

Model to Predict a Split Range Control of Hot and Cold Water System

Ukpaka Chukwuemeka Peter^{1,*}, Orie Kenneth Eze², Dike Blessing³

Abstract

Mathematical model was developed to monitor, predict, and stimulate hot and cold water system using split range control application. Computer program language of MATLAB software of ODE function was applied to monitor the trend of temperature parameter in the system. Result obtained revealed decrease in temperature value of the hot water and increase in the temperature value of cold water until equilibrium was attained at 132°F at > 1.0 h. A constant in the degree of temperature of the cold water was obtained at 135°F as well as for hot water is 53°F. The research work demonstrates the application of split range control in monitoring, predicting and simulating the hot to cold water system.

Keywords: Model, predict, split range, control, hot, cold water system

INTRODUCTION

Temperature control model was considered in this research and the problems also considered in a split range control action of hot and cold water system. The temperature of both cold and hot water plays an active role in examining the effect of the input temperature as well as the output temperature. Two valves are controlled—one for cooling and one for heating, as in a reactor. The temperature controller controls the cold water valve or signal in the range 0–50%, and it controls the hot water valve for control signal in the range 50–100 [1].

Split range control refers to a situation where two final control elements are handled by a single controller. Usually, different fluid flows through each of two valves, for example, steam and cold water, acid and base [2]. Split range is usually misapplied to a common situation where a large valve and a small valve supply different amount of the same fluid. (The situation is better handled by other strategies such as “mid range or coarse - fine control”).

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Received Date: February 26, 2021

Accepted Date: April 14, 2021

Published Date: May 21, 2021

Citation: Ukpaka Chukwuemeka Peter, Orie Kenneth Eze, Dike Blessing. Model to Predict a Split Range Control of Hot and Cold Water System. Journal of Water Pollution & Purification Research. 2021; 8(1): 32–42p.

In these, one may have one signal control and more than one final control elements control valve (cv). In the case were one controller controls two CV's. Thus: Normal control signal ranges from 4–20 mA. One can set the zero of the first valve as 4 mA and 12 mA as span of the same, the second one zero may be set at 12 mA and span of 20 mA [3].

Split Range Control

In a split range control, the output of the controller is split and sent to two or more manipulated variables for controlling various output devices as well as when its output is in the range of 50–100%. Split range control is used when we need to control more than one parameter using a signal control. There are many process

control application in the industry where it is desirable to have multiple control valves respond to the output of a common controller [4]. Control valve configured to follow the command of the same controller are said to be split-range, or sequenced. Split-range control valves may take different forms of sequencing. Below are some sequencing seen in the industry: Complementary, exclusive and progressive.

Complementary Valve Sequencing

This type of control valve sequencing is made where two valve serve to proportion a mixture of two fluid streams is illustrated in Figure 1 shown.

Exclusive Valve Sequencing

This is a type sequencing were both valve are fully closed at a 50% controller output signal, with one valve opening fully as the controller output drives towards 100% and the other valve opening fully as the controller output goes to zero 0%. The nature of the valve sequencing is to an “either-or” throttled path or process fluid. This, either processed fluid flows through one valve or through the other both never through the both at the same time [5]. Figure 2 demonstrates the exclusive control value of proportion mixture of two fluid streams of 50:100% with flow diagram showcases, pH analyzer and controlling unit, temperature analyzer, acid input, caustic input etc.

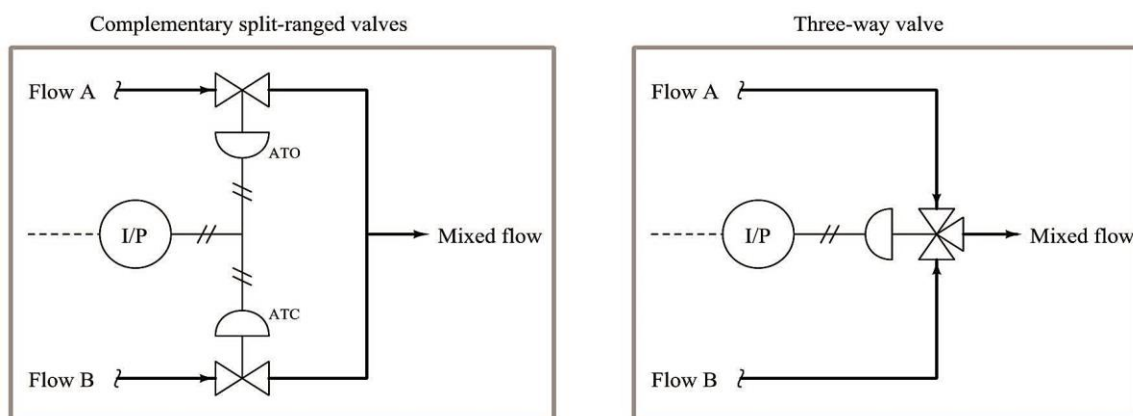


Figure 1. A Control Value of Proportion Mixture of Two Fluid Streams [5].

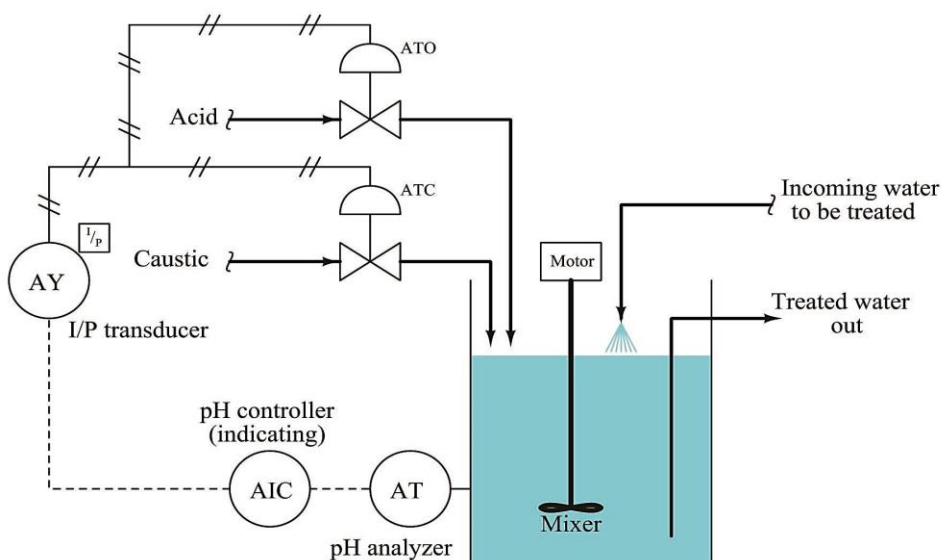


Figure 2. Exclusive Control Value of Proportion Mixture of Two Fluid Streams of 50:100% [6].

Sequencing of Progressing Valve

Mechanism which control flow in terms of fluid beyond the application of single control valves; example is the pH control with a range 1 to 6 acidic 7 neutral and above 7 to 14 is alkaline.

Split Rang Control Working Principle

For better understanding, split range control is illustrated by applying it to temperature control. In such an application, the process requires to be heated or cooled depending on the product temperature as well as the temperature transmitter, the controller, and two control valves are connected or split-range control in a typical temperature control application.

In the above diagram, the 0%–100% range of the controller output is split in two between the two valves. If the controller between 0%–50%, it is the cooling valve that operates. The valve is fully open when the output is 0% and fully closed when the output is 50%. If the controller output is between 50% and 100%, it is the heating valve that is in operation [6]. At 50%, the heating valve starts to open and is it fully open at 100% of the controller output. The first converter responds to the current from 4–12 mA, while the other operates at in a range from 12–20 mA.

Split Range Control Using Pneumatic Signal

In this case, the signal from controller is a pneumatic signal. In such a setup, the two valves are mechanically different [5]. The spring or diaphragm actuators of the two valves are selected so that their ranges of operation are different.

Using Split Range to Control Input and Output of a Reactor

In this case, chemical reactant enters a reactor via the first control valve that limits the input flow. Another control valve limits the output flow of the reactor. To have a high efficiency chemical reaction and a uniform product, the pressure inside the reactor must be kept above a given level. If the pressure is still low, the controller closes the output valve [4].

Gas flaring is common in refinery and chemical industry. These devices are mainly utilized to burn the excess of flammable gas in a vessel and to prevent an eventual explosion due to over-pressure. Many flare system depend on split range control for the evacuation of the gas [7–10].

In an application, a fluid or a gas is processed in a vessel. The chemical reaction or physical condition to which the fluid is exposed allows obtaining the desired product plus an (undesired) excess of flammable gas [11–12].

MATERIALS AND METHODS

The Model

The tube and shell exchanger is illustrated in Figure 3 as presented.

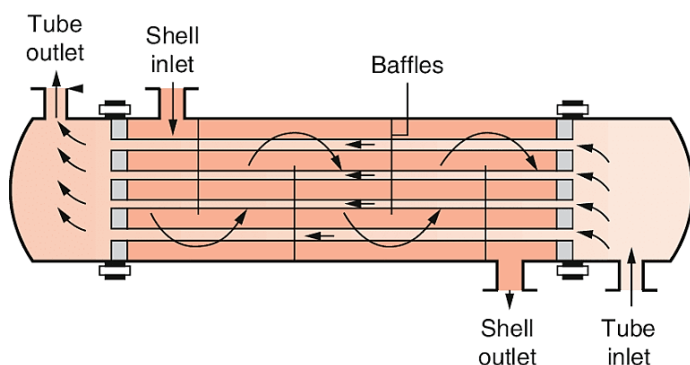


Figure 3. Tube and Shell Heat Exchanger [7].

There are two methods to determine the model to predict a split range control

1. Application of passivity concept
2. Use of proportional, PI, PID controller

The Steps Involve

- Develop the open loop model for a heat exchanger
- Determine the manipulated and disturbance variable
- Determine the stability of the model
- If not stable, add P-controller. Then develop the closed model using PI controller

Open Loop Model for A Heat Exchanger

The cold and hot fluid can either be in shell or tube side, it does not affect the model. We will consider two parts in this model (hot and cold side). In the hot side, the temperature of the hot stream will be the control variable and in the cold side, the temperature of the cold stream will be the control variable [7].

Hot Side Model

Rate of inflow of energy – rate of outflow of energy ± rate of generation of energy = rate of accumulation of energy

In the heat exchanger system, no generation nor consumption of energy because there is no reaction. Also in the heat exchanger system, the temperature in the system is not uniform but changes with length so we consider a boundary where the temperature.

$$\frac{V}{F} \times \frac{\partial^2 T_h}{\partial t^2 \partial l} + \frac{\partial T_h}{\partial l} + \frac{US(T_h - T_{cin})}{\rho_h F_h C_{ph}} = 0 \quad (1)$$

The above equation is the mathematical model to predict how far temperature changes with length and time partially.

When the temperature in the heat exchanger is uniform, the equation becomes:

$$\rho_h F_h C_{ph} (T_{hin} - T_{ref}) - (\rho_h F_h C_{ph} (T_{hout} - T_{ref}) + Q) = \frac{d(\rho_h V_h C_{ph} (T_h - T_{ref}))}{dt}$$

Let $T_{ref}=0$ and $Q = UA(T_h - T_{cin})$

$$\rho_h F_h C_{ph} T_{hin} - (\rho_h F_h C_{ph} T_h + UA(T_h - T_{cin})) = \rho_h V_h C_{ph} \frac{dT_h}{dt} \quad (2)$$

Making $\frac{dT_h}{dt}$ subject of formula in equation (2) we have

$$\frac{dT_h}{dt} = \frac{\rho_h F_h C_{ph} T_{hin}}{\rho_h V_h C_{ph}} - \frac{\rho_h F_h C_{ph} T_h}{\rho_h V_h C_{ph}} - \frac{UA(T_h - T_{cin})}{\rho_h V_h C_{ph}}$$

$$\frac{dT_h}{dt} = \frac{F_h T_{hin}}{V_h} - \frac{F_h T_h}{V_h} - \frac{UA(T_h - T_{cin})}{\rho_h V_h C_{ph}}$$

Collecting like terms

$$\frac{dT_h}{dt} = \frac{F_h}{V_h} (T_{hin} - T_h) - \frac{UA}{\rho_h V_h C_{ph}} (T_h - T_{cin}) \quad (3)$$

Let $\frac{F_h}{V_h} = \frac{1}{\tau}$ and $\frac{UA}{\rho_h V_h C_{ph}} = k$

$$\frac{dT_h}{dt} = \frac{1}{\tau} (T_{hin} - T_h) - k(T_h - T_{cin}) \quad (4)$$

Opening the bracket from equation (4) we have

$$\frac{dT_h}{dt} = \frac{1}{\tau} T_h - \frac{1}{\tau} T_h - kT_h + kT_{cin}$$

Re-arranging

$$\frac{dT_h}{dt} + \left(\frac{1}{\tau} + k\right)T_h = \frac{1}{\tau}T_{hin} + kT_{cin} \quad (5)$$

Let $a = \frac{1}{\tau} + k$, substituting a into equation (5)

$$\frac{dT_h}{dt} + aT_h = \frac{1}{\tau}T_{hin} + kT_{cin} \quad (6)$$

Where $T_h \rightarrow$ controlled variable

$T_{hin} \rightarrow$ Manipulated variable

$T_{cin} \rightarrow$ Disturbance variable

Equation (6) represent input-output model to show how the temperature of hot stream changes with time.

Cold Stream Model

$$\frac{dT_c}{dt} = \frac{F_c}{V_c}(T_{cin} - T_c) - \frac{UA}{\rho_c V_c C_{pc}}(T_{hin} - T_c) \quad (7)$$

Let $\frac{dT_c}{dt} = \frac{1}{q}(T_c - T_c) - m(T_{hin} - T_c)$

$$\frac{dT_c}{dt} = \frac{1}{q}T_{cin} - \frac{1}{q}T_c - mT_{hin} + mT_c$$

$$\frac{dT_c}{dt} + \left(\frac{1}{q} - m\right)T_c = \frac{1}{q}T_{cin} - mT_{hin}$$

Let $u = \frac{1}{q} - m$

$$\frac{dT_c}{dt} + uT_c = \frac{1}{q}T_{cin} - mT_{hin} \quad (8)$$

Where $T_c \rightarrow$ Controlled variable, $T_{cin} \rightarrow$ Manipulated variable and $T_{hin} \rightarrow$ Disturbance variable
 Equation (8) is the model that shows how the temperature of the cold stream changes with time.

$$\rho_h F_h C_{ph}(T_h - T_{ref}) - (\rho_h F_h C_{ph}(T_h + dT_h - T_{ref}) + dQ) = \frac{d\rho_h V_h C_{ph}(dT_h - T_{ref})}{dt}$$

Let $T_{ref} = 0$

$$\rho_h F_h C_{ph}T_h - \rho_h F_h C_{ph}T_h - \rho_h F_h C_{ph}dT_h - dQ = \frac{d\rho_h V_h C_{ph}dT_h}{dt}$$

But $dQ = dUA(T_h - T_{cin})A = \pi D dl$

$$dQ = U dA(T_h - T_{cin})dA = \pi D dl$$

$$dQ = U(T_h - T_{cin})dAD \rightarrow \text{Diameter of heat exchanger}$$

$$dQ = U(T_h - T_{cin})\pi D dl \pi D = S \rightarrow \text{perimeter of heat exchanger}$$

$$dQ = US(T_h - T_{cin})dl$$

$$-\rho_h F_h C_{ph}dT_h - US(T_h - T_{cin})dl = \frac{d\rho_h V_h C_{ph}dT_h}{dt}$$

Divide through by dl

$$-\rho_h F_h C_{ph} \frac{dT_h}{dl} - US(T_h - T_{cin}) = \rho_h V_h C_{ph} \frac{\partial^2 T}{\partial t^2 \partial l}$$

Divide through by $\rho_h F_h C_{ph}$

$$-\frac{dT_h}{dl} - \frac{US(T_h - T_{cin})}{\rho_h F_h C_{ph}} = \frac{\rho_h V_h C_{ph} \partial^2 T}{\rho_h F_h C_{ph} \partial t^2 \partial l}$$

$$-\frac{dT_h}{dl} - \frac{US(T_h - T_{cin})}{\rho_h F_h C_{ph}} = \frac{V_h}{F_h} \frac{\partial^2 T}{\partial t^2 \partial l}$$

From equation (6) we have

$$\frac{dT_h}{dt} + aT_h = \frac{1}{\tau} T_{hin} + kT_{cin}$$

At steady state

$$\frac{dT_{h(s)}}{dt} + aT_{h(s)} = \frac{1}{\tau} T_{hin(s)} + kT_{cin(s)}$$

Subtract 1(a) from 1

$$\frac{d(T_h - T_{h(s)})}{dt} + a(T_h - T_{h(s)}) = \frac{1}{\tau} (T_{hin} - T_{hin(s)}) + k(T_{cin} - T_{cin(s)})$$

Let $T_h - T_{h(s)} = T'_h$, $T_{hin} - T_{hin(s)} = T'_{hin}$, $T_{cin} - T_{cin(s)} = T'_{cin}$

$$\frac{dT'_h}{dt} + aT'_h = \frac{1}{\tau} T'_{hin} + kT'_{cin}$$

Laplace both side of the above equation

$$sT'_{hs} - T'_{h(0)} + aT'_{hs} = \frac{1}{\tau} T'_{hin(s)} + kT'_{cin(s)}$$

Let $T'_{h(0)} = 0$

$$sT'_{hs} + aT'_{hs} = \frac{1}{\tau} T'_{hin(s)} + kT'_{cin(s)}$$

$$(s + a)T'_{hs} = \frac{1}{\tau} T'_{hin(s)} + kT'_{cin(s)} \quad (9)$$

Recall from equation (8)

$$\frac{dT_c}{dt} + uT_c = \frac{1}{q} T_{cin} - mT_{hin}$$

Expressing in terms of derivation variable and Laplace transform

$$sT'_{cs} - T'_{c(0)} + T'_{c(s)} = \frac{1}{q} T'_{cin(s)} - mT'_{hin(s)}$$

$T'_{c(0)} = 0$

$$(s + u)T'_{cs} = \frac{1}{q} T'_{cin(s)} - T'_{hin(s)} \quad (10)$$

From equation (9)

$$T'_{hs} = \frac{1}{(s+a)\tau} T'_{hin(s)} + \frac{kT'_{cin(s)}}{(s+a)} \quad (11)$$

$$T'_{h(s)} = G_{p(s)1} T'_{hin(s)} - G_{d(s)1} T'_{cin(s)}$$

$$G_{p(s)1} = \frac{1}{(s+a)}, G_{d(s)1} = \frac{k}{s+a}$$

From equation (10)

$$T'_{c(s)} = \frac{1}{q(s+u)} T'_{cin(s)} - \frac{mT'_{hin(s)}}{s+u} \quad (12)$$

$$T'_{c(s)} = G_{p(s)2} T'_{cin(s)} + G_{d(s)2} T'_{hin(s)}$$

Determination of Stability of Open Loop Model

From equation (11)

$$T'_{h(s)} = \frac{1}{(s+a)\tau} T'_{hin(s)} + \frac{kT_{cin(s)}}{(s+a)}$$

$$(s+a) = 0, s = -a$$

$$s+a = 0, s = -a$$

Condition for Stability

A stable system must not have any real root. The value of 'a' will always be positive because $a = \frac{1}{\tau} + k$ which are all positive values and now make the root of the process is stable. Equation (11) is stable.

From equation (12), we have

$$T'_{c(s)} = \frac{1}{q(s+u)} T'_{cin(s)} - \frac{mT_{hin(s)}}{s+u}$$

The poles of the transfer function

$$s+u = 0$$

$s = -u$, but $u = \frac{1}{q} - m$ This will always give a negative answer and thereby making the poles positive. Therefore equation (12) is unstable.

The Closed Loop Model Adding Proportional Controller

$$y_s = \frac{k_c}{q(s+u)+k_c} y_{sp(s)} - \frac{mq}{q(s+u)+k_c} d(s)$$

The model will stable at some condition for k_c

$$q(s+u) + k_c < 0$$

$$q(s+u) < -k_c$$

$$s+u < \frac{-k_c}{q}$$

$$s < \frac{-k_c}{q} - v < \frac{-k_c - uq}{q}$$

$$s < \frac{-k_c - uq}{q}$$

For the system to stable $k_c < -uq$

Where $u = \frac{1}{q} - m$

$$u = \frac{F_c}{V_c} - \frac{UA}{\rho_c V_c C_c}$$

The general closed loop model is given by;

$$Y(s) = \frac{G_p(s)G_f(s)G_c(s)}{1+G_p(s)G_m(s)G_f(s)G_c(s)} Y_{sp}(s) + \frac{G_d(s)}{1+G_p(s)G_m(s)G_f(s)G_c(s)} d(s)$$

$$Y(s) = \frac{\frac{k_c}{q(s+u)}}{1+\frac{k_c}{q(s+u)}} Y_{sp}(s) + \frac{\frac{-m}{s+u}}{1+\frac{k_c}{q(s+u)}} d(s)$$

$$Y(s) = \frac{\frac{k_c}{q(s+u)}}{\frac{q(s+u)+k_c}{q(s+u)}} Y_{sp}(s) + \frac{\frac{-m}{s+u}}{1+\frac{k_c}{q(s+u)}} d(s)$$

$$Y(s) = \frac{k_c}{q(s+u)+k_c} Y_{sp}(s) - \frac{mq}{q(s+u)+k_c} d(s)$$

Closed Loop Model for Controlling Temperature of The Cold Stream Transfer Function

- Process: $G_p(s) = \frac{1}{q(s+u)}$
- Disturbance/ load: $G_d(s) = -\frac{m}{s+u}$
- Controller(p-controller): $G_c(s) = k_c$
- Measuring device and final control element can have transfer function to be equal to 1 since they have mutual or no effect on the system.
 $G_m(s) = G_f(s) = 1$

Definition of Terms

ρ_c and ρ_h → density of cold and hot fluid
 F_h and F_c → volumetric flow rate of hot and cold fluid
 C_{pc} and C_{ph} → specific heat capacity of hot and cold fluid
 T_c and T_h → outlet temperature of hot and cold fluid
 T_{cin} and T_{hin} → inlet temperature of the hot and cold fluid
 S → Perimeter of heat exchanger
 U → Overall heat transfer coefficient
 K_c → Proportional gain
 T_{ref} → Reference temperature

RESULTS AND DISCUSSION

The result obtained from the simulation is presented in Figure 4 as well as the Computer program language [11, 12].

SIMULATION

```
function [dTdt] = Group2( t,T)
dTdt=zeros(2,1)
cph=0.74
U=120
A=178.7
rho_h=750
Vh=78
Fh=700
k=(U*A)/(rho_h*Vh*cph)
r=Vh/Fh
a=(1/r)+k
```



```

Thin=140
cpc=1
U=120
A=178.7
rho_c=997
Vc=50
Tcin=50
Fc=680
q=(Vc/Fc)
m=(U*A)/(rho_c*Vc*cpc)
u=(1/q)-m
dTdt(1)=-a*T(1)+(1/r)*Thin+(k*Tcin)
dTdt(2)=-u*T(2)+(1/q)*Tcin-(m*Thin)

end

%Callup
clear
clc
h=0.2
tspan=0:h:12
T0=[70 160]
[t,T]=rk4sys2(@Group2,tspan,T0,h)
plot(t,T(:,1),'-*b')
holdon
plot(t,T(:,2),'->r')
xlabel('Time(hr)')
ylabel('Temperature (F)')
legend('Cold Side Temp','Hot side Temp')

```

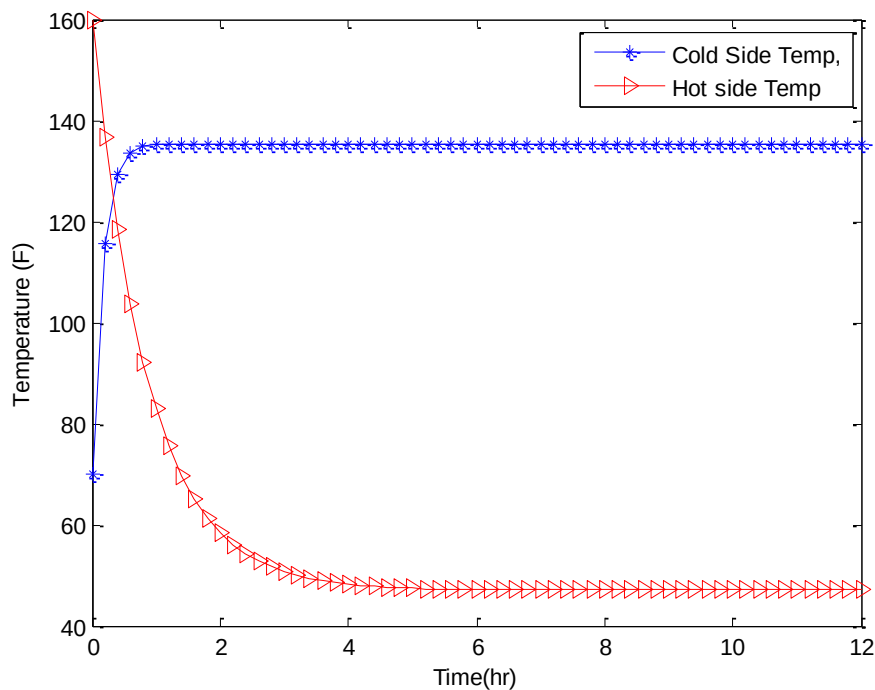


Figure 4. Simulated Plot of Temperature against Time.

Figure 4 demonstrate the characteristics of the hot and cold water interaction in a system. Increase in temperature was observed with increase in time for cold water as well as decrease in the temperature was observed with increased in hot water until an equilibrium was attained at 132°F. The variation in temperature can be attributed to the variation of the hot and cold water as well as the variation of time of contact.

ODE FUNCTION

```
function [tp,yp] = rk4sys2(dydt,tspan,y0,h)
% [t,y] = rk4(dydt,tspan,y0,h):
% input:
% tspan = [ti, tf] where ti and tf = initial and
% final values of independent variable
% y0 = initial value of dependent variable
% h = step size
% output:
% t = vector of independent variable
n=length(tspan)
ti = tspan(1);
tf = tspan(n);
if n==2
t = (ti:h:tf)';
n = length(t);
% so that range goes from t = ti to tf
if t(n)<tf
t(n+1) = tf;
n = n+1;
end
else
t=tspan
end
tt=ti;y(1,:)=y0;
np=1;tp(np)=tt;yp(np,:)=y(1,:);
i=1;
while (1)
tend=t(np+1);
hh=t(np+1)-t(np);
if hh>h,hh=h,end
while (1)
if tt+hh>tend,hh=tend-tt; end
k1=dydt(tt,y(i,:))
ymid=y(i,:)+k1*(hh./2)
k2=dydt(tt+hh/2,ymid)
ymid=y(i,:)+k2*hh/2;
k3=dydt(tt+hh/2,ymid)
yend=y(i,:)+k3*hh;
k4=dydt(tt+h/2,yend)
phi=(k1+2*(k2+k3)+k4)/6;
y(i+1,:)=y(i,:)+phi*hh;
tt=tt+hh;
i=i+1;
if tt>=tend, break,end
end
```

```
np=np+1;tp(np)=tt;yp(np,:)=y(i,:);  
if tt>=tf,break,end  
end
```

CONCLUSION

In conclusion, we have successfully developed the mathematical model to be used in predicting the split range control of both HOT AND COLD water systems, with the help of the heat exchanger which is termed as one of the most important instrument needed in achieving this. In the course of the research for this project, dynamic models were out-rightly developed for both manipulations of values and control which was then added to make the heat exchanger network a more passive system.

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