

Continuous-Time Modelling of the Two-Input Fourth Order DC_DC Converter

B. Amarendra Reddy^{1*}, P. Vamsi Krishna²

Abstract

Two-input fourth-order integrated (TIFOI) converter is designed to process the power from two different sources V_{g1} and V_{g2} . This converter facilitates regulation of dc-bus voltage and LVS current. In the power conversion process it performs bucking operation for first dc source V_{g1} , buck-boost operation for the second source V_{g2} . Power stage dynamics of the TIFOI converters is modeled using the state-space analysis. The system modelling plays an important role in the design and implementation of the integrated converter control systems. The TFM of an integrated converter can be obtained mainly in two ways as these are widely used and accepted methodologies. These are (i) small signal continuous-time model and (ii) small-signal discrete-time models. These modelling methodologies were reported in literature for single-input dc-dc converters, but their application in the two-input dc-dc converters has not been exhaustively addressed yet. The transfer function matrix (TFM) of the TIFOI converter are obtained using small-signal continuous-time model and the detailed mathematical aspects are presented in this paper. Using this TFM of the TIFOI dc-dc converter is obtained.

Keyword: Photo voltaic, State-space averaged models, fixed-frequency PWM control signal, duty ratio control signals, State Space Matrix, small signal model, equilibrium point, converter dynamics.

INTRODUCTION TO CONVERTER MODELLING

The switched mode dc-dc converters are designed using either with a single source or with multiple sources. Multiple sources are interfaced in dc-dc converters to allow both flexibility and reliability. A single integrated converter that accommodates different types of sources forms a multi-input/multi-port converter (MICs/MPCs) [2]. Such a MIC can continue to operate even if one of the dc sources has failed. Different sources that can be used in MICs are wind turbines, fuel cells, photo voltaic (PV) panels, batteries, super capacitor/ultra-capacitor/electric double layer capacitor. The advantages of MICs include (i) input sources can deliver power to load either individually or simultaneously, (ii) possibility of having lesser number of components, (iii) better steady-state performance, (iv) different types of sources with different magnitudes can be integrated.

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Steady-state analysis of the two-input integrated converters gives deep understanding of multi-input converter design while the dynamic analysis gives the interaction effects, pairing of the input-output variables, controller structure selection and controller design aspects. In case of power processing using two dc sources, a two-input integrated converter is a viable option for power control as it uses less number of components and is simple in structure. A two-input integrated converter for the optimal power distribution was proposed in [2], where the modelling aspects were

explored in detail. In this work, two-input fourth-order integrated (TIFOI) dc-dc converters is designed to process the power from two different sources V_{g1} and V_{g2} . These two converters facilitate regulation of dc-bus voltage and LVS current. The system modelling plays an important role in the design and implementation of the integrated converter control systems. In the following paragraph the literature relevant to converter system modelling are discussed.

Mathematical modelling of a converter is obtained in several ways for example state-space averaged models [1], sampled-data model [3], PWM switch model [4], and dynamic phasors model [19]. Though many modelling methods were reported in literature to define dynamics of the dc-dc integrated converters, but the state-space averaging model [1], [2], [5], [6] is popularly used. PWM switch model using averaging method is reported in [4]. Low frequency sampled data models of switched mode converters are discussed in [3]. State-space modelling aspects of multi-input converters have been discussed in ref. [8], [9]. Overview of modelling aspects applicable to dc-dc converter systems is reported in [5]. An exact small-signal discrete-time model for multiphase converter is proposed in [11]. Exact small-signal discrete-time analysis have been used to formulate the model of digitally controlled boost converters [7], two-input buck-SEPIC converter [12], two-input integrated converters [13–18], and for multi-state dc-dc converter [10].

SIGNIFICANCE OF MODELING

Modeling is an essential and most important aspect for such converter systems. It helps the designer in many ways: (1) To understand the operating principles of a converter in various modes. (2) For analysing the dynamics of integrated dc-dc converter. (3) Thorough steady-state and transient analysis. (4) Useful for design of converter control system. If the exact model is not known, the controller designed may not be effective in regulating the integrated converter variables and the resultant system may exhibit poor dynamic performance characteristics.

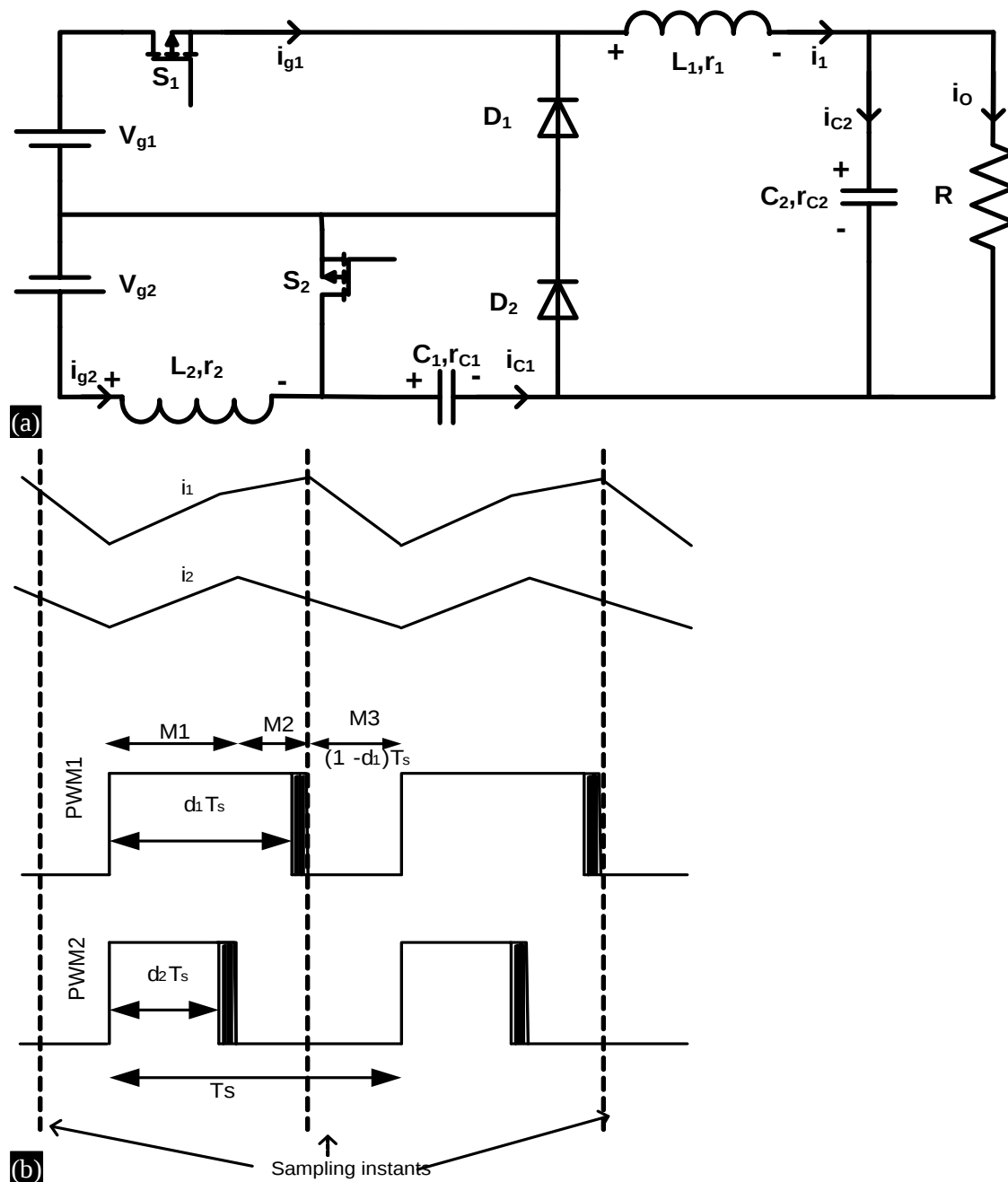
For the case of TII converter there are two input variables and two output variables and each output quantity is a function of both of the controlling inputs. Hence, this dependency is mathematically represented by a set of transfer functions and its compact form is transfer function matrix (TFM), wherein the transfer functions are the elements. The TFM of the TII dc-dc converters is useful (i) to analyse control-loop interactions, (ii) pairing of the input-output variables, (iii) controller structure selection, (iv) controller design and (v) to analyse stability and robustness (vi) steady-state characteristics of the system.

The TFM of an integrated converter can be obtained mainly in two ways as these are widely used and accepted methodologies. These are (i) small signal continuous-time model and (ii) small-signal discrete-time models. These modelling methodologies were reported in literature for single-input dc-dc converters, but their application in the two-input dc-dc converters has not been exhaustively addressed yet. Thus, in this work an attempt has been made to extend the studies of the single-input dc-dc converters into two-input dc-dc converter system. The dynamics of the converter in each mode of operation is described using state-space model and the respective state-space matrices are used in small-signal transfer functions (continuous-time) formulation.

TIFOI DC-DC CONVERTER

The proposed two-input two-output fourth-order integrated (TIFOI) dc-dc converter [14–18], shown in Figure 1(a), is suitable for power processing from two sources V_{g1} , which is a high voltage source (HVS) and V_{g2} , which is a low voltage source (LVS). It has two diodes, four energy storage elements, and two switching devices, hence it forms a fourth-order system. During this power conversion process it performs bucking operation for HVS, V_{g1} , buck-boost operation for the LVS, V_{g2} . The converter is having the following salient features (a) Both sources V_{g1} and V_{g2} are supplying power to down-stream loads by drawing continuous currents from their respective sources and hence the corresponding source ripple currents are low. (b) automatic load transfer on to the first dc source if the second dc source capacity, weak source with limited power supplying capacity, is less than the load demand, and (c) simple control strategy, with or without overlap of duty ratio signals, as there are only two switching devices need to be controlled.

The two switches of this converter turns-ON and turns-OFF using the fixed-frequency PWM control signals d_1 , d_2 , and the constant switching frequency is $1/T_s$. Here, the integrated circuit is analysed by considering $d_1 > d_2$, using the trailing-edge synchronized switching signals. For $d_1 > d_2$, case the switching sequence in one cycle is: (i) mode-1: S_1 and S_2 both ON, D_1 and D_2 both are in off-state (ii) mode-2: S_1 -ON and D_2 -ON, S_2 -OFF and D_1 -OFF (iii) mode-3: S_1 -OFF and D_2 -ON, S_2 -OFF and D_1 -ON. The conduction time periods in each mode of operation is obtained using PWM gating signals diagram as shown in Figure 1(b). This figure is also gives the information regarding the nature of the inductor currents and using this charging and discharging periods of the magnetic field can be obtained. Based on the conduction periods of switches S_1 , S_2 the effective operating times of the integrated circuit during mode-1, mode-2, and mode-3 are obtained as $(0 < t < d_2 T_s)$, $(d_2 T_s < t < d_1 T_s)$, and $(d_1 T_s < t < T_s)$ respectively. The operating circuit diagrams of the integrated converter for $d_1 > d_2$ within one switching cycle are shown in Figure 1(c)-Figure 1(e).



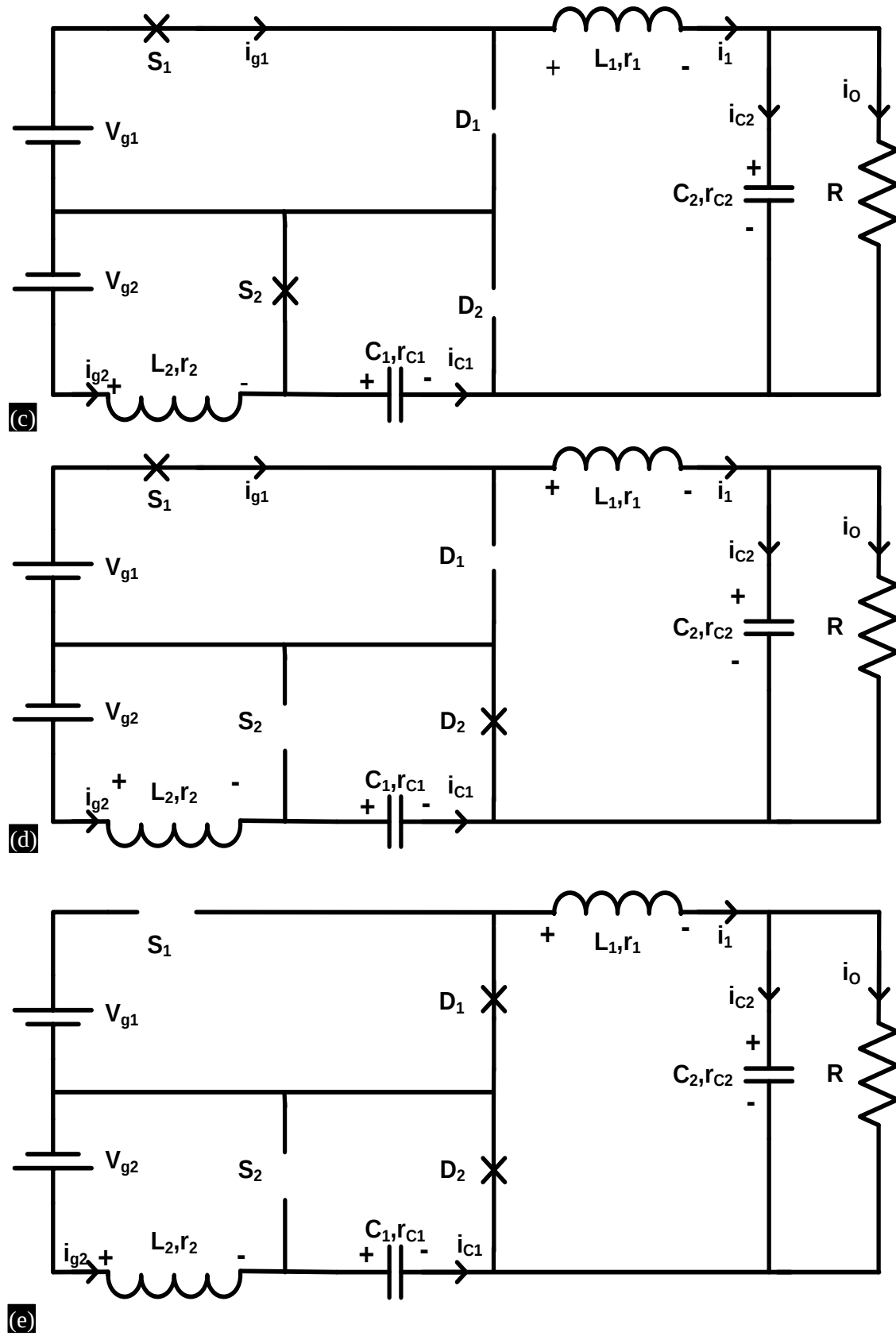


Figure 1. (a) Circuit Diagram of Fourth Order Converter, (b) PWM gating signals, (c) Mode-1 Circuit Diagram, (d) Mode-2 Circuit Diagram, (e). Mode-3 Circuit Diagram.

The dynamics of power conversion process in mode-1 of operation is described using the equations 1(a), 1(b) to 1(d).

$$\dot{x} = A_1 x + B_1 u \quad 1(a)$$

$$\hat{y} = E_{01} \hat{x} \quad 1(b)$$

$$\hat{y} = \begin{bmatrix} \hat{v}_0 & \hat{i}_{g2} \end{bmatrix}^T \quad 1(c)$$

$$\begin{bmatrix} \hat{v}_0 \\ \hat{i}_{g2} \end{bmatrix} = \begin{bmatrix} E_1 \\ P_1 \end{bmatrix} \hat{x} = E_{01} \hat{x} \quad 1(d)$$

$$\text{Where } A_1 = \begin{bmatrix} -\frac{k_1}{L_1} & 0 & \frac{1}{L_1} & -\frac{b}{L_1} \\ 0 & -\frac{r_2}{L_2} & 0 & 0 \\ -\frac{1}{C_1} & 0 & 0 & 0 \\ \frac{b}{C_2} & 0 & 0 & -\frac{b}{RC_2} \end{bmatrix}, B_1 = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} a & 0 & 0 & b \end{bmatrix} \quad F_1 = \begin{bmatrix} 0 \end{bmatrix} \quad P_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

The dynamics of power conversion process is in mode-2 of operation are described using equations 2(a) to 2(c).

$$\dot{x} = A_2 x + B_2 u \quad 2(a)$$

$$\hat{y} = E_{02} \hat{x} \quad 2(b)$$

$$\begin{bmatrix} \hat{v}_0 \\ \hat{i}_{g2} \end{bmatrix} = \begin{bmatrix} E_2 \\ P_2 \end{bmatrix} \hat{x} = E_{01} \hat{x} \quad 2(c)$$

$$A_2 = \begin{bmatrix} -\frac{(r_1 + a)}{L_1} & 0 & 0 & -\frac{b}{L_1} \\ 0 & -\frac{(r_2 + r_{C_1})}{L_2} & -\frac{1}{L_2} & 0 \\ 0 & \frac{1}{C_1} & 0 & 0 \\ \frac{b}{C_2} & 0 & 0 & -\frac{b}{RC_2} \end{bmatrix}, B_2 = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} a & 0 & 0 & b \end{bmatrix} \quad F_2 = \begin{bmatrix} 0 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

The dynamics of power conversion process is in mode-3 of operation described using equations 3(a) to 3(c).

$$\dot{x} = A_2 x + B_2 u \quad 3(a)$$

$$y = E_{03} x \quad 3(b)$$

$$\begin{bmatrix} v_0 \\ i_{g2} \end{bmatrix} = \begin{bmatrix} E_3 \\ P_3 \end{bmatrix} x = E_{03}x \quad (3c)$$

$$A_3 = \begin{bmatrix} -\frac{(r_1 + a)}{L_1} & 0 & 0 & -\frac{b}{L_1} \\ 0 & -\frac{(r_2 + r_{C_1})}{L_2} & -\frac{1}{L_2} & 0 \\ 0 & \frac{1}{C_1} & 0 & 0 \\ \frac{b}{C_2} & 0 & 0 & -\frac{b}{RC_2} \end{bmatrix} \quad B_3 = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad E_3 = [a \quad 0 \quad 0 \quad b]$$

$$F_3 = [0] \quad P_3 = [0 \quad 1 \quad 0 \quad 0]$$

Voltage conversion ratio of TIFOI dc-dc converter

The load voltage dependency on the source voltages V_{g1} and V_{g2} of the integrated converter is described using the voltage conversion ratio or voltage gain. It is obtained by using the steady-state time-domain analysis. The voltage gain of this system can be derived by applying volt-second balance principle for the inductors L_1 and L_2 . The principle of inductor volt-second balance states that average value of the periodic inductor voltage waveforms is zero, when the converter operates in steady-state. It is observed from equation (4) that the voltage gain expression of the TIFOI dc-dc converter is a function of V_{g1} , V_{g2} and d_1 , and d_2 . Therefore, during this power conversion process it performs bucking operation for HVS, V_{g1} , buck-boost operation for the LVS, V_{g2} .

$$V_o = V_{g1}d_1 + \frac{V_{g2}d_2}{1-d_2} \quad (4)$$

Design Equations of the TIFOI dc-dc converter

The design equations for the L_1 , L_2 , C_1 , and C_2 of the TIFOI dc-dc converter can be obtained using time-domain analysis. Design equations for the converter circuit elements are given in eqn.(5)

$$L_1 = \frac{V_o(1-d_1)}{\Delta i_{L1} f_s} \quad , \quad L_2 = \frac{V_{g2}d_2}{\Delta i_{L2} f_s} \quad (5)$$

$$C_1 = \frac{I_2(1-d_2)}{f_s \Delta v_{C1}} \quad , \quad C_2 = \frac{(1-d_1)^2 V_o}{4 \Delta V_{C2} L_1 f_s^2}$$

TIFOI dc-dc Converter Parameters

To extract the salient features of the TIFOI dc-dc converter and to verify load distribution together with dc-bus regulation a 24V, 96 W converter system is considered and parameters considered for 96 W, are tabulated in Table 1.

Table 1. TIFOI dc-dc converter.

Parameter	Value
V_{g1}	60 V
V_{g2}	24 V
V_0	45 V
R	10 Ω +/- 50%
P_o	200 W
L_1	407 μ H
L_2	620 μ H
C_1	9.813 μ F
C_2	45.04 μ F
f_s	50 KHz

DERIVATION OF TFM USING SMALL-SIGNAL CONTINUOUS-TIME MODEL

The TFM of the integrated converter is formulated using a small-signal continuous-time model, which is useful to identify control loop interactions. The small-signal continuous-time model describes the local behaviour of the integrated converter about the steady-state operating point. At this operating point, it is formulated using the procedure steps given below by considering the following assumptions: (a) The TII converter considered here operates in three different modes of operation for $d_1 > d_2$, where d_1, d_2 are fixed-frequency PWM control signals / duty ratio control signals. (b) The two switches present in this converter are regulated using two independent duty ratio control signals. (c) During each mode of operation of the switching cycle, the source voltages are assumed to be constant. (d) In each mode of the operation, the structure of integrated converter is different therefore, respective state-space matrices are different. In each mode of operation, the power stage dynamics can easily be described by a set of state equations given by $\dot{x} = A_k x + B_k u$, and the output equation given by $y = E_{0k} x + F_k u$ where

$$y = \begin{bmatrix} v_0 \\ i_{g2} \end{bmatrix}, E_{0k} = \begin{bmatrix} E_k \\ P_k \end{bmatrix}, F_k = \begin{bmatrix} F_{vk} \\ F_{pk} \end{bmatrix}$$

where $k=1, 2, 3$ for mode-1, mode-2 and mode- 3 respectively.

The state-space averaging method is the most acceptable and commonly used method to model this integrated converter by considering the three different modes of operation. Its formulation is based on the average conduction periods of the switches (S_1 & S_2) and these are denoted using d_1 and d_2 . The state-space averaging model of the integrated converter is used to derive the small-signal TFM model. The state-space averaged model at equilibrium point is given in equation 6(a). At equilibrium point the rate of change of state variables are equals to zero i.e., $\frac{dx}{dt} = 0$.

$$\begin{aligned} 0 &= AX + BU \\ Y &= E_0 X + FU \end{aligned} \tag{6(a)}$$

The averaged matrices A , B , E_0 and F of the integrated converter for $d_1 > d_2$ are defined using eqn. 6(b).

$$\begin{aligned} A &= d_2 A_1 + (d_1 - d_2) A_2 + (1 - d_1) A_3 \\ B &= B_1 d_2 + B_2 (d_1 - d_2) + B_3 (1 - d_1) \\ E_0 &= E_{01} d_2 + E_{02} (d_1 - d_2) + E_{03} (1 - d_1) \\ F &= F_1 d_2 + F_2 (d_1 - d_2) + F_3 (1 - d_1) \end{aligned} \tag{6(b)}$$

where A =system matrix, B =input matrix, E_0 =output matrix, F = transmission matrix

The following steps in sequence are needed to formulate the small signal TFM model.

(i) Consider the state equation, output equation and substitute the averaged values of the system matrix, input matrix, output matrix and transmission matrix and these are described using the eqn.7(a) and eqn.7(b).

$$\frac{dx}{dt} = (d_2 A_1 + (d_1 - d_2) A_2 + (1 - d_1) A_3) x(t) + (B_1 d_2 + B_2 (d_1 - d_2) + B_3 (1 - d_1)) u(t) \tag{7(a)}$$

$$Y = (d_2 E_{01} + (d_1 - d_2) E_{02} + (1 - d_1) E_{03}) x(t) + (F_1 d_2 + F_2 (d_1 - d_2) + F_3 (1 - d_1)) u(t) \tag{7(b)}$$

(ii) To construct a small-signal model at a steady-state point, assume that the output variables, state variables and the input variables (duty cycles) of TII converter model are equal to steady-state values plus superimposed small ac variations in respective signals as defined in equation 8.

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{X} + \hat{\mathbf{x}} \quad , \quad \mathbf{u}(t) = \mathbf{U} + \hat{\mathbf{u}} \quad , \quad \mathbf{y}(t) = \mathbf{Y} + \hat{\mathbf{y}} \\ d_1 &= D_1 + \hat{d}_1 \quad , \quad d_2 = D_2 + \hat{d}_2 \quad , \quad 1 - d_1 = d_1' = D_1' - \hat{d}_1 \end{aligned} \quad (8)$$

$\mathbf{X}$ =Steady-state state vector $\mathbf{U}$ = Steady-state input vector

$\mathbf{Y}$ = Steady-state output vector D_1, D_2 = Steady-state duty cycles

$\hat{\mathbf{x}}$ =small ac variations in state vector $\hat{\mathbf{u}}$ = small ac variations in input vector

$\hat{\mathbf{y}}$ =small ac variations in output vector $\hat{d}_1, \hat{d}_2$ =small ac variations in duty cycles

(iii) Substitute the steady-state plus small ac variations of each variable in the state and output equations. These are given in eqn.9(a) and eqn.9(b).

$$\begin{aligned} \frac{d(\mathbf{X} + \hat{\mathbf{x}})}{dt} &= (D_2 + \hat{d}_2)A_1(\mathbf{X} + \hat{\mathbf{x}}) + (D_1 + \hat{d}_1)A_2(\mathbf{X} + \hat{\mathbf{x}}) \\ &\quad - (D_2 + \hat{d}_2)A_2(\mathbf{X} + \hat{\mathbf{x}}) + (D_1' - \hat{d}_1)A_3(\mathbf{X} + \hat{\mathbf{x}}) \\ &\quad + (D_2 + \hat{d}_2)B_1(\mathbf{U} + \hat{\mathbf{u}}) + (D_1 + \hat{d}_1)B_2(\mathbf{U} + \hat{\mathbf{u}}) \\ &\quad - (D_2 + \hat{d}_2)B_2(\mathbf{U} + \hat{\mathbf{u}}) + (D_1' - \hat{d}_1)B_3(\mathbf{U} + \hat{\mathbf{u}}) \end{aligned} \quad 9(a)$$

$$\begin{aligned} (\mathbf{Y} + \hat{\mathbf{y}}) &= (D_2 + \hat{d}_2)E_{01}(\mathbf{X} + \hat{\mathbf{x}}) + (D_1 + \hat{d}_1)E_{02}(\mathbf{X} + \hat{\mathbf{x}}) \\ &\quad - (D_2 + \hat{d}_2)E_{02}(\mathbf{X} + \hat{\mathbf{x}}) + (D_1' - \hat{d}_1)E_{03}(\mathbf{X} + \hat{\mathbf{x}}) \\ &\quad + (D_2 + \hat{d}_2)F_1(\mathbf{U} + \hat{\mathbf{u}}) + (D_1 + \hat{d}_1)F_2(\mathbf{U} + \hat{\mathbf{u}}) \\ &\quad - (D_2 + \hat{d}_2)F_2(\mathbf{U} + \hat{\mathbf{u}}) + (D_1' - \hat{d}_1)F_3(\mathbf{U} + \hat{\mathbf{u}}) \end{aligned} \quad 9(b)$$

(iv) Separating the terms of steady-state values, the linear terms of small ac variations, and higher order non-linear (H.O.NL) terms in state and output equations. It is described using eqn.10(a) and eqn.10(b).

$$\begin{aligned} 0 + \frac{d\hat{\mathbf{x}}}{dt} &= \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{U} + \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\hat{\mathbf{u}} + ((A_2 - A_3)\mathbf{X} + (B_2 - B_3)\mathbf{U})\hat{d}_1 \\ &\quad + ((A_1 - A_2)\mathbf{X} + (B_1 - B_2)\mathbf{U})\hat{d}_2 + \text{H.O.NL.Terms} \end{aligned} \quad 10(a)$$

$$\text{H.O.NL.Terms} = (A_1 - A_2)\hat{d}_2\hat{\mathbf{x}} + (A_2 - A_3)\hat{d}_1\hat{\mathbf{x}} + (B_1 - B_2)\hat{\mathbf{u}}\hat{d}_2 + (B_2 - B_3)\hat{\mathbf{u}}\hat{d}_1$$

$$\begin{aligned} \mathbf{Y} + \hat{\mathbf{y}} &= \mathbf{E}_0\mathbf{X} + \mathbf{F}\mathbf{U} + \mathbf{E}_0\hat{\mathbf{x}} + \mathbf{F}\hat{\mathbf{u}} + ((E_{02} - E_{03})\mathbf{X} + (F_2 - F_3)\mathbf{U})\hat{d}_1 \\ &\quad + ((E_{01} - E_{02})\mathbf{X} + (F_1 - F_2)\mathbf{U})\hat{d}_2 + \text{H.O.NL.Terms} \end{aligned} \quad 10(b)$$

$$\text{H.O.NL.Terms} = (E_{01} - E_{02})\hat{\mathbf{x}}\hat{d}_2 + (E_{02} - E_{03})\hat{\mathbf{x}}\hat{d}_1 + (F_1 - F_2)\hat{\mathbf{u}}\hat{d}_2 + (F_2 - F_3)\hat{\mathbf{u}}\hat{d}_1$$

(v) The small signal model is obtained by considering only linear terms of small variations by dropping the nonlinear product terms. These are given in eqn.11(a) and 11(b).

$$\frac{d\hat{\mathbf{x}}}{dt} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\hat{\mathbf{u}} + ((A_2 - A_3)\mathbf{X} + (B_2 - B_3)\mathbf{U})\hat{d}_1 + ((A_1 - A_2)\mathbf{X} + (B_1 - B_2)\mathbf{U})\hat{d}_2 \quad 11(a)$$

$$\hat{y} = E_0 \hat{x} + F \hat{u} + ((E_{02} - E_{03})X + (F_2 - F_3)U)\hat{d}_1 + ((E_{01} - E_{02})X + (F_1 - F_2)U)\hat{d}_2 \quad 11(b)$$

(vi) For the case of TII converter, switch duty control signals, d_1 and d_2 , are the input variables, v_0 and i_{g2} are the two output variables. The output vector is defined using $E_{0k} = [E \ P]^T$, where E , P are row vectors corresponding to V_0 and i_{g2} current.

$$\text{where } E_0 = E_{01}d_2 + E_{02}(d_1 - d_2) + E_{03}(1 - d_1), \quad E_0 = \begin{bmatrix} E \\ P \end{bmatrix}$$

$$E = E_1d_2 + E_2(d_1 - d_2) + E_3(1 - d_1), \quad P = P_1d_2 + P_2(d_1 - d_2) + P_3(1 - d_1)$$

The load voltage ' $\hat{v}_0(s)$ ', depends on both of the controlling duty ratios (d_1, d_2). The transfer functions corresponding to V_0 are obtained by considering d_1 and d_2 independently, these are given using eqn.12(a) and 12(b).

$$\text{Define } \gamma_1 = [(A_2 - A_3)X + (B_2 - B_3)U], \quad \Psi_1 = [(E_2 - E_3)X + (F_{2v} - F_{3v})U]$$

$$\text{and } \gamma_2 = [(A_1 - A_2)X + (B_1 - B_2)U], \quad \Psi_2 = [(E_1 - E_2)X + (F_{1v} - F_{2v})U]$$

$$\frac{v_0(s)}{d_1(s)} = E * (sI - A)^{-1} * \gamma_1 + \Psi_1 \quad 12(a)$$

$$\frac{v_0(s)}{d_2(s)} = E * (sI - A)^{-1} * \gamma_2 + \Psi_2 \quad 12(b)$$

The LVS current ' $\hat{i}_{g2}(s)$ ', depends on both of the controlling duty ratios (d_1, d_2). Hence, the transfer functions corresponding to this output due to each input are given in eqn.13(a) and eqn.13(b).

$$\text{Define } \Psi_3 = [(P_2 - P_3)X + (F_{2p} - F_{3p})U], \quad \Psi_4 = [(P_1 - P_2)X + (F_{1p} - F_{2p})U]$$

$$\frac{i_{g2}(s)}{d_1(s)} = P * (sI - A)^{-1} * \gamma_1 + \Psi_3 \quad 13(a)$$

$$\frac{i_{g2}(s)}{d_2(s)} = P * (sI - A)^{-1} * \gamma_2 + \Psi_4 \quad 13(b)$$

The above equations, eqn..12(a),.12(b),13(a) and 13(b) are combined and expressed in compact form as given below. These equations describe the interdependence of input, and output variables of this converter system, in the small-signal sense. This dependence is represented mathematically in the continuous-time domain using eqn.14. And these equations describe the converter dynamics and collectively represent the TFM.

$$\begin{bmatrix} \hat{v}_0(s) \\ \hat{i}_g(s) \end{bmatrix} = \begin{bmatrix} E * [sI - A]^{-1} * \gamma_1 + \Psi_1 & E * [sI - A]^{-1} * \gamma_2 + \Psi_2 \\ P * [sI - A]^{-1} * \gamma_1 + \Psi_3 & P * [sI - A]^{-1} * \gamma_2 + \Psi_4 \end{bmatrix} \begin{bmatrix} \hat{d}_1(s) \\ \hat{d}_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \hat{d}_1(s) \\ \hat{d}_2(s) \end{bmatrix} \quad (14)$$

In the above equation (14) is the TFM and defines integrated converter dynamics. It is a function of plant parameters, controlling quantities, source voltages, and load resistance. In the TIFOI converter, any change either in load or source voltage (i.e., $R \in [R_{\min}, R_{\max}]$, $V_{g1} \in [V_{g1\min}, V_{g1\max}]$, $V_{g2} \in [V_{g2\min}, V_{g2\max}]$)

alters various current/voltages within the converter. To understand the system performance (both steady-state and dynamic conditions) against these variations, ϕ, γ_1, γ_2 and TFM should be computed.

Computation of TFM for TIFOI converter

Time-domain analysis is carried out for obtaining the design equations of the various parameters of the TIFOI dc-dc converter system. These are utilised to obtain the design values of the parameters. These are: $L_1=407 \mu\text{H}$; $L_2=620 \mu\text{H}$; $C_1=9.813 \mu\text{F}$; $C_2=45.04 \mu\text{F}$; (specifications: $R=10 \text{ ohms}$; $V_{g1}=60 \text{ V}$; $V_{g2}=24 \text{ V}$; $V_0=45 \text{ V}$; $f_s=50,000 \text{ Hz}$; $T_s=1/f_s$). Using these parameters, TFM of the TIFOI dc-dc converter system ('G') is obtained in the MATLAB environment and It is given in eqn.(15).

$$\begin{aligned} G_{11}(s) &= \frac{6.98 \times 10^4 s^3 + 3.136 \times 10^9 s^2 + 6.409 \times 10^{12} s + 2.511 \times 10^{17}}{s^4 + 3802 s^3 + 1.565 \times 10^8 s^2 + 3.525 \times 10^{11} s + 4.244 \times 10^{15}} \\ G_{12}(s) &= \frac{3.935 \times 10^4 s^3 + 1.536 \times 10^9 s^2 - 5.447 \times 10^{12} s + 1.997 \times 10^{17}}{s^4 + 3802 s^3 + 1.565 \times 10^8 s^2 + 3.525 \times 10^{11} s + 4.244 \times 10^{15}} \\ G_{21}(s) &= \frac{5.088 \times 10^{12} s + 1.076 \times 10^{16}}{s^4 + 3802 s^3 + 1.565 \times 10^8 s^2 + 3.525 \times 10^{11} s + 4.244 \times 10^{15}} \\ G_{22}(s) &= \frac{5.461 \times 10^4 s^3 + 9.442 \times 10^8 s^2 + 9.632 \times 10^{12} s + 4.809 \times 10^{16}}{s^4 + 3802 s^3 + 1.565 \times 10^8 s^2 + 3.525 \times 10^{11} s + 4.244 \times 10^{15}} \end{aligned} \quad (15)$$

CONCLUSIONS

TIFOI dc-dc converter system is designed to process power from two different sources V_{g1} and V_{g2} . These two converters facilitate the regulation of dc-bus voltage and LVS current. The dynamic behaviour of these converters are described using state-space models. These state-space models are used for deriving the TFM. These TFMs are derived using small signal continuous-time model. These transfer functions are used to analyze the steady-state and dynamic behavior of the system and these are also used for designing the controllers. The mathematical procedures to obtain the TFM for these integrated converter systems using this methodology is discussed.

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