_2^2t^2)^{-1}\}-3\xi(mm_2^2t^3)^{-1}] \tag{13}$$

ENERGY CONDITION

In theoretical cosmology, four fundamental energy conditions are used to understand the behaviour of matter and energy in the universe:

1. *Weak Energy Condition (WEC):* States that the energy density measured by any observer is non-negative for any time like vector.
2. *Null Energy Condition (NEC):* Requires that the energy density is non-negative for any null vector.
3. *Dominant Energy Condition (DEC):* Ensures that the energy density is non-negative and that energy propagation does not exceed the speed of light.
4. *Strong Energy Condition (SEC):* Asserts that the combined energy density and pressure are non-negative, and that gravity is always attractive.

Collectively known as energy conditions (ECs), these principles are vital for maintaining the stability and consistency of cosmological models. Notably, a violation of the SEC can highlight significant issues related to rapid expansion in theoretical models, especially those involving exotic matter or dark energy [37-41].

In the context of *f(R, T)* gravity theory, where (*R*) is the Ricci scalar and *(T)* is the trace of the energy-momentum tensor, [42-45] the energy conditions have been extensively studied. It has been demonstrated that the *(T)* sector cannot be chosen arbitrarily due to its specific form, resulting from the conservation of the energy-momentum tensor being integral to the calculations [46-49].

An anisotropic universe is the foundation upon which the cosmos is built [50]. ECs in the *f(R, T)* gravity theory are published in the literature [51], and it has been shown that the T sector cannot be selected freely since it has a specific shape [52]. This is because the conservation of energy-momentum tensor is incorporated into the calculations. When it comes to modified gravitational field equations, the ECs are presented in the following manner [53].

When dealing with modified gravitational field equations, the energy conditions are redefined to incorporate the changes in the gravity theory. The ECs in *f(R, T)* gravity consider the additional terms and constraints introduced by the modified theory, ensuring adherence to conservation laws and physical plausibility [54-56]. This approach is crucial for evaluating the feasibility and implications of these modified theories in describing the universe's dynamics.

In Figure 1 the scale factor is a mathematical concept that describes how much something is scaled up or down. It is used in various fields such as mathematics, physics, and engineering. Here are a few key points about the variation of scale factor: In Figures 2–4, the behaviours of the ECs mentioned above are illustrated, with the values of m = 0.7 and $\gamma = 0.5$. In Figure 5 the density of different components of the universe (such as dark matter, dark energy, and radiation) changes with redshift. Redshift, denoted by z, measures how much the wavelength of light has been stretched due to the expansion of the universe. In Figure 6 The densities of different components of the universe (such as matter, radiation, and dark energy) relate to the Hubble parameter (HHH), which measures the rate of expansion of the universe. The observation that all of the energy criteria are broken for the fixed values of the free parameters may be made from the data presented in Figures 7 and 8 [57, 58]. By constraining the generated model with observational data and ensuring positive energy density, the values of the free parameters can be effectively determined. Both figures are shows the accelerating expansion of the universe [59, 60].

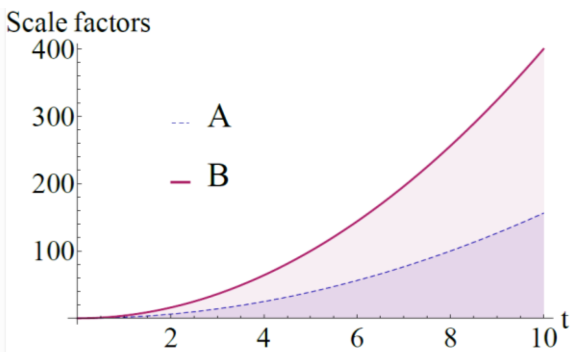

Figure 1. Variation of scale factor

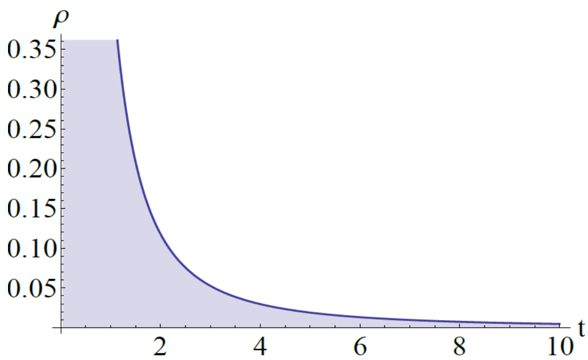

Figure 2. Density Vs Time.

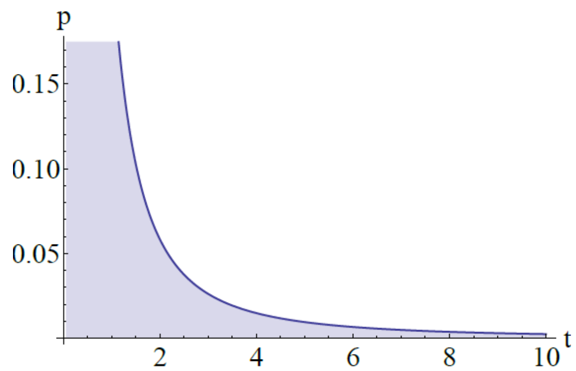

Figure 3. Pressure Vs Time.

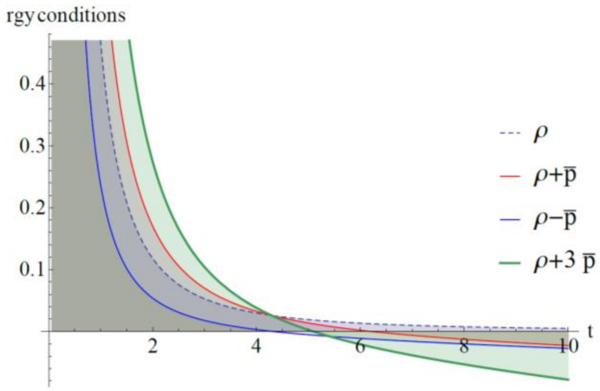

Figure 4. Variation of Energy conditions.

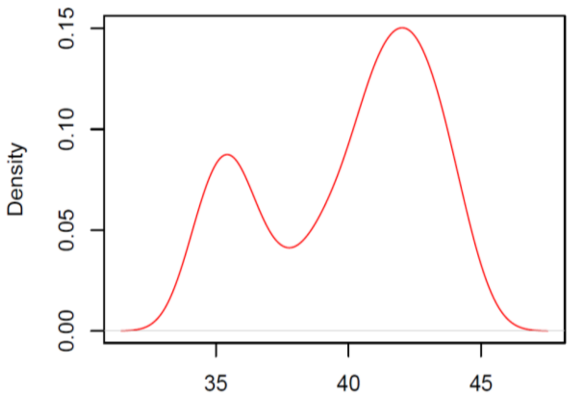

Figure 5. Density Vs Redshift.

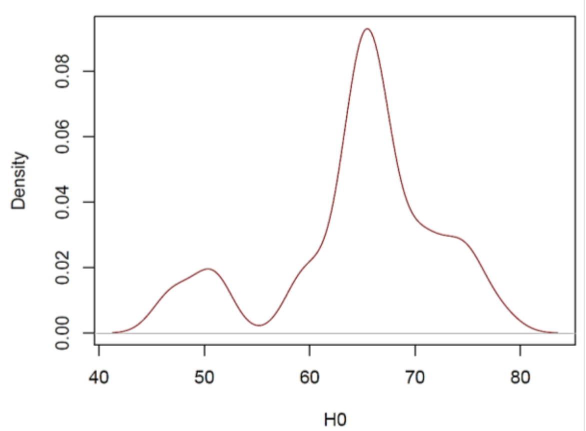

Figure 6. Density Vs Hubble parameter.

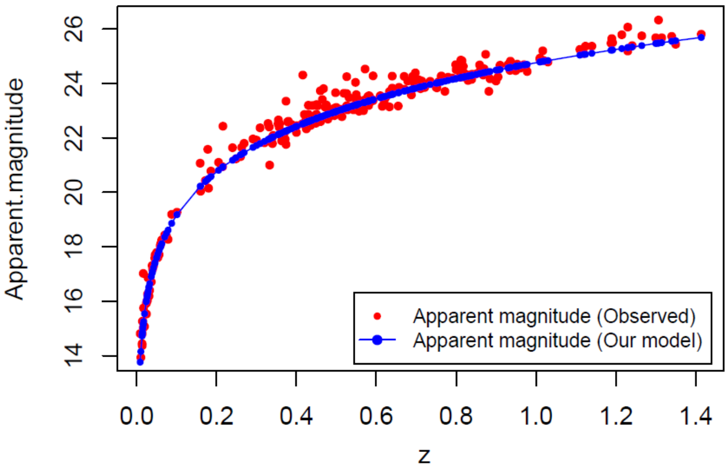

Figure 7. Redshift Vs Apparent magnitude.

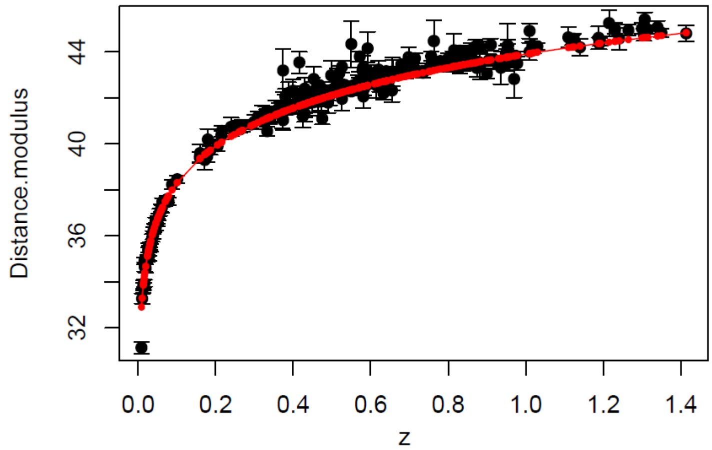

Figure 8. Redshift Vs Distance modulus.

CONCLUSIONS

Within the scope of this research, we have bound the resulting model with observational data in order to estimate the numerical values of the model parameters. The following are some of the most important findings from the current study:

- The estimated value of the deceleration parameter for the developed model is q0 = -0.30 ± 0.05, with a confidence level of 1σ.
- According to the developed best fit model, the distance modulus (μ(z)) and apparent distance (mb(z)) are well-suited to the observational data points obtained from astronomical observations (Figures 6 and 7 for more explanation).
- LRS BI anisotropic universe is the model that has been derived. This model tends to become isotropic as t → ∞, which means that the anisotropy is null for longer time periods.
- We discover that the value of $(\Omega_b)_0$ is 0.68 ± 0.06 at the 1σ level of 36 OHD points in the model that we have built. It asserts that the bulk viscosity play a key role in the cosmos that we are now seeing.
- Within the framework of our current model, the NEC, WEC, DEC, and SEC are all violated (Figure 4). It is important to point out that the SEC must be broken by the current fast expansion of the cosmos. This is something that deserves mention here. As a result, we are able to draw the conclusion that the model that we have obtained is consistent with the current fast expansion of the universe.

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